MILNE'S PROGRESSIVE ARITHMETICS

SECOND BOOK

PROGRESSIVE ARITHMETIC

THIRD BOOK

BY

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PREFACE

This book, which is intended for use in the highest grades in our grammar schools, has been constructed upon a broader and more comprehensive plan than the preceding books in the series. The desirability of gathering together in a somewhat complete whole the knowledge that students have previously acquired, and of including the essentials of arithmetic in a single volume, has led to the adoption of the purely topical method of presentation.

Great care has been expended on the method of presenting the various principles and on the solutions, with the aim of making them in the highest degree clear, concise, accurate, and practical. By treating together in some instances subjects which though logically related are usually separated, it has been possible to simplify considerably the methods of solution generally employed.

Analysis of problems has received adequate attention, and algebraic methods of solution have been introduced for the purpose of simplifying the processes in certain classes of problems.

The problems have been framed with the greatest care. They are not numerical puzzles, nor are they based on unreal conditions. They have been made both rational and practical,

and they relate to a wide range of subjects drawn from modern life and industries. The information they embody has been gathered from reliable sources. The prime object of the book, however, is not to convey information on extraneous topics, but to teach the processes of arithmetic in the best way possible.

WILLIAM J. MILNE.

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PROGRESSIVE ARITHMETIC

THIRD BOOK

NOTATION AND NUMERATION

1. Any one thing is called a **unit**.

One, a book, a dollar, a gallon, are units.

2. A unit or a collection of units is called a **number**.

Numbers are represented by characters, as *figures* or *letters*, and are expressed by *words*.

3. Any method of representing numbers by characters is called **notation**.

The notation by *figures*, in common use among civilized nations, is the **Arabic**, named from the Arabs who first introduced it into Europe. It is sometimes called the *Indian*, or *Hindu*, *notation* because it originated in India.

For certain purposes we use a notation by *letters* called the **Roman notation**. It was named from the ancient Romans who used it.

4. Any method of naming, or reading, numbers represented by characters is called **numeration**.

The method in common use in America is known as the *French numeration*.

ARABIC SYSTEM

5. In counting a large number of objects it is natural to arrange them in equal groups. When the number of the first groups becomes large they may be gathered into larger groups, and these again into still larger groups, and so on.

The system of grouping that is most commonly used is that of grouping by *tens*, called the **decimal** system from the Latin word *decem* meaning *ten*.

In such a system 10 units make a group, 10 of these groups make a larger group, 10 of these larger groups make the next larger group, and so on.

6. The Arabic system of notation is a decimal system. It represents numbers by means of the following ten figures combined according to certain principles:

one	two	three	four	five	six	seven	eight	nine	naught
1	2	3	4	5	6	7	8	9	0

Naught is also called **zero** and **cipher**. It represents no value and is used to fill vacant places.

The other nine figures are called *significant* figures, because each represents a value.

7. In this system a group of ten is called **one ten**, written 10; a group of 10 tens is called **one hundred**, written 100; a group of 10 hundreds, **one thousand**, 1000; etc.

8. In the number 1111, the first 1 beginning at the right stands for 1 unit, the others in order toward the left for 1 ten, 1 hundred, and 1 thousand.

What does each 4 in 4444 represent? each 7 in 7777?

9. The decimal system may be extended indefinitely to the left or to the right. Before extending it to the right, a dot called the **decimal point** is placed after the figure that represents units.

10. Since in a number like 1111.111, each 1 stands for 10 times as much as the next 1 toward the right, the first 1 following the decimal point stands for 1 of the 10 equal parts of a unit, called **one tenth**; the second for 1 of the 10 equal parts of one tenth, or 1 of the 100 equal parts of a unit, called **one hundredth**; the third for 1 of the 1000 equal parts of a unit, called **one thousandth**; etc.

What does each 3 in 33.333 represent? each 5 in 555.55? each 8 in 8888.888?

11. All figures on the left of the decimal point represent whole units, called **integers.**

All figures on the right of the decimal point represent parts of units, called **decimals.**

12. Figures in *units*' place express *units of the first order;* in *tens*' place, *units of the second order;* in *hundreds*' place, *units of the third order;* etc.

The decimal orders are numbered *first*, *second*, *third*, etc., from the decimal point toward the right.

13. The largest number that can be represented by *three* figures is 999; the smallest number that requires *four* figures to represent it is 1000.

Thousands are named in the same way as units; thus, 1 thousand, 2 thousand, 3 thousand, etc., up to 999 thousand; 1000 thousand is called **one million** (1,000,000).

Millions are named in the same way, 1000 million being called **one billion** (1,000,000,000); 1000 billion, **one trillion** (1,000,000,000,000); etc.

14. For convenience in writing and reading large numbers, the figures are separated by commas into groups of three figures each, called **periods**.

The highest or left-hand period may contain less than three figures.

15. The first period of integers, counting from the right, is used to denote any number from 1 to 999 *units;* the second, any number from 1 to 999 *thousand;* the third, any number from 1 to 999 *million;* etc.

16. The general arrangement of the orders of units and their separation into periods is illustrated by the following table:

TRILLIONS' PERIOD		BILLIONS' PERIOD			MILLIONS' PERIOD			THOUSANDS' PERIOD			UNITS' PERIOD						
Ten-trillions	Trillions	Hundred-billions	Ten-billions	Billions	Hundred-millions	Ten-millions	Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Units	Decimal Point	Tenths	Hundredths	Thousandths
3	7,	2	4	0,	0	8	0,	0	0	0,	4	3	8	.	0	7	5
				6,	3	7	2,	4	0	8,	0	0	0				
														.	0	6	6
		3	8	9,	0	0	0,	2	7	4,	0	2	7	.	4	5	

The first number in the table is read, "37 trillion, 240 billion, 80 million, 438, *and* 75 thousandths."

Read the other numbers in the table.

17. Observe that

1. Each period beginning at the left is read as if it stood alone, the *name* of units' period, only, being omitted.
2. The figure 0 or a period of 0's is not read.
3. The word "and" is not used, except between an integer and a decimal.
4. The decimal point precedes tenths' figure in *all* decimals.

A decimal point may be assumed to follow the units' figure of *all* integers.

18. Other orders of decimals to the right of those in the table are *ten-thousandths*, *hundred-thousandths*, *millionths*, etc.

19. A number that is expressed by an integer and a decimal is a **mixed decimal**. It is also called a **mixed number** (§ 129).

20. Following is the fundamental principle of any decimal system:

Ten units of any order, integral or decimal, make one unit of the next higher order.

EXERCISES

21. **1.** Copy, point off, and read:

4710	32256	6445386	1234567890
2323	45045	7777412	406725304
6106	6300	1496300	9625148
7000	20721	530005	8000001
9960	4245	65198	643250699
2505	92007	9101	375416826
1206	5402	460743	5004008010
3458	16010	9702000	4647101566
4383	87000	565990	8766776677
1439	29564	6080040	21348613259
9574	66813	78621	10000000000
2492	48267	4000782	862415731897
6741	90415	9909009	2765007829345

2. Read:

.483	125.4	28.003	.000002
.574	62.875	5.0602	.00000075
.625	9.505	8.0075	.10605022
.007	28.56	0.00382	.95361485
.312	381.284	11.58109	.000012764
.074	109.326	37.43692	.000007065

WRITTEN EXERCISES

22. Express by figures:

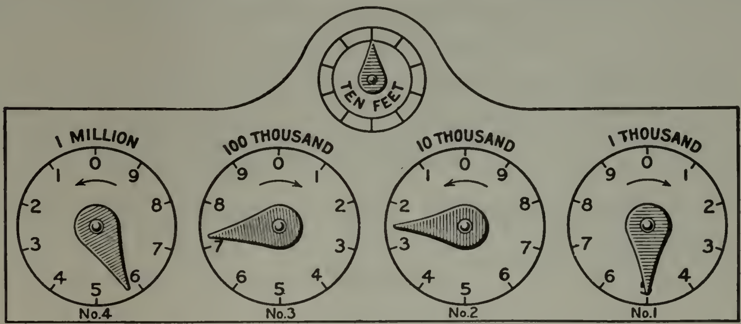
1. Twelve thousand, two hundred sixty.
2. Six hundred thousand, seven hundred one.
3. Nineteen thousand, eight hundred forty-two.
4. Seven hundred one thousand, ninety-eight.
5. Nine hundred ninety-nine thousand, seven.
6. Three million, three hundred thousand.
7. Sixty million, sixty thousand, forty-five.
8. Forty-six million, eight hundred sixty-two.
9. Thirteen million, fifty-five.
10. Two hundred seven million, five hundred seventy-one thousand, three hundred eight.
11. Five hundred two million, seven hundred six thousand, nine hundred twenty-three.
12. One hundred one million, three hundred four thousand, six hundred eighty-nine.
13. Four hundred two, and sixty-eight thousandths.
14. Ninety-six, and ninety-one ten-thousandths.
15. Two hundred eighty, and seventy-one millionths.
16. One million, one thousand, one, and one thousandth.
17. Six billion, four hundred seventy-one million, eight hundred thirty-six thousand.
18. Seventy-five billion, forty million, nine hundred eighty-one thousand, three hundred four.
19. Two million, and seven tenths.
20. Seventy-two thousand, and sixteen thousandths.
21. Four hundred, and one hundred six millionths.
22. Two, and one thousand ninety-six ten-millionths.

EXERCISES

23. The dials of a gas meter, and of other kinds of meters, give readings in the *decimal scale.*

Each division on dial No. 1 represents 100 cubic feet of gas, and each *complete* revolution of the hand records the passage of "1 thousand" cubic feet of gas through the meter, as indicated over the top of the dial. With each complete revolution of the hand of No. 1, the hand of No. 2 moves from one division to the next, and 10 revolutions of the hand of any dial produce 1 revolution of the hand of the dial next on the left.

Each division on the small *unit* dial, used for testing, represents 1 cubic foot.



1. How much gas passes through the meter while the hand of No. 1 moves from 0 to 1? from 1 to 2?

2. How much gas is No. 1 recording?

3. How much gas passes through the meter while the hand of No. 2 moves from 0 to 1? from 0 to 2?

4. How much gas are No. 2 and No. 1 together recording?

5. How many complete revolutions has the hand of No. 2 made while the hand of No. 3 has been moving from 0 to its present position? Read No. 3, No. 2, and No. 1 together.

6. Read all the dials. How much gas is the meter recording now? What is the greatest amount of gas it can record?

UNITED STATES MONEY

24. In the money of the United States:

10 mills = 1 cent	**10 dimes = 1 dollar**
10 cents = 1 dime	**10 dollars = 1 eagle**

The **dollar sign** is $; the sign for cent or cents is ¢.

25. Since 10 units of one denomination make 1 unit of the next higher denomination, United States money is conveniently represented in figures by the decimal system, dimes and cents being written as hundredths, and mills as thousandths of a *dollar*, which is the unit of the system.

Thus, 4 eagles, 6 dollars, 7 dimes, 8 cents, 5 mills, is written $46.785, and read "46 dollars, 78 cents, 5 mills."

Also, $.01 is read "1 cent"; $.001, "1 mill"; $.256, "25 cents 6 mills"; 25¢, "25 cents"; and 56.7¢, "56 and 7 tenths cents."

26. In business, the seller usually regards any part of a cent as an additional cent.

EXERCISES

27. 1. Read as dollars and cents:

$7.42	$11.25	$256.30	$100.10	$12,645.32
$2.87	$28.06	$809.66	$650.03	$75,050.90

2. Read as dollars, cents, and mills:

$3.845	$1.264	$24.056	$17.283	$100.084
$9.206	$7.605	$30.015	$89.617	$620.908

3. Write in the decimal form, using the sign $:

19 dollars 7 cents	500 dollars 16 cents 3 mills
63 dollars 82 cents	88 dollars 37 cents 5 mills
80 cents ; 1 cent 5 mills	740 cents; 23,765 cents

4. Write to the nearest cent: $1.667 ; $.264; $4.56; $.812.

ROMAN SYSTEM

28. The Roman notation uses seven letters, namely:

I	V	X	L	C	D	M
1	5	10	50	100	500	1000

29. These letters are combined to represent other numbers according to the following principles:

1. *When a letter is followed by the same letter or a letter of less value, the values of the letters are to be united.*

Thus, II represents 2; XXX, 30; VI, 6; DC, 600.

2. *When a letter is followed by a letter of greater value, its value is to be taken from that of the greater.*

Thus, IV represents 4; IX, 9; XL, 40; CD, 400.

3. *A letter with a bar placed over it represents a thousand times as much as it does without the bar.*

Thus, $\overline{\text{V}}$ represents 5000; $\overline{\text{X}}$, 10,000; $\overline{\text{IV}}$, 4000.

30. This table further illustrates the method of combination:

III, 3	XIV, 14	XLI, 41	XCIX, 99	MC, 1100
XII, 12	XIX, 19	LXX, 70	DCCC, 800	$\overline{\text{XX}}$, 20,000

EXERCISES

31. 1. Read the following:

$\overline{\text{M}}$	LXXIX	CCV	MDXC
XCIX	CXVIII	DXIX	$\overline{\text{MD}}$CCC
DLXXV	CCXXVI	DCXL	MMDC
CXXXII	CDLXIII	$\overline{\text{D}}$CCL	$\overline{\text{IV}}$CCXL

2. Express the following in the Roman notation:

55	61	101	496	1607
66	58	114	509	5000
79	99	325	1900	10200

FUNDAMENTAL PROCESSES

32. You have learned how to *add*, *subtract*, *multiply*, and *divide*, but you should practice on these operations until you can perform them **rapidly** and with **absolute accuracy**.

ADDITION OF INTEGERS AND DECIMALS

33. The process of finding a number that is equal to two or more given numbers is called **addition**.

34. The result obtained by adding is called the **sum**, or **amount**.

35. The numbers added are called **addends**.

36. The sign + indicates *addition* and is read **plus**.

37. The sign = indicates *equality* and is read **equals**, or **is equal to**.

38. Numbers that are made up of the same kind of units are called **like numbers**.

$4 and $6 are like numbers; also 8 feet and 5 feet.

39. Numbers that are made up of different kinds of units are called **unlike numbers**.

7 pounds and 9 gallons are unlike numbers.

40. 1. Can you add $4 and $6? 7 pounds and 9 gallons?

Only like numbers can be added.

2. What is the sum of 6 and 2? of 2 and 6? of 3, 4, and 2? of 2, 4, and 3? of 4, 3, and 2?

The sum of two or more numbers is the same in whatever order they are added.

WRITTEN EXERCISES

41. **1.** Add 3.5, 3.6, 6.2, 7.4, 6.3, 8.4, and 7.8.

12	3.5	11
	3.6	
	6.2	9
13	7.4	
	6.3	
15	8.4	12
	7.8	
	3.2	
	40	
	43.2	

We write units of the same order in the same column.

Beginning at the bottom of the right-hand column to add, we *mentally* separate the figures into convenient groups (as indicated), thinking of the *total* of each and naming *results only:* "12, 21, 32."

Since this is tenths' column, the footing is not 32, but 32 tenths, or 3.2. This we write below with its right-hand figure under the column just added.

Proceeding in a similar manner, we add the units' column: "15, 28, 40."

Writing 40 below the footing of the previous column and one place to the left, we add the footings and place the decimal point in the sum under the other decimal points.

Test. — Add each column in the reverse order. If the results agree, the work is probably correct.

Drill on exercises 2–5 until you can add and test them in 8 minutes or less:

2.	**3.**	**4.**	**5.**
42,653	.00625	2.743	3,745,876,200
8,557	.42038	5.492	917,582,688
2,941	.09416	0.3845	16,382,496,177
6,248	.00064	4.6287	8,465,923,587
19,167	.76258	11.4955	22,348,796,135
32,584	.17592	28.3766	85,375,469
62,108	.30906	9.58482	56,399,851
4,844	.49718	3.91764	47,235,565,695
75,276	.48376	18.71485	87,356,929,847

In books of account, dollars and cents are separated by a line, as in the following exercises, instead of by decimal points.

Add and test in 9 minutes or less:

6.		7.		8.		9.		10.	
$78	50	$325	63	$2493	67	$3879		$105092	25
99	37	106	25	678	10	172	56	827219	40
29	17	204	16	3429	12	8	94	274416	29
4	25	75	24	117		5	62	98270	
	14		09	5276	30	476	28	7630	81
75	46	234	75	9417	37	2630	06	1192	11
7	44	308	42	148	04	5523		124	16
86	93	54	50	171	14	89649	44	796	96
17		4	19	9	18	24763	84	2643	10
63	22	2	43	481	92	5169	36	23344	66
23	68	192	05	25	42	18371	88	648872	18
50	29	416	23	8345	68	539	42	985020	60

11. Add 481 thousand 2 hundred, 7 million 375 thousand 289, 25 million 875, 58 million 5 thousand 96, 32 million 630 thousand 88.

12. Add 37 thousand 250 and 8 hundredths, 64 thousand 208 and 9 tenths, 10 thousand and 70 thousandths, 856 thousand 926 and 25 ten-thousandths, 648 and 126 ten-thousandths, 70 and 2069 ten-thousandths.

13. Add $87.65, $19.16, $45.07, $92.95, $168.20, $256.48, $925.18, $374.77, $410.85, $24.32, $528.80, $7.95, $1244.50, $3265.25, $2481.17.

14. Find the sum of six hundred fifty-two thousand five hundred nine, twenty-one thousand eighty-five, seven hundred ninety-one thousand ninety-nine, three hundred eighty-seven thousand six hundred forty, six hundred seventy-five thousand four hundred ninety-six, and seventy thousand twenty.

15. Mr. Belding bought a house for $4500, and sold it at a gain of $173.50. What was the selling price?

16. A lady had a hat made, costing as follows: frame, $.25; plume, $5; ribbon, $.75; velvet, $2.25; buckle, $1; and making, $1. How much did she expend for the hat?

17. A man bought the following articles to furnish his office: desk, $28.50; desk chair, $5.75; typewriter, $85; table, $12; and chairs, $25.50. Find the cost of all.

18. In 1905 the United States navy consisted of 265 vessels fit for sea service, 47 under construction, and 15 unfit for sea service. Find the total number of vessels.

19. In the United States one year, live stock valued at $129,955 and other property valued at $3,016,520 were destroyed by lightning. Find the total loss by lightning that year.

20. Recently the world produced 4,908,000 tons of cane sugar and 6,990,000 tons of beet sugar in a year. How much sugar of both kinds did the world produce that year?

21. A transatlantic steamer carried, on one trip, 467 first cabin passengers, 300 second cabin passengers, 300 steerage passengers, and a crew of 550 men. Find the number of people on board.

22. For a single trip of a steamer, 19,800 pounds of fresh meat and 14,300 pounds of salt beef and mutton are provided. How many pounds of meat are provided?

23. A freight vessel carrying wheat from Duluth to Liverpool, reaches the ocean at the Straits of Belle Isle, 2384 miles from Duluth and 2234 miles from Liverpool. How many miles is the wheat carried in going to Liverpool?

24. The cable from San Francisco to Manila is laid in sections having the following lengths: San Francisco to Hawaii, 2089 miles; Hawaii to Wake Island, 2040 miles; Wake Island to Guam, 1290 miles; Guam to Manila, 1520 miles. Find the total length of the cable.

25. Part of the educational system of Freiburg, Germany, is a public theater. One year the receipts were: season tickets, $15,923; single tickets, $10,711; and extra performances, $7414. This did not pay all the expenses, but the city made up the deficit, $32,606. Find the total annual expense.

26. Find the weight of a train comprising a locomotive weighing 156,000 pounds, a tender weighing 85,000 pounds, and loaded cars weighing 98,200 lb., 112,500 lb., 96,840 lb., 104,360 lb., 99,760 lb., 109,580 lb., 122,300 lb., 118,650 lb., 92,850 lb., 103,590 lb., 114,250 lb., 105,680 lb., 112,610 lb., 99,150 lb., 120,540 lb., 119,730 lb., 98,600 lb., and 115,950 lb., respectively.

27. According to the census report there were in the United States 226,477 blacksmiths, 290,611 iron and steel workers, 283,145 machinists, 33,046 boiler makers, 12,473 stove, furnace, and grate makers, 28,122 tool and cutlery makers, 13,505 wheelwrights, and 18,487 wire workers. Find the total number of persons engaged in these industries.

28. A telephone company's receipts for a year were $2,934,075.59 for exchange service, $818,459.73 for toll service, $68,225.36 for private line service, $51,778.14 for messenger service, $1,802.92 for rents, $50,402.50 for interest, and $21,610.39 from miscellaneous sources. Find the total receipts for the year.

29. The losses in a coal strike were estimated as follows:

Loss of the miners in wages	$28,930,000
Loss of other workers	6,457,000
Loss of mine operators	52,250,000
Loss of business men in anthracite region	10,280,000
Loss of outside business	9,790,000
Damage to mines and machinery	7,200,000
Loss of coal-carrying railroads	26,000,000

Find the total strike losses.

SUBTRACTION OF INTEGERS AND DECIMALS

42. The process of taking part of a number away from it, or of finding how much greater one number is than another, is called **subtraction.**

43. The number from which another is subtracted is called the **minuend.**

44. The number subtracted is called the **subtrahend.**

45. The result obtained by subtracting is called the **remainder,** or **difference.**

46. The sign – signifies *subtraction* and is called **minus.** It is read *less* or *minus.*

47. Can you subtract \$7 from \$10? 8 feet from 12 gallons? 9 minutes from 14 minutes? 3 cents from 10 pounds?

Only like numbers can be subtracted.

48. Find the difference between 7 and 5; 7 + 3 and 5 + 3; 7 – 3 and 5 – 3. How do these differences compare?

Increasing or decreasing the minuend and subtrahend by the same number does not change the difference, or remainder.

WRITTEN EXERCISES

49. 1. From 83.9 subtract 26.5.

$$\begin{array}{r} 83.9 \\ 26.5 \\ \hline 57.4 \end{array}$$

5 tenths from 9 tenths leaves 4 tenths, or .4, which we write under the tenths.

Since 6 units cannot be taken from 3 units, we take 1 ten, equal to 10 units, from the 8 tens to unite with the 3 units. The sum is 13 units; 6 units from 13 units leaves 7 units, which we write under the units.

1 ten has already been taken from the 8 tens, leaving 7 tens. 2 tens from 7 tens leaves 5 tens, which we write under the tens.

The remainder is 57.4.

Another method of subtraction, known as the *Austrian* method, is based on the idea of finding what number added to the subtrahend will produce the minuend.

83.9
26.5
57.4

Since **.4** added to .5 equals .9, we write .4 in the remainder under tenths.

Since **7** added to 6 equals 13, we write 7 under the units and *carry* 1 ten to the next column. (We carry 1 ten because having added 10 to the minuend to make 13, by § 48 we must add 1 ten to the subtrahend.)

Since **5** added to the sum of 1 and 2 equals 8, we write 5 under the tens.

NOTE. — The teacher is advised not to confuse pupils by teaching both of these methods to the same class. The one that is preferred should be adopted and followed.

Test. — Add the remainder to the subtrahend. If the sum is equal to the minuend, the work is probably correct.

Subtract and test:

2. 46.3 23.9	**3.** 4321 2560	**4.** $ 241.30 175.60	**5.** 38.735 16.912
6. 7.41 2.74	**7.** 6340 5824	**8.** $ 544.56 273.89	**9.** 625.75 92.46
10. .625 .467	**11.** 7582 906	**12.** $ 842.71 84.16	**13.** .84691 .07540
14. 5.27 .99	**15.** 8412 5369	**16.** $ 920.50 760.50	**17.** .00825 .00098
18. 8.23 5.63	**19.** 7852 2987	**20.** $ 638.19 338.28	**21.** 2.5361 .4795
22. 9.48 3.89	**23.** 9888 3999	**24.** $ 744.36 560.85	**25.** 721.68 639.08

26. From 700 subtract 85.38.

700
85.38
614.62

Without writing them we may consider 0's to occupy tenths' and hundredths' places of the minuend.

Since there are no significant figures in the minuend until we reach hundreds' place, we take 1 hundred from the 7 hundreds and change it to 9 tens, 9 units, 9 tenths, and 10 hundredths.

Then we subtract, writing the result for each order in the proper column: "8 from 10, 2; 3 from 9, 6; 5 from 9, 4; 8 from 9, 1; and nothing from 6, 6."

Subtract and test:

27. 4200 1875	**28.** 5000 1282	**29.** 1700 699	**30.** $40 3.65	**31.** $100 42.80
32. 7160 4962	**33.** .9 .7431	**34.** 7.96 2.345	**35.** $75 19.27	**36.** $545 76.25
37. .608 .0031	**38.** 52.6 14.38	**39.** 1 0.576	**40.** $90 0.88	**41.** $250 132.14

42. 50,000 – 16,582

43. 61,348 – 22,548

44. 84,164 – 27,629

45. 90,000 – 49,167

46. 84,000 – 62,675

47. $193.80 – $54.95

48. $246.20 – $86.77

49. 6.25 – .0055

50. 77.7 – 7.777

51. 4.81 – 1.3486

52. 6.0075 – 2.649

53. 20.431 – 7.456

54. $62,956 – $438.88

55. $46,814 – $587.95

How many years have elapsed since the following events?

56. Atlantic cable, 1858.

57. Morse telegraph, 1844.

58. Marconi telegraph, 1896.

59. First cotton gin, 1794.

60. First life-preserver, 1805.

61. First steamboat in U.S., 1786.

62. First railroad in U.S., 1826.

63. First newspaper in U.S., 1704.

64. The area of the United States in 1790 was 827,844 square miles, and in 1900 it was 3,025,600 square miles. Find the increase in territory.

65. One year the total number of immigrants into the United States was 1,027,421. Of these 788,239 entered at the port of New York. How many entered at other ports?

66. The highest mountain in the Appalachian Mountain system is Mt. Mitchell, 6703 feet above sea level. The highest mountain in the Rocky Mountain system is Mt. McKinley, elevation 20,464 feet. Find the difference in elevation.

67. A man bought a launch and paid $6775 for it, which was $3140 more than he paid for an automobile. Find the cost of the automobile.

68. The heaviest bell in the world weighs 432,000 pounds, and is in Moscow, Russia. How much heavier is it than the bell in the City Hall, New York, which weighs 22,500 pounds?

69. The daily travel in all countries is about 813,000 miles by rail and 361,832 miles by water. How much does the daily travel by rail exceed that by water?

70. A month's receipts of oranges and lemons at New York City consisted of 275,372 boxes. All of the oranges and some boxes of lemons came from California. The total receipts of lemons were 142,822 boxes. How many boxes of oranges came from California?

71. How many more boxes of lemons were received than of oranges?

72. Two men took a 5-day trip in an automobile. Monday morning, before they started, the cyclometer registered 482.5 miles. Monday night it registered 541.7 mi., Tuesday night 603.4 mi., Wednesday night 661.2 mi., Thursday night 754.9 mi., and Friday night 836.3 mi. Find the whole distance traveled and the distance traveled each day.

73. Hook-and-ladder truck No. 1 weighs 9655 pounds. Engine No. 1 weighs 7985 pounds. Find the difference in weight.

74. If the tank in the tender of a locomotive has a capacity of 7500 gallons, but contains only 875 gallons, how many gallons are required to fill it?

75. A locomotive weighing 138,460 pounds was so constructed that all of the weight except 21,180 pounds was carried on the driving wheels. Find the weight on the driving wheels.

76. Two hundred twenty-nine thousand sixty-eight persons in Chicago voted for the adoption of voting machines, and twenty-six thousand nine hundred ninety voted against their adoption. By what majority did the measure pass?

77. A mechanic drilled two holes with their centers 2.45 inches from either end of a steel bar 8.25 inches long. How far apart were the holes from center to center?

78. During April a poultry man received \$571.18 from sales of eggs and \$202.35 from sales of fowls. His expenses for the month were: feed, \$117.56; coal, \$20.20; lighting, \$5.96; advertising, printing, stationery, and postage, \$133.15; express, \$4.20; wages, \$185; other expenses, \$72. Find his profit.

79. A city orchestra in Germany cost the city \$1190 for first conductor, \$714 for second conductor, \$500 for concert master, \$452 for harpist, \$15,227 for other musicians, and \$5397 for supplies, etc. The receipts were: symphony concerts, \$2292; theater music, \$12,614; city park concerts, \$2292; other receipts, \$7004. Find the surplus.

80. The iron used at the stove foundry is not pure iron. A stove that weighs 400 lb. contains about 11.96 lb. free carbon, 1.48 lb. combined carbon, 10.08 lb. silicon, 4.32 lb. phosphorus, .08 lb. sulphur, 2.84 lb. manganese, and the rest pure iron. Find the weight of pure iron in such a stove.

Accounts

50. A record of business transactions is called an **account.**

51. The business of the day is usually recorded, as the transactions occur, in a book called a **daybook.** The records of the several transactions are afterward transferred, or **posted**, into another book called a **ledger**, where they are arranged in groups, each containing the transactions that belong to one class, as, for example, those relating to one person or firm.

These groups, or **ledger accounts**, headed by the name of the person, are arranged so that the amounts charged against him, called **debits**, appear on the *left* side and those in his favor, called **credits**, on the *right* side.

52. A person who owes a debt is called a **debtor**, and a person to whom a debt is owed is called a **creditor.**

53. The difference between the sum of the debits and the sum of the credits of an account is called the **balance.**

54. *Computing* the balance, *entering* it on the lesser side, and *ruling* up the account, is called **closing**, or **balancing**, the account.

Some business men do this at stated times, as once a month, the balance being *brought down* to continue the account. It is also done when an account is settled.

55. The following are a few abbreviations in common use:

Company,	Co.	Credit or creditor,	Cr.	Paid,	Pd.
Balance,	Bal.	Debit or debtor,	Dr.	Received,	Rec'd.
Account,	Acc't.	Merchandise,	Mdse.	Payment,	Pay't.

The symbol # placed *after* a number stands for *pounds*, but if placed *before*, it is an abbreviation for the word "number."

Thus, 24 # means "24 pounds," but # 24 means "number 24."

The symbol @, meaning *at*, is always followed by the *price of a unit.*

Thus, 7# sugar @ 5¢, means "7 pounds of sugar at 5 cents a pound"; also, 6 doz. eggs @ 19¢, means "6 dozen eggs at 19 cents a dozen."

56. The following form illustrates a closed ledger account with John Graham, the balance being brought down to continue the account into the next month:

John Graham

Dr.											Cr.
190–			*			190–			*		
Jan.	*4*	*Flour*	*1*	*$15*	*40*	*Jan.*	*4*	*Cash*	*1*	*$10*	
"	*7*	*Oats*	*4*	*8*	*70*	"	*12*	*Carting*	*8*	*12*	*50*
"	"	*Cash*	"	*5*		"	*19*	"	*15*	*10*	
"	*15*	*Corn Meal*	*11*	*17*	*50*	"	*24*	"	*21*	*17*	
"	*24*	*Hay*	*21*	*24*		"	*31*	*Balance*		*32*	*47*
"	*29*	*Oats*	*34*	*11*	*37*						
				81	*97*					*81*	*97*
190–											
Feb.	*1*	*Balance*		*32*	*47*						

1. What are the debits in this account? Find their sum.

2. Name the credits. Find their sum.

3. How much is the balance? How is it found? See whether it has been correctly computed.

4. Is the balance in favor of or against John Graham?

On which side of the account would it have been entered, if it had been in his favor?

On which side would it have been brought down?

5. What is the *footing* of the debit side of the account? of the credit side? How do the footings compare?

Test. — When an account is properly closed, the footing of one side is equal to the footing of the other.

* These columns contain numbers of the daybook pages upon which the several transactions were entered.

WRITTEN EXERCISES

57. Prepare a ledger form for each of the following accounts, supply the year, enter the items, close and test the account:

1. George Griffin.

Debits. — Jan. 8, Groceries, \$10.37; Jan. 22, Groceries, \$18.91; Feb. 10, Groceries, \$7.89; Feb. 24, Groceries, 48¢; Mar. 6, Groceries, \$15.75; Mar. 20, Groceries, 66¢; Apr. 4, Groceries, \$9.72; Apr. 23, Groceries, \$13.96.

Credits. — Jan. 16, Cash, \$5; Feb. 15, Cash, \$14.50; Mar. 8, Labor, \$6.30; Mar. 14, Cash, \$17; Apr. 15, Cash, \$20; May 3, Labor, \$8.75; May 17, Cash, \$5.

2. Thomas Hinds & Co.

Debits. — July 12, Mdse., \$185.50; July 19, Cash, \$210; July 25, Mdse., \$119.63; July 30, Mdse., \$223.29; Aug. 11, Mdse., \$88.72; Aug. 18, Cash, \$335; Sept. 10, Mdse., \$47.54.

Credits. — July 3, Mdse., \$237.45; July 19, Mdse., \$149.80; July 28, Cash, \$125; Aug. 6, Mdse., \$92.24; Aug. 14, Mdse., \$422.18; Aug. 29, Mdse., \$116.40; Sept. 4, Cash, \$25; Sept. 8, Mdse., \$173.91; Sept. 24, Mdse., \$275.16.

3. Johnson & Mason.

Debits. — May 6, 7 Carriages, \$875; May 14, 5 Double Wagons, \$450; May 20, 4 Runabouts, \$210; May 25, 3 Surreys, \$525; June 1, 3 Single Wagons, \$225; June 8, 1 Phaeton, \$200; and 6 Open Buggies, \$480; June 15, 2 Buckboards, \$100; and 2 Road Carts, \$70.

Credits. — Jan. 4, White Ash Lumber, \$449.25; Jan. 9, Hub Timber (elm), \$375.40; Feb. 4, Spoke Timber (hickory), \$274.65; Apr. 8, Basswood Lumber, \$124.80; May 10, Cash, \$400; May 18, Cash, \$325; June 4, Cash, \$560; June 12, Cash, \$350.

MULTIPLICATION OF INTEGERS AND DECIMALS

58. The process of taking one number as many times as there are units in another is called **multiplication.**

59. The number multiplied is called the **multiplicand.**

60. The number by which we multiply is called the **multiplier.**

The multiplier shows how many times the multiplicand is to be taken.

61. The result obtained by multiplying is called the **product.**

62. The multiplicand and multiplier are called **factors of the product.**

63. The sign × indicates *multiplication.* It is read **times** when the multiplier precedes it, and **multiplied by** when the multiplier follows it.

Thus, 3 × 5 is read "3 times 5," if 3 is the multiplier, but "3 multiplied by 5," if 5 is the multiplier.

When it is important to know which number is the multiplier, and it would otherwise be in doubt, this book uses *times* or *multiplied by* instead of the sign.

64. A number that is used with reference to some particular kind of object or unit is called a **concrete number.**

6 pears, $12, 40 miles, are concrete numbers.

65. A number that is used without reference to any particular kind of object or unit is called an **abstract number.**

6, 12, 40, are abstract numbers.

66. How many dollars are 4 times $5? $5 multiplied by 4?

Can you find $5 times 4? Can you multiply 4 by $5?

The multiplier must be an abstract number or be regarded as abstract.

67. Complete and read:

1. 8×3 pounds = ?
2. 7×6 bushels = ?
3. 3 pounds $\times 8$ = ?
4. 6 bushels $\times 7$ = ?

How does the name of the unit in the product compare with the name of the unit in the multiplicand?

The product and multiplicand are like numbers.

68. How does 5 times 11 compare with 11 times 5? 2 times 17 with 17 times 2? 3 times 21 with 21 times 3?

The product is numerically the same whichever number is regarded as the multiplier.

In practice it is generally more convenient to use the smaller number as the multiplier.

Multiplication by Integers

69. Every cipher that is annexed to an integer moves each figure one place toward the left, and therefore (§ 20) changes the units to tens, the tens to hundreds, the hundreds to thousands, etc. Hence,

To multiply an integer by 10, *annex one* 0; *by* 100 *two* 0's; *by* 1000, *three* 0's; *etc.*

70. In a decimal or a mixed number, every removal of the decimal point one place toward the right corresponds to removing each figure one place toward the left. Hence,

To multiply a decimal by 10, *move the decimal point toward the right one place; by* 100, *two places; by* 1000, *three places; etc.*

If necessary, annex 0's to give the multiplicand enough places.

EXERCISES

1. Multiply by 10: 64; 352; 19.5; .33; .004.
2. Multiply by 100: 81; 5160; 3.28; .0688; .00025.
3. Multiply by 1000: 371.5; .56; 7.0125; .003625.
4. Multiply by 1,000,000: 5.5; 3.75; 16.875; .0006483.

WRITTEN EXERCISES

71. **1.** Multiply 46 by 80; 9.875 by 300.

46	9.875
80	300
3680	2962.5

Since 80 equals 8×10, we multiply by 8 and then by 10; that is, we multiply by 8 and annex *one* 0.

We first multiply by 3 and then by 100; that is, we multiply by 3 and move the decimal point *two* places toward the right.

Multiply:

2. 46 by 40

3. 75 by 60

4. .38 by 70

5. .74 by 90

6. 5.6 by 60

7. 9.9 by 200

8. 8.7 by 500

9. .43 by 600

10. .65 by 800

11. 345 by 200

12. 614 by 700

13. .075 by 300

14. .048 by 500

15. 2.57 by 6000

16. .0515 by 12,000

17. 81.76 by 110,000

18. .0436 by 500,000

19. .000005625 by 8 million

20. Multiply 83.6 by 428.

	83.6	83.6
	428	428
8 times 83.6 =	668.8	668 8
20 times 83.6 =	1672.0	1672
400 times 83.6 =	33440.0	3344
428 times 83.6 =	35780.8	35780.8

The full *partial products* are 668.8, 1672.0, and 33440.0; but in practice we write them in brief form, omitting the ciphers and decimal points as in the process on the right.

Observe that the right-hand figure of each abbreviated partial product is directly under the figure of the multiplier used to obtain it.

Multiply:

21. 482 by 65
22. 715 by 88
23. 932 by 49
24. 656 by 72
25. 5.75 by 48
26. .496 by 25
27. 34.5 by 76
28. 5.28 by 144
29. .756 by 625
30. $32.16 by 55
31. $18.75 by 36
32. $24.13 by 41
33. $75.20 by 111
34. $32.84 by 685
35. $12.96 by 875
36. $29.99 by 792
37. $155.72 by 81
38. $436.14 by 78

39.	**40.**	**41.**
27.64	6.895	24.68
5 06	4 007	4800
165 84	48 265	197 44
13820	27580	987 2
13985.84	27628.265	1184 64

Multiply:

42. 742 by 402
43. 386 by 605
44. 9.24 by 801
45. 42.33 by 407
46. 8.834 by 705
47. .3875 by 108
48. 7.246 by 305
49. 91.31 by 4062
50. 41.82 by 1001
51. 7.965 by 6004
52. .0775 by 8008
53. 42.62 by 2500
54. 85.8 by 16,000
55. 4.19 by 84,000
56. 3.763 by 9600
57. 9.044 by 7500
58. .0807 by 4300
59. $428.50 by 180
60. $269.75 by 240
61. $177.90 by 1600
62. $24.065 by 40,000
63. $0.567 by 260,000
64. .004065 by 52,000
65. .000076 by 440,000

66. If a fall of snow averaged .597 of an inch an hour, how much snow fell in 8 hours?

67. It costs $1.22 per word to cable from New York to Hong-Kong. How much does it cost to send a message of 27 words?

68. If a bushel of wheat absorbs 1.184 pounds of potassium from the soil, how many pounds will 125 bushels absorb?

69. The United States cabinet officers receive a salary of $12,000 per year, each. What are the combined salaries of the 9 cabinet officers for a term of 4 years?

70. The daily pay of a locomotive engineer was $4.10. What was his salary for a year of 340 working days?

71. A pack train carrying lumber and provisions to the *Golden Age* mine consisted of 28 burros. If the average load was 485 pounds apiece, how many pounds did all carry?

72. A flask of quicksilver is 76.5 pounds. In a recent year the production of quicksilver in the United States was 34,291 flasks. Express the quantity produced in pounds.

73. A hoisting engine consumed 40 pounds of coal per hour for 10 hours a day. Find the cost of 1 day's supply of coal at $.145 per hundred pounds.

74. On one trip of an ocean steamer 70 cases of eggs, each case containing 30 dozen, were consumed. How many eggs were consumed on the trip?

75. How much will it cost to keep 48 barrels of poultry in cold storage for 2 months at 30¢ per barrel per month?

76. A jobber bought 724 boxes of oranges at $2.25 and repacked them, obtaining 716 boxes of sound fruit which he sold at $2.50 each. How much did he gain?

77. If it costs 32¢ per can of 40 quarts to ship milk from Little Falls to New York, how much will it cost to ship 87 such cans? How many quarts are there in such a shipment?

Multiplication by Decimals

72. Since 10 units of any order make 1 unit of the next higher order, 1 unit of any order is 1 tenth, or .1, of a unit of the next higher order; that is, it is a unit of the next higher order multiplied by .1.

Consequently, every removal of a figure one place toward the right, or of the decimal point one place toward the left, changes tens to units, units to tenths, tenths to hundredths, etc. Hence,

To multiply an integer or a decimal by .1 *move the decimal point toward the left one place; by* .01, *two places; by* .001, *three places, etc.*

If necessary, prefix 0's to the multiplicand; thus, 2.5 multiplied by .001 = .0025.

EXERCISES

1. Multiply by .1: 7; 45; 369; 8.2; .6; .75.
2. Multiply by .01: 56; 7492; 37.5; 8.75; .432.
3. Multiply by .001: 965; 18,609; 75; 6.25; .4.
4. Multiply by .0001: 2387; 96381.6; 8.46.
5. Multiply by .00001: 46,394; 276.375; 92.65.
6. Multiply by .000001: 324,625; 55,650; 75.96.

WRITTEN EXERCISES

73. 1. Multiply 235 by .3; 74.2 by .07.

$$\begin{array}{r} 235 \\ .3 \\ \hline 70.5 \end{array} \qquad\qquad \begin{array}{r} 74.2 \\ .07 \\ \hline 5.194 \end{array}$$

Since .3 equals 3 × .1, we multiply by 3 and then by .1; that is, we multiply by 3 and point off *one* decimal place.

Since .07 = 7 × .01, we multiply by 7 and then by .01; that is, we multiply by 7 and move the decimal point *two* places toward the left.

Multiply:

2. 36 by .4	**8.** 854 by .05
3. 41 by .7	**9.** 792 by .002
4. 5.8 by .3	**10.** 91.6 by .0008
5. .28 by .4	**11.** 42.85 by .007
6. 75 by .06	**12.** 682.75 by .0004
7. 6.7 by .06	**13.** 24876 by .00009

14. Multiply .4365 by .058.

$$\begin{array}{r} .4365 \\ \underline{.058} \\ 34920 \\ \underline{21825} \\ .025317\cancel{0} \end{array}$$

Since .058 = 58 × .001, we multiply by 58 and then by .001; that is, we multiply by 58 and move the decimal point *three* places toward the left, prefixing one 0 to give places enough.

The product has *four* decimal places for the decimal in the multiplicand, and *three* more for the decimal in the multiplier, or *four* plus *three* decimal places. Since 0's on the right of a decimal do not affect its value, we may omit the 0 on the right.

Observe that *the number of decimal places in the product is equal to the number in the multiplicand plus the number in the multiplier*.

Test. — Multiply the multiplier by the multiplicand. If the results agree, the work is probably correct.

Multiply and test:

15. 6.4 by 2.2	**24.** 428 by .75	**33.** 246.5 by 10.2
16. 3.7 by 5.6	**25.** 36.5 by .32	**34.** 33.75 by 8.04
17. 21 by .38	**26.** 22.1 by 1.8	**35.** 720.6 by .035
18. 84 by 7.5	**27.** 4.17 by .72	**36.** 32.15 by .068
19. 96 by .605	**28.** 1228 by .45	**37.** 70.41 by 70.4
20. 7.4 by 3.02	**29.** 1655 by .32	**38.** 9074 by .0345
21. .35 by 40.8	**30.** 874.6 by 6.5	**39.** 2768 by .1625
22. 2.9 by .106	**31.** 591.8 by .85	**40.** .3455 by 166.6
23. .44 by 8.05	**32.** 41.76 by 4.2	**41.** 448.5 by 7.404

Multiply :

42. 665.5 by 3.044
43. 71.62 by 51.85
44. 4637 by .1842
45. 2.936 by 464.1
46. 75 by 3.0065
47. 48 by 9.0107
48. 9.2 by 36.502
49. .47 by 100.64
50. 78.256 by 8.75
51. 4.6254 by 32.5
52. 738.49 by .496
53. 55.410 by .855
54. 82 by .00045
55. 64 by .000235
56. 96 by 1.00465
57. 4.9 by 25.3726
58. 32,000 by 3.0045
59. 75,600 by .00088
60. 42,720 by .00705
61. 81.625 by .00404
62. 3.6875 by .80608
63. 16 million by .15625
64. 7.5 million by 1.0844
65. 36.25 million by .0075

66. One year 74,510,064 pounds of paper were used in the United States for job printing. Find the value of this quantity of paper at 8.4¢ per pound.

SUGGESTION. — The paper was worth 74,510,064 times \$.084. Since 74,510,064 times .084 is equal to 74,510,064 multiplied by .084, we may find the number of dollars in the product by using the number having the fewer figures as the multiplier.

67. If 15,917,000 school children in the United States contribute 5¢ each to relieve suffering in a famine, how much money will they contribute to the relief fund?

68. The following tickets were sold for the Symphony Club's concert: 420 @ \$2.50, 366 @ \$2, 474 @ \$1.25, and 474 @ \$.75. What were the box-office receipts?

69. Edward bought a box containing 126 oranges and another containing 150 oranges, at \$3.25 per box. He sold 9 dozen at 35¢ per dozen, 4 dozen at 40¢ per dozen, 75 oranges at 4¢ each, and the rest at 3¢ each. How much did he gain?

70. A paper mill contained 14 paper-making machines each of which made 1.32 tons of paper per hour. Find the output for a day of 24 hours.

71. For every ton of paper made, 1.95 tons of water had to be evaporated. How many tons were evaporated per day?

72. The hot air used to evaporate the water and carry it off was supplied at the rate of 440 cubic feet per minute for each ton of paper. How much hot air was supplied per hour?

73. A bar of iron 1 foot long expands .000006822 of a foot for every degree of rise in temperature. If a bar of iron is 60 feet long at 32° F., what will be its length at 33° F.? at 62° F.?

74. A strip of zinc 1 foot long expands .00001653 ft. for every degree of rise in temperature. If a strip of iron and a strip of zinc are each 1 foot long at 32° F., which has the greater length at 650° F., and how much greater?

75. Steel expands .00001093 ft. per foot of length for every degree of rise in temperature. If the length of a steel rail in freezing weather is 30 feet, how many inches longer will it be on a day when the temperature is 105° F.?

76. It is estimated that each person in New York State consumes .473 of a barrel of flour and meal per year. Find the amount consumed per year in a city of 3.5 million people.

77. A steamer on the enlarged Erie Canal will carry 900 tons of wheat and tow 3 barges, each carrying 100 tons, on the trip from Buffalo to New York. If the average load on the return trip is 1250 tons, how many tons of freight will the steamer and barges transport in a season of 10 round trips?

78. How much less will it cost to transport a cargo of wheat weighing 78,000 hundredweight from Chicago to New York by the Great Lakes and the Erie Canal at 7.85¢ per hundredweight than by rail at 16.5¢?

Bills

74. A detailed written statement of indebtedness for goods sold or services rendered is called a **bill.**

A bill should be dated and should state the names of the parties concerned (debtor and creditor); the terms of credit; the name, quantity, and price of a unit of each item; the whole value of each item; and the entire amount, or footing.

75. When a bill is paid, the creditor writes "Received Payment" or "Paid" near the bottom and signs his name. This is called **receipting** the bill.

The creditor may authorize some person to act as his agent in receipting bills. In this case the agent signs the creditor's name and below that his own surname or initial preceded by "Per" or "By."

76. The following form illustrates a bill receipted by an authorized agent of the creditor:

HENRY MAXWELL JOHN R. GREEN

BUFFALO, N.Y., *April 1*, 190*6*.

Mr. *Chas. R. Davis,*
36 West Main St.

Bought of MAXWELL & GREEN
214 SWAN STREET

HARDWARE, IRON, TOOLS, IMPLEMENTS, WAGONS, ETC.

TERMS: *Cash*

		10 kegs 20-penny nails 2.40	*24*			
		25 kegs 8-penny nails 2.50	*62*	*50*		
		12 Disston saws 1.25	*15*			
		27 Stanley planes 1.10	*29*	*70*	*131*	*20*
		Rec'd pay't,				
		Maxwell & Green.				
		Per B.				

WRITTEN EXERCISES

77. Make out bills for the following, supplying the date and places of business of both debtor and creditor, find the footings, and receipt in due form :

1. H. H. Moore bought of Lackawanna Coal Co. 1343 tons of coal @ $ 3.40.

2. H. H. Moore sold to President of Board of Contract and Supply (your city), 1260 tons of coal @ $ 4.25.

3. Hayes & Reynolds sold to Empire Milling Co. 1000 bu. No. 1 hard Manitoba wheat @ 87 ¢, 1200 bu. No. 2 red wheat @ 85¢, and 5000 bu. No. 2 corn @ 46 ¢.

4. George Saxe bought of John Stillman 2 brass bedsteads @ $ 36.50, 2 bureaus @ $ 28, 1 mahogany chiffonier @ $ 75, and 1 golden oak toilet table @ $ 42.50.

5. The Champlain Canning Co. is debtor to Stephen A. Dole, for carting and advancing freight charges, as follows :

Jan. 6, 2 loads @ $ 1.50; freight charges $ 8.50
Jan. 7, 3 loads @ $ 1.50; freight charges $ 12.00
Jan. 8, 1 load @ $ 1.50; freight charges $ 7.50
Jan. 9, 5 loads @ $ 1.50; freight charges $ 16.25

6. Albert Perkins bought of the Continental Hardware Co. 15 Excelsior ranges @ $ 38, 12 Alps ranges @ $ 32.25, 16 Jewel gas ranges @ $ 13.50, 42 cook stoves @ $ 23, 18 Safety oil stoves @ $ 7.75, 2 parlor stoves @ $ 35, 6 parlor stoves @ $ 31.50, 27 double boilers @ $ .89, and 4 dozen razors @ $ 11.50.

7. Allen Bros. bought of Storrs & Stoneman 6 doz. blank books @ $ 1.40, 4 doz. blank books @ $ 2.25, 72 pints ink @ 35¢, 64 steel rulers @ 39 ¢, 15 cameras @ $ 4.95, 3 doz. trays @ $ 1.50, 5 M clips @ $ 1.10, 16 doz. blotting pads @ 49 ¢, and 48 bottles library paste @ 19 ¢.

DIVISION OF INTEGERS AND DECIMALS

78. How many 4's are there in 16? How many times is 4 contained in 16?

Separate 24 cent-pieces into 4 equal piles. How many are there in each pile?

79. The process of finding how many times one number is contained in another is called **division.**

Division is also the process of separating a number into equal parts. This kind of division is sometimes called **partition.**

80. The number divided is called the **dividend.**

81. The number by which we divide is called the **divisor.**

82. The result obtained by dividing is called the **quotient.**

83. When all the dividend is divided and the quotient is an integer, the division is **exact.**

84. The part of the dividend that is left when the division is not exact is called the **remainder.**

85. The sign ÷ indicates *division.* It is read **divided by.**

Division is also indicated by writing the dividend above the divisor with a straight line between them; or by writing the dividend at the right of the divisor with a curved line between.

Thus, $\frac{24}{6}$, or 6)24, indicates that 24 is to be divided by 6.

86. Complete and read:

1. $20 ÷ $4 = ?
2. 36¢ ÷ 9¢ = ?
3. 18 rods ÷ 6 rods = ?
4. 56 feet ÷ 8 feet = ?

If the dividend and divisor are like numbers, the quotient is an abstract number.

87. Complete and read:

1. $\$20 \div 5 = ?$
2. $36¢ \div 4 = ?$
3. 18 rods $\div 3 = ?$
4. 56 feet $\div 7 = ?$

If the dividend is concrete and the divisor is abstract, the dividend and quotient are like numbers.

Division by Integers

WRITTEN EXERCISES

88. 1. Divide 1346 by 5; 13.46 by 5.

$$5\overline{)1346}$$
$$269, 1 \text{ rem.}$$
$$\text{or} \quad 269\tfrac{1}{5}$$

The remainder may be written in either of these ways. The latter way makes it a part of the quotient, the division of 1 by 5 being indicated.

$$5\overline{)13.46}$$
$$2.69\tfrac{1}{5}$$
$$\text{or} \quad 2.69, .01 \text{ rem.}$$

The quotient is $2.69\frac{1}{5}$.

By annexing a decimal 0 to the dividend, the division may be completed, the quotient being 2.692.

When the divisor is so small that the work may be done mentally, and only the figures of the quotient are written, the process is called **short division**.

Divide:

2.	3.	4.	5.	6.
$4\overline{)1776}$	$6\overline{)4074}$	$7\overline{)31.43}$	$5\overline{)6442}$	$2\overline{)31.75}$
7.	**8.**	**9.**	**10.**	**11.**
$3\overline{)4.16}$	$8\overline{)7000}$	$4\overline{)811}$	$8\overline{).2504}$	$6\overline{).0785}$
12.	**13.**	**14.**	**15.**	**16.**
$5\overline{)84.2}$	$6\overline{)33.75}$	$8\overline{)4.262}$	$7\overline{)629.3}$	$9\overline{).3762}$

17. 1000 by 6
18. 82.94 by 7
19. 2.643 by 2
20. 473.3 by 3
21. .2896 by 8
22. 654.5 by 11
23. 3.456 by 12
24. 40.04 by 11
25. 7.235 by 12

89. Cutting off one figure from the right of an integer corresponds to moving each figure one place toward the right and, therefore (§ 20), changes the tens to units, the hundreds to tens, etc. Hence,

To divide an integer by 10, *cut off one figure from the right; by* 100, *two figures; by* 1000, *three figures; etc.*

When the figures cut off are all 0's, the division is exact; when any are significant, the number represented by the figures cut off is the remainder.

$3800 \div 100 = 38|00 = 38$; and $3827 \div 100 = 38|27 = 38$, remainder 27, or 38.27.

90. In a decimal or a mixed number, every removal of the decimal point one place toward the left corresponds to removing each figure one place toward the right. Hence,

To divide a decimal by 10 *move the decimal point toward the left one place; by* 100, *two places; by* 1000, *three places, etc.*

If necessary, prefix 0's; thus, $5.6 \div 100 = .056$.

EXERCISES

1. Divide by 10: 320; 46,000; 167; 4.67; 82.4; .775.
2. Divide by 100: 66,400; 3250; 4182; 62.81; 206; .089.
3. Divide by 1000: 4,360,000; 8720; 485; 37.61; .32; 6.25.
4. Divide by 1,000,000: 12,820,000; 422,300,000; 66.6.

WRITTEN EXERCISES

91. 1. Divide 9384 by 400; 938.4 by 400.

$$4|00\overline{)\,93|84}$$
$$23\tfrac{184}{400}$$

Since $400 = 100 \times 4$, we divide by 100 and then by 4; that is, we cut off two figures and divide the rest of the dividend by 4, giving a quotient of 23 and a remainder of 1 hundred plus the 84 cut off, or 184.

$$4\overline{)\,9.384}$$
$$2.346$$

We divide by 100 and then by 4; that is, we move the decimal point *two* places toward the left, obtaining 9.384; then we divide this quotient by 4, obtaining 2.346.

Divide:

2. 5760 by 60	**7.** 3524 by 400	**12.** 83,550 by 5,000
3. 91.35 by 70	**8.** 263.2 by 500	**13.** 31,635 by 6,000
4. 3160 by 80	**9.** 8.267 by 700	**14.** 41,620 by 80,000
5. 761.7 by 90	**10.** 2.464 by 1100	**15.** 25,110 by 90,000
6. 2847 by 120	**11.** 38.46 by 1200	**16.** 57,200 by 110,000

17. Divide 56,249 by 68.

	827	827
	68)56249	68)56249
800 times 68 =	54400	544
	1849 undivided	184
20 times 68 =	1360	136
	489 undivided	489
7 times 68 =	476	476
827 times 68,	plus 13 rem. = 56,249	13

Since $50,000 \div 68$ is less than any number of *ten-thousands* and $56,000 \div 68$ is less than any number of *thousands*, but $56,200 \div 68$ equals about 800, write 8 in the quotient over the hundreds of the dividend, multiply 68 by 800, and subtract the product from the dividend, leaving 1849 to be divided. 1849 contains 68 about 20 times. Subtract 20 times 68; then the part of the dividend still undivided is 489. This contains 68 about 7 times. Subtract 7 times 68; the remainder is 13.

The quotient is 827.

Since 827 times 68, plus 13 = 56,249 (the dividend) the work is correct.

In practice we write the work in brief form as in the process on the right, dividing thus: $562 \div 68$ = about 8; write 8 in the quotient over 2, *the last figure of the part of the dividend used;* 8 times 68 = 544; subtract and bring down 4, the next figure of the dividend; $184 \div 68$ = about 2; write 2 in the quotient; 2 times 68 = 136; subtract and bring down 9; $489 \div 68$ = about 7; 7 times 68 = 476; subtract.

Test. — Multiply the divisor by the quotient and to the product add the remainder, if there is one. If the work is correct, the result will equal the dividend.

When the divisor is so large that each step in the work must be written, the process is called **long division**.

Divide and test :

18. 133,400 by 256

19. 263,016 by 312

20. 205,387 by 481

21. 630,143 by 567

22. 400,000 by 755

23. 2,231,650 by 508

24. 1,275,000 by 3400

25. 3,424,000 by 1720

26.

$$\begin{array}{r}
\$5.376 \\
265)\overline{\$1424.64} \\
\underline{1325} \\
996 \\
\underline{795} \\
2014 \\
\underline{1855} \\
1590 \\
\underline{1590}
\end{array}$$

The quotient, carried to mills by annexing a decimal 0 to form the last partial dividend, is \$5.376. To the nearest cent it is \$5.38.

27.

$$\begin{array}{r}
.001648^{+} \\
57)\overline{.09395} \\
\underline{57} \\
369 \\
\underline{342} \\
275 \\
\underline{228} \\
470 \\
\underline{456}
\end{array}$$

The sign + shows that the quotient is a little more than .001648. To five decimal places it is a little less than .00165, indicated .00165−.

Divide :

28. \$743.12 by 56

29. \$873.25 by 35

30. \$1181.84 by 44

31. \$1696.32 by 72

32. \$5534.02 by 89

33. \$3465.50 by 58

34. \$5346.70 by 127

35. 2515.6 by 95

36. 321.86 by 76

37. 372.012 by 58

38. 43.4172 by 746

39. 3297.18 by 614

40. 364.175 by 875

41. .015912 by 255

Find quotients :

To nearest tenth	To nearest hundredth	To nearest thousandth
42. $44.0 \div 7$	**50.** $1.70 \div 6$	**58.** $26 \div 11$
43. $125 \div 8$	**51.** $100 \div 9$	**59.** $11 \div 26$
44. $100 \div 37$	**52.** $100 \div 12$	**60.** $75 \div 87$
45. $62.5 \div 16$	**53.** $480 \div 66$	**61.** $50 \div 38$
46. $275 \div 175$	**54.** $815 \div 48$	**62.** $100 \div 55$
47. $1000 \div 28$	**55.** $600 \div 43$	**63.** $27.24 \div 65$
48. $55{,}600 \div 732$	**56.** $2870 \div 275$	**64.** $34.482 \div 821$
49. $90{,}000 \div 925$	**57.** $568.5 \div 615$	**65.** $319.59 \div 728$

66. If there are 18,000 men in 24 battalions of soldiers, how many men are there in 1 battalion?

67. The monthly rent receipts from an apartment house amounted to $513. There were 19 apartments. What was the average rent of each?

68. It takes a letter 45 days to go from New York to Shanghai, China, a distance of 14,445 miles. How many miles is it carried per day?

69. One year, only 94 ounces of platinum were produced in the United States. The estimated value was $1814. Find the value per ounce, to the nearest cent.

70. A year's production of crude borax amounted to 2600 tons, valued at $91,000. Find the average value per ton.

71. A blast furnace in continuous operation for 1475 days produced 493,120 tons of pig iron. Find the average daily production; the average hourly production to the nearest .01 ton.

72. The operating expenses of a street railway for a year of 365 days were $125,092.80. Find the average daily expenses.

73. The total daily travel of all the cars was equivalent to a daily travel for 1 car of 2880 miles. Find the average expense per car per mile, or per "car-mile."

74. A large shipment of salt from Silver Springs, N.Y., consisted of 4.2 million pounds of salt in 118 cars. Find the average car load to the nearest hundred pounds.

75. How many barrels of salt, of 280 pounds each, were there?

76. A stock raiser shipped to Chicago a car load of cattle comprising 18 head and weighing 23,859 pounds. Find the average weight per head.

77. Find to the nearest cent the whole amount received for the cattle at $5.86 per 100 pounds, and the average amount per head.

Division by Decimals

92. **1.** Divide 12 by 4; 2×12 by 2×4. How do the quotients compare?

2. Compare $6 \div 2$ with $60 \div 20$; with $600 \div 200$.

3. Divide .35 by 5; 3.5 by 50; 35 by 500.

In $.35 \div 5$, what effect does it have on the quotient to multiply both dividend and divisor by 10? by 100?

Multiplying both dividend and divisor by the same number does not change the quotient.

WRITTEN EXERCISES

93. **1.** Divide 7.6912 by 3.68.

$$7.6912 \div 3.68$$

$$\begin{array}{r} 2.09 \\ 368\overline{)769.12} \\ \underline{736} \\ 3312 \\ \underline{3312} \end{array}$$

Before dividing we change the divisor to an *integer* by multiplying it by 100, but to avoid changing the quotient, we must also multiply the dividend by 100.

When we bring down 1 to form the *second partial dividend*, we find that 331 does not contain 368; therefore we write 0 in the quotient and bring down the next figure of the dividend, forming the partial dividend, 3312.

NOTE. — Observe that the number of decimal places in the quotient is equal to the number in the dividend less the number in the divisor.

2. Divide 17.2 by .016

$$\begin{array}{r} 1075 \\ 16\overline{)17200} \\ \underline{16} \\ 120 \\ \underline{112} \\ 80 \\ \underline{80} \end{array}$$

We multiply both dividend and divisor by 1000.

3. Divide .0469 by 1.75

$$\begin{array}{r} .0268 \\ 175\overline{)4.69} \\ \underline{3\;50} \\ 1\;190 \\ \underline{1\;050} \\ 1400 \\ \underline{1400} \end{array}$$

We multiply both dividend and divisor by 100.

Divide as indicated:

4. $65 \div .64$
5. $15 \div .9375$
6. $64 \div .0625$
7. $17.4 \div 19.2$
8. $12.6 \div 2.88$
9. $8.64 \div 3.84$
10. $39.6 \div 6.875$
11. $8.236 \div .145$
12. $319.5 \div .075$
13. $6.305 \div 3.25$
14. $945 \div .448$
15. $918 \div .53125$
16. $33.176 \div .0464$
17. $10.2382 \div 49.7$
18. $5453.37 \div .118$

Find quotients to the nearest fourth decimal place:

19. $10 \div 3.265$
20. $24 \div 57.51$
21. $1.6803 \div 23.8$
22. $70.822 \div 87.5$
23. $19.776 \div 3.55$
24. $3.1418 \div 27.5$

25. How long will it take a gasoline launch that has a speed of 6.75 miles per hour to travel 20.25 miles?

26. How long will it take an automobile that has a speed of 41.7 miles per hour to go 150.12 miles?

27. When wheat costs 62.5¢ per bushel, how many bushels can be bought for $1000?

28. A cubic inch of cast iron weighs .2607 of a pound. How many cubic inches to the nearest tenth of a cubic inch are there in a mass that weighs 10 pounds?

29. Ice is .92 as heavy as water, and glass is 2.89 times as heavy as water. How many times as heavy as ice is glass? Obtain the answer correct to the nearest hundredth.

COMBINATION OF PROCESSES

94. When + and − are the only signs in an expression, the operations are performed in order beginning at the left.

$$2 + 8 - 6 + 3 = 10 - 6 + 3 = 4 + 3 = 7.$$

95. When × occurs in an expression in connection with +, −, or both, the indicated *multiplications* must be performed first.

$$8 + 2 \times 3 - 9 = 8 + 6 - 9 = 5.$$

96. When ÷ occurs in an expression in connection with +, −, or both, the indicated *divisions* must be performed first.

$$10 - 8 \div 2 + 5 = 10 - 4 + 5 = 11.$$

97. When × and ÷ are the only signs in an expression or are succeeding signs in any expression, the indicated multiplications and divisions are usually performed in order from the left.

$$7 + 10 - 6 \div 3 \times 4 = 7 + 10 - 2 \times 4 = 7 + 10 - 8 = 9.$$

98. The parentheses ( ), brackets [ ], braces { }, and the vinculum ——, called **signs of aggregation**, are used to group expressions, each group being regarded as a single number.

All operations within groups should be performed first.

$$10 \times \overline{6 - 2} + [9 \div (2 + 1)] = 10 \times 4 + 9 \div 3 = 40 + 3 = 43.$$

WRITTEN EXERCISES

99. Find the value of:

1. $5 \times 10 - 7$
2. $5 \times (10 - 7)$
3. $2 \times .5 + .3 \times .4$
4. $16 - 2 \times 2 \times 12 \div 4$
5. $(2.5 - 1.3) \div 4 \times 2$
6. $6 + 2 \times 8 - 4 \div 2$
7. $(6 + 2) \times 8 - 4 \div 2$
8. $(6 + 2 \times 8 - 4) \div 2$
9. $6 + 2 \times (8 - 4) \div 2$
10. $6 + 2 \times (8 - 4 \div 2)$
11. $3 + [2 + \overline{10 - 8} + \overline{4 \times 3 - 2 \times 5}] \div 2$
12. $24 - \{16 - 6 \div 3 \times (5 \times 5 - 6 \times 4)\}$

REVIEW PROBLEMS IN INDUSTRIES

SUGGESTION. — When the problems are related to one another, the answer to each should be kept until the series is completed.

100. **1.** A salesman bought a mileage book in New York, boarded the 8 : 30 A.M. Empire State Express, and rode to Buffalo. The next day he returned on the same train, leaving Buffalo at 1 : 00 P.M. Find his fare both ways at 2 ¢ per mile.

Miles	West, read downward.			East, read upward.		Miles
0	8:30	Lv.	New York	Ar.	10:00	440
143	11:10	Ar.	Albany	Lv.	7:00	297
143	11:13	Lv.	Albany	Ar.	6:57	297
238	12:55	Lv.	Utica	Lv.	5:02	202
291	1:58	Ar.	Syracuse	Lv.	3:57	149
291	2:01	Lv.	Syracuse	Ar.	3:54	149
371	3:25	Lv.	Rochester	Lv.	2:24	69
440	4:45	Ar.	Buffalo	Lv.	1:00	0

2. How long did it take him to go to Buffalo? to return?

3. Find the average speed per hour returning.

4. The train left New York with 320 passengers; at Albany 75 got off and 69 got on; at Utica 32 got off and 46 on; at Syracuse 40 got off and 48 on; and at Rochester 55 got off and 21 on. How many passengers went to Buffalo?

5. All passengers paid fare at the rate of 2 ¢ per mile, except 133 purchasers of regular tickets in New York, who paid \$.24 each in addition to the 2 ¢ per mile. Find the receipts from fares.

6. The parlor car company sold 21 seats to Albany @ \$1, 17 to Utica and Syracuse @ \$1.50, 38 to Rochester and Buffalo @ \$2, and the exclusive use of 2 drawing-rooms to Buffalo @ \$7. In addition they received 1 ¢ per mile from the railroad company for the rent of the car. Find the total parlor car receipts.

7. Find the pay of the engineers for a trip at 3.5 ¢ per mile.

8. It takes a steel steamship loaded with wheat 140 hours to go from Chicago to Montreal via the Welland Canal, and 160 hours to make the return trip. Sixty hours are spent in port. How many days does it take for the round trip?

9. The distance between the two ports is 1266 miles. How many miles does the vessel travel in a season of 13 round trips?

10. A vessel of this type carries, on an average, 75,000 bushels of wheat (60 pounds to the bushel) on each trip to Montreal and 1125 tons of other commodities on the return trip. How many more tons of freight does the vessel carry on a voyage to Montreal than on the return voyage?

11. Find the freight earnings for a season, at 5¢ per bushel "down" and $1 per ton "up."

12. How much more does the vessel earn going down than going up, each time? during the whole season?

13. The season's expenses are about as follows:

Wages and board, $7350	Customs fees, etc., $500
Coal, 230 tons per round trip @ $3	Outfit and repairs, $2875
Engine expenses, oil, etc., $500	Insurance, $4600
Elevating 975,000 bu. wheat @ ¼¢	Management, $2000
Shoveling wheat, at $4 per 1000 bu.	General expenses, $2000
Wheat lost and damaged, $1950	Wear of vessel, $5750

Find the total expenses for the season.

14. Find the profit for the season; for 1 round trip.

15. Going up the St. Lawrence River the freight vessel must pass through canals. Their lengths are as follows: Lachine, 8.5 mi.; Soulanges, 14 mi.; Cornwall, 11 mi.; Farrans Point, 1 mi.; Rapide Plat, 3.5 mi.; Galops, 7.125 mi.; Murray, 5.167 mi. The Welland Canal is 26.75 mi. long. How many miles must the vessel pass through canals to reach Lake Erie?

16. The Welland Canal has 27 locks, giving a total lift from Lake Ontario to Lake Erie of 326.75 feet. Find the average lift.

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17. Find the freight charges on 136,000 pounds of farm machinery shipped from Chicago to Kansas City at 30¢ per 100 pounds.

18. Find the cost of transporting a stationary engine with boiler, weight 45,000 pounds, from Galveston to Denver at 74¢ per 100 pounds.

19. A bushel of shelled corn weighs 56 pounds. Find the freight on 8000 bushels of shelled corn, shipped by rail to Toledo at 12¢ per 100 pounds, and thence by water to Buffalo at $3\frac{1}{2}$¢ per 100 pounds.

20. A grain dealer shipped 5 car loads of shelled corn, 212,450 pounds, and 3 car loads of wheat, 189,700 pounds, from Omaha to Chicago. The freight tariff on corn was 12¢, and on wheat 11¢, per 100 pounds. Find the freight charges, including $3 per car for switching.

21. How much must a man pay to ship 700 bags of coffee, averaging 130 pounds each, from New Orleans to Chicago at 25¢ per hundred pounds? from New Orleans to Omaha at 35¢ per hundred? to Denver at 75¢ per hundred? to Memphis at 14¢ per hundred? to Salt Lake City at $1.50 per hundred?

22. The shipments of cotton from Mississippi during one season were as follows:

Shipped to	Bales	Shipped to	Bales
New Orleans	527,598	Alabama mills	2,710
Memphis	284,655	Georgia mills	2,349
New England mills	228,619	Ohio River points	2,208
Mobile	54,791	Virginia mills	2,092
Louisville	53,949	Bayou Sara, La.	1,579
Norfolk	29,345	Canada	1,574
Boston	18,074	Philadelphia	1,543
Pensacola	16,504	Western mills	1,337
North Carolina mills	15,458	Charleston, S.C.	1,335
Interior Alabama points	12,600	St. Louis	1,150
Savannah	11,508	Helena, Ark.	1,104
Baltimore	9,440	South Carolina mills	1,100
Brunswick, Ga.	6,321	Interior Tennessee points	1,040
Baton Rouge	5,185	Tennessee mills	931
New York City	4,806	Other places	1,515

Of this cotton all but 76,942 bales was grown in Mississippi. In addition 21,650 bales of cotton grown in Mississippi were consumed by mills in the state.

How many bales of cotton did Mississippi produce?

23. How many more bales were shipped to New England mills than to North Carolina mills?

24. How much more cotton was shipped to New Orleans and Memphis than to all other places?

25. More than 6 times as many bales were shipped to Norfolk as to New York City. Find how many more.

26. Oct. 1 the reading of Mr. Hardy's gas meter was 15,200 (cubic feet); Jan. 1 it was 22,100; Apr. 1, 29,900; July 1, 34,300; and Oct. 1, a year after he began to use gas, 39,600. Find the cost of gas, for each quarter and for the whole year, at $1.20 per 1000 cubic feet.

27. Find the average cost per month.

28. Each of these heavy lines represents the population of a country. The number of *millions* is written above the line.

France	38.962
Great Britain	41.961
Japan	45.862
Germany	58.549
United States	80.372
Russia	141.000

How much greater or less is the population of the United States than that of each of the other countries?

29. How much does the population of Russia exceed the combined population of France, Great Britain, and Japan?

30. One district of France produced 3,525,000 tons of coal one year, having an average value of $ 3.06 per ton. Find the value of the coal mined.

31. Find the cost at current prices in your home market, of 5 lb. butter, 6 qt. milk, 8 lb. beef, 12 lb. sugar, and 1.75 lb. tea.

Find the cost in Paris at prices equivalent in our money to 50 ¢ per lb. for butter, 10 ¢ per qt. for milk, 40 ¢ per lb. for beef, 10 ¢ per lb. for sugar, and $1.40 per lb. for tea.

32. Recently .32 of the 260,000 seamen employed on British merchant vessels were foreigners. How many were foreigners?

33. If the flour mills near Liverpool are capable of producing each hour 625 sacks of flour of 280 pounds each, how many sacks can they produce in 7 hours? how many pounds?

To how many barrels of 196 pounds is this equal?

34. If the match factories in the Japanese city of Kobé one year manufactured matches to the value of 9,450,000 yen, what was this value, in United States money, a yen being worth 49.8 ¢?

35. Of the 28,000 Germans who left their native land to live across the ocean one year, .93 came to the United States. Find the number of our German immigrants that year.

36. If the sum paid for the steerage passage of 240 Germans sailing from Bremen to New York was \$9708, what was the price of passage per person?

37. One season this country received German toys worth \$4,500,000. If all the toys exported by Germany were worth 3.4 times this amount, find their total value.

38. How much do girls who work in a German doll factory earn in 6 days of 10 hours each, if they receive 3.5¢ per hour?

39. Our sales to the people of Great Britain one year amounted to \$580,322,098, and our purchases from them amounted to \$191,219,295. How much money was required to balance the account and to which country was it paid?

40. During a recent tourist season in Switzerland, 26,569 Germans stayed at Geneva hotels, 9618 Englishmen, 35,114 Swiss, 68,513 Frenchmen, 14,177 Americans, and 23,094 others. How many visitors were there?

41. The value of 15,000,000 pounds of chocolate exported from Switzerland in a year was \$5,100,000. The value of that exported to Great Britain was \$2,125,000; of that exported to the United States, \$578,000. How many pounds were exported to each of these countries?

42. One year 388,500,000 bushels of grain were sent from Russia. Find its value at 64¢ per bushel.

43. India obtained in a season 222,200,000 pounds of tea from the 524,500 acres under tea cultivation. Find to the nearest tenth of a pound the average yield per acre.

44. On a South African ostrich farm of 48 acres there were 5 ostriches per acre. If each bird produced \$28.75 in feathers during the year, what was the income?

45. Find the total length of railways in the world, divided among the continents thus: America, 270,386 miles; Europe, 187,776; Asia, 46,592; Africa, 15,649; Australasia, 16,702.

FACTORS AND DIVISORS

101. 1. How many are 3×4? 6×2? $3 \times 2 \times 2$?

2. Name *two* numbers whose product is 16; 24; *four* numbers whose product is 16; 24.

102. The integers that multiplied together produce a given number are called its **factors**.

103. An integer that will divide a number without leaving a remainder is called an **exact divisor** of the number.

The factors of a number are exact divisors of it.

2, 3, 4, and 6 are exact divisors of 12. *Divisible* means *exactly divisible.*

104. Two or more numbers that are divisible by the same number are said to have a **common divisor**, or a **common factor**.

The greatest number that will exactly divide all of them is called their **greatest common divisor (g. c. d.)**.

105. A number that has no factors except itself and 1 is called a **prime number**.

5, 7, 11, are prime numbers.

106. A number that has other factors than itself and 1 is called a **composite number**.

4, 9, 12, are composite numbers.

107. 1. Tell which of the following are divisible by 2:

2, 4, 5, 8, 10, 13, 17, 18, 20, 21, 42, 50.

2. Which of these numbers are not divisible by 2?

108. A number that is divisible by 2 is called an **even number**; a number that is not divisible by 2 is called an **odd number**.

DIVISIBILITY OF NUMBERS

109. The figures that are used to represent a number are called its **digits**.

The digits of 358 are 3, 5, and 8.

110. Let the student illustrate with numbers each of these useful **tests of divisibility**.

A number is divisible by

2, *if the units' figure is* 2, 4, 6, 8, *or* 0.
5, *if the units' figure is* 5 *or* 0.
3, *if the sum of the digits is divisible by* 3.
9, *if the sum of the digits is divisible by* 9.

EXERCISES

111. By applying the preceding tests tell which of the numbers 2, 5, 3, 9, are exact divisors of :

1. 24	**4.** 225	**7.** 567	**10.** 3705	**13.** 7964
2. 35	**5.** 374	**8.** 654	**11.** 4839	**14.** 8730
3. 72	**6.** 460	**9.** 864	**12.** 7080	**15.** 9828

112. Illustrate these additional tests.

A number is divisible:

1. By **6**, if it is *even* and the sum of its digits is divisible by 3.

2. By **4**, if its *two* right-hand digits are 0's or if the number expressed by them is divisible by 4.

3. By **25**, if its *two* right-hand digits are 0's or if the number expressed by them is divisible by 25.

4. By **8**, if its *three* right-hand digits are 0's or if the number expressed by them is divisible by 8.

5. By **11**, if the *difference* between the *sums* of its *alternate* digits is zero or is divisible by 11.

EXERCISES

113. Find which of the numbers 2, 3, 4, 5, 6, 8, 9, 11, 25, are exact divisors of:

1. 17,418	3. 23,512	5. 210,705	7. 568,392
2. 97,317	4. 58,914	6. 105,372	8. 890,550

9. If an *even* number is divisible by an *odd* number, it is divisible by *twice* that number. Illustrate.

10. An exact divisor of a number is an exact divisor of any number of times that number. Illustrate.

11. An exact divisor of each of two numbers is an exact divisor of their *sum* and of their *difference*. Illustrate.

FACTORING

114. 1. What numbers are exact divisors of 18?

2. If 3 is taken as one of two factors of 18, what is the other factor? How is it found?

In separating a number into two factors, any exact divisor may be taken for one factor and the quotient for the other.

115. The process of separating a number into its factors is called **factoring.**

116. Factors that are prime numbers are called **prime factors.**

2, 2, and 3 are the prime factors of 12.

117. When numbers have no common factor except 1 they are **prime to each other.**

4 and 21 are prime to each other, though neither is a prime number.

118. A small figure written at the right of a number and a little above, to indicate how many times the number occurs as a factor, is called an **exponent.**

In $16 = 2^4$, the 4 is an exponent, indicating that 2 occurs 4 times as a factor of 16; that is, 2^4 means $2 \times 2 \times 2 \times 2$.

WRITTEN EXERCISES

119. **1.** Find the prime factors of 2295.

$$\begin{array}{r|r} 5 & 2295 \\ \hline 9 & 459 \\ \hline 3 & 51 \\ \hline & 17 \end{array}$$

$$2295 = 5 \times 3^3 \times 17$$

Since the units' figure of 2295 is 5, by what number is 2295 divisible? Dividing by 5, we find the other factor of 2295 to be 459.

Since the sum of the digits of 459 is divisible by 9, we divide by 9 and find the other factor to be 51. The sum of the digits of 51 is divisible by 3; then 3 is one factor of 51 and 17 is the other.

Hence the factors of 2295 found are 5, 9, 3, and 17, but the *prime factors* are 5, 3, 3, 3, and 17; that is, $2295 = 5 \times 3^3 \times 17$.

2. Find the prime factors of 7000; of 2880; of 8250.

$$\begin{array}{r|r} 10 & 7000 \\ \hline 10 & 700 \\ \hline 10 & 70 \\ \hline & 7 \end{array} \qquad \begin{array}{r|r} 10 & 2880 \\ \hline 9 & 288 \\ \hline 8 & 32 \\ \hline & 4 \end{array} \qquad \begin{array}{r|r} 10 & 8250 \\ \hline 25 & 825 \\ \hline 3 & 33 \\ \hline & 11 \end{array}$$

$7000 = 2^3 \times 5^3 \times 7$ $\quad 2880 = 5 \times 3^2 \times 2^6$ $\quad 8250 = 5^3 \times 2 \times 3 \times 11$

NOTE. — Do not try divisors greater than half the number to be factored.

3. Factor all the composite numbers from 1 to 100 and make a list of the prime numbers.

Find the *prime factors* of:

4. 144	**7.** 576	**10.** 1050	**13.** 64,640
5. 260	**8.** 891	**11.** 9702	**14.** 30,888
6. 315	**9.** 672	**12.** 4620	**15.** 44,000

16. Find the greatest common divisor of 105 and 231.

SOLUTION. — Factoring, we find that $105 = 3 \times 5 \times 7$ and $231 = 3 \times 7 \times 11$; hence, the *common* divisors of 105 and 231 are 3, 7, and 3×7, or 21, and their *greatest* common divisor is 21.

Find the greatest common divisor of:

17. 24 and 120	**19.** 210 and 350	**21.** 16, 24, and 40
18. 96 and 168	**20.** 352 and 384	**22.** 48, 60, and 96

CANCELLATION

120. Since 8×5 is contained in 16×5 the same number of times that 8 is contained in 16, and since 8 is contained in 16 the same number of times that 1 is contained in 2, it is evident that the factors 5 and 8 may be omitted or *canceled* from *both* dividend and divisor *without changing the quotient.*

121. The process of shortening computations in division by rejecting equal factors from both dividend and divisor is called **cancellation.**

WRITTEN EXERCISES

122. 1. Divide $8 \times 77 \times 15$ by $4 \times 44 \times 20$.

$$\frac{\overset{\cancel{2}}{\cancel{8}} \times \overset{7}{\cancel{77}} \times \overset{3}{\cancel{15}}}{\cancel{4} \times \underset{\underset{2}{\cancel{4}}}{\cancel{44}} \times \underset{4}{\cancel{20}}} = \frac{21}{8} = 2\tfrac{5}{8}$$

We indicate the division by writing $8 \times 77 \times 15$ above a line and $4 \times 44 \times 20$ below it (§ 85).

Since dividing both dividend and divisor by 4 does not change the quotient, the factor 4 is canceled from 8 and 4, leaving 2 in the dividend and 1 (not written) in the divisor.

Similarly, 11 is canceled from 77 and 44; 5 from 15 and 20; and 2 from 2, left in the dividend, and from one of the 4's left in the divisor.

Since the factors now left in the dividend are prime to those left in the divisor, we have 7×3 divided by 2×4, or $\frac{21}{8}$. The quotient is $2\frac{5}{8}$.

NOTE. — When all the factors of the dividend are canceled, the resulting dividend is 1. The same is true of the divisor. When all the factors of *both* dividend and divisor are canceled, the quotient is $\frac{1}{1}$, or 1.

Divide, using cancellation:

2. $14 \times 27 \times 24 \times 80$ by $32 \times 63 \times 45$

3. $25 \times 42 \times 18 \times 54$ by $12 \times 70 \times 30 \times 9$

4. $16 \times 28 \times 72 \times 50$ by $35 \times 64 \times 24 \times 15$

5. $11 \times 81 \times 26 \times 100$ by $39 \times 15 \times 90 \times 22$

6. $5000 \times 810 \times 1750$ by $625 \times 45 \times 30 \times 50$

7. Divide $.25 \times 6.3 \times 35$ by $4.9 \times .12 \times 7.5$.

$$\frac{\cancel{25}\times\overset{\overset{\cancel{3}}{\cancel{9}}}{\cancel{63}}\times\overset{5}{\cancel{35}}\times\overset{5}{\cancel{10}}}{\underset{\cancel{7}}{\cancel{49}}\times\underset{\underset{2}{\cancel{4}}}{\cancel{12}}\times\underset{\cancel{3}}{\cancel{75}}}=\frac{25}{2}=12.5$$

Omitting decimal points from the dividend multiplies it by 100×10, or by 1000. Omitting decimal points from the divisor multiplies it by $10 \times 100 \times 10$, or by 10,000. Consequently, to make both dividend and divisor integral *without changing the quotient*, after omitting decimal points, we must multiply the dividend by 10.

By cancellation the quotient is then found to be $\frac{25}{2}$, or 12.5.

Divide, using cancellation:

8. $3.3 \times .5$ by $.11 \times 12$

9. $7 \times .48$ by 42×1.6

10. $8.8 \times .6$ by $.08 \times 2.4$

11. 2.5×4.2 by $.75 \times 3.5$

12. 9.6×2.8 by $3 \times .064$

13. $25 \times .81$ by 1.25×9

14. $4.5 \times .56 \times 6.8 \times 9.2$ by $.7 \times 9 \times 4.6$

15. $39 \times 144 \times 49 \times 36$ by $156 \times 56 \times 27$

16. $63 \times 33 \times 51 \times 480$ by $27 \times 280 \times 340$

17. $7.92 \times 2.4 \times .035 \times 756$ by $5.04 \times .018 \times 800 \times .33$

18. $432 \times 8.4 \times .72 \times 2.16 \times 48.4$ by $.56 \times 19.8 \times 57.6 \times 16.2$

19. $243 \times 210 \times 147 \times 78 \times 88$ by $162 \times 63 \times 231 \times 32 \times 130$

20. How much will 7500 bricks cost at $8.40 per 1000?

21. A man gave 12 days' service for 24 bushels of potatoes worth 56¢ a bushel. How much money did he earn per day?

22. When eggs are worth 32¢ per dozen, how many dozen eggs will pay for 16 pounds of tea at 40¢ per pound?

23. A miller bought 14,400 feet of lumber at $20 per 1000 feet, and paid for it in flour at $6 a barrel. How many barrels were required?

24. A teamster carted 4 loads of sand daily for 14 days and was allowed $.90 per load for the work. He took his pay in corn at the market price and received 84 bushels. How much was the corn worth per bushel?

FRACTIONS

123. **1.** If any one thing is divided into 8 *equal* parts, what is one part called? What are 3 parts called? 5 parts? 7 parts?

2. One eighth is written $\frac{1}{8}$; three fourths, $\frac{3}{4}$.

Write: three eighths; five eighths; seven twelfths.

3. In the fraction $\frac{5}{6}$, what does 6, the number below the line, show? 5, the number above the line?

124. One or more of the equal parts of a unit is called a **fraction.**

125. The number that shows into how many equal parts the unit is divided is called the **denominator.**

126. The number that shows how many parts form the fraction is called the **numerator.**

127. The numerator and denominator of a fraction are called its **terms.**

128. What are the terms of the fraction $\frac{4}{5}$? Which is the numerator? the denominator?

A fraction is read by reading the numerator (the number of parts) and then the denominator (telling the kind of parts).

Thus, $\frac{4}{5}$ is read, "four fifths"; $\frac{27}{20}$ is read, "twenty-seven twentieths."

129. A number expressed by an integer and a fraction is called a **mixed number.**

The mixed number $8\frac{4}{5}$ is read, "eight *and* four fifths"; \$4.33$\frac{1}{3}$ is read, "four dollars, thirty-three and one third cents."

130. Read the following:

1. $\frac{13}{24}$	4. $\frac{37}{144}$	7. $16\frac{1}{2}$ feet	10. $\$24.37\frac{1}{2}$
2. $\frac{47}{96}$	5. $\frac{79}{160}$	8. $45\frac{3}{4}$ pounds	11. $\$39.66\frac{2}{3}$
3. $\frac{83}{50}$	6. $\frac{167}{300}$	9. $72\frac{5}{8}$ gallons	12. $\$90.49\frac{5}{6}$

WRITTEN EXERCISES

131. Write in figures:

1. Three fortieths.
2. Eight twenty-fifths.
3. Nineteen twentieths.
4. Thirty-five ninetieths.
5. Seventy-five hundredths.
6. Eight and seven twelfths.
7. Eight hundred four-thousandths.
8. Sixty-four one-hundred-seventy-fifths.
9. Six hundred five four-thousand-eight-hundredths.
10. Forty-five dollars eighty-seven and one half cents.
11. Two hundred twenty-five and ninety-eight one-hundred-forty-fourths.

REDUCTION

132. The process of changing the form of *any* number without changing its value is called **reduction**.

133. Reduction of fractions to higher or lower terms.

1. Draw a line and mark it into halves; then into fourths; then into eighths.

$\frac{1}{2} =$ —— fourths $\qquad \frac{1}{2} =$ —— eighths

$\frac{2}{4} =$ —— halves $\qquad \frac{4}{8} =$ —— halves

2. How may the terms of $\frac{2}{4}$ be obtained from those of $\frac{1}{2}$? from those of $\frac{4}{8}$?

3. Which has the larger, or *higher*, terms, $\frac{1}{2}$ or $\frac{2}{4}$? the *lower* terms, $\frac{2}{4}$ or $\frac{4}{8}$? the *lowest* terms, $\frac{4}{8}$, $\frac{2}{4}$, or $\frac{1}{2}$?

134. *Multiplying or dividing both terms of a fraction by the same number does not change its value.*

135. When the terms of a fraction are prime to each other, the fraction is expressed in its **lowest terms**.

EXERCISES

136. 1. Change to twelfths: $\frac{1}{2}$; $\frac{1}{3}$; $\frac{2}{3}$; $\frac{1}{4}$; $\frac{3}{4}$; $\frac{5}{6}$.

2. Change to sixteenths: $\frac{1}{2}$; $\frac{1}{4}$; $\frac{3}{4}$; $\frac{1}{8}$; $\frac{3}{8}$; $\frac{5}{8}$; $\frac{7}{8}$.

3. Change to twenty-fourths: $\frac{1}{2}$; $\frac{2}{3}$; $\frac{3}{4}$; $\frac{5}{6}$; $\frac{7}{8}$; $\frac{11}{12}$.

4. Reduce $\frac{5}{10}$ to halves; $\frac{8}{12}$ to thirds; $\frac{12}{16}$ to fourths.

5. Reduce to lower terms: $\frac{3}{9}$; $\frac{6}{10}$; $\frac{4}{12}$; $\frac{8}{12}$; $\frac{10}{20}$; $\frac{12}{16}$.

6. Reduce to lowest terms: $\frac{6}{8}$; $\frac{6}{12}$; $\frac{8}{16}$; $\frac{12}{18}$; $\frac{10}{20}$; $\frac{16}{24}$.

Express, at sight, each of the following in lowest terms:

7.	$\frac{5}{25}$	$\frac{8}{32}$	$\frac{9}{45}$	$\frac{7}{35}$	$\frac{8}{80}$	$\frac{6}{36}$	$\frac{9}{36}$	$\frac{8}{40}$	$\frac{9}{27}$
8.	$\frac{4}{48}$	$\frac{8}{48}$	$\frac{6}{48}$	$\frac{8}{64}$	$\frac{7}{56}$	$\frac{9}{54}$	$\frac{6}{72}$	$\frac{9}{72}$	$\frac{8}{96}$
9.	$\frac{15}{30}$	$\frac{16}{32}$	$\frac{18}{36}$	$\frac{13}{26}$	$\frac{14}{28}$	$\frac{12}{48}$	$\frac{16}{48}$	$\frac{24}{48}$	$\frac{15}{45}$
10.	$\frac{12}{60}$	$\frac{25}{75}$	$\frac{12}{96}$	$\frac{20}{30}$	$\frac{60}{40}$	$\frac{30}{45}$	$\frac{24}{80}$	$\frac{24}{32}$	$\frac{24}{36}$

WRITTEN EXERCISES

137. 1. Change $\frac{3}{4}$ to ninety-sixths.

$$\frac{3\times 24}{4\times 24}=\frac{72}{96}$$

From § 134 we know that it will not change the value of $\frac{3}{4}$ to multiply both terms by the same number. Since $96 \div 4 = 24$, we must multiply both terms by 24.

Reduce:

2. $\frac{7}{16}$ to 96ths
3. $\frac{15}{18}$ to 90ths
4. $\frac{14}{36}$ to 108ths
5. $\frac{3}{4}$ to 144ths
6. $\frac{7}{8}$ to 136ths
7. $\frac{5}{6}$ to 150ths
8. $\frac{11}{12}$ to 156ths
9. $\frac{15}{16}$ to 208ths
10. $\frac{19}{20}$ to 300ths

11. Reduce $\frac{2}{3}$ and $\frac{5}{12}$ each to a fraction whose denominator is 48; 96; 144; 240; 384; 528.

12. Reduce $\frac{135}{180}$ to its lowest terms.

$$\frac{5)135}{5)180} = \frac{27}{36}$$

$$\frac{9)27}{9)36} = \frac{3}{4}$$

Or, $$\frac{45)135}{45)180} = \frac{3}{4}$$

From § 134 we know that it will not change the value of $\frac{135}{180}$ to divide both terms by the same number. By § 110 we see that 5 and also 9 will exactly divide both terms of $\frac{135}{180}$.

Dividing both terms by 5, $\frac{135}{180} = \frac{27}{36}$; dividing both terms of $\frac{27}{36}$ by 9, $\frac{27}{36} = \frac{3}{4}$.

The terms of $\frac{3}{4}$ are prime to each other; consequently, $\frac{135}{180}$ reduced to lowest terms is equal to $\frac{3}{4}$.

Or we may directly divide both terms of the fraction $\frac{135}{180}$ by the greatest common divisor, 45 (§ 119).

Reduce to lowest terms:

13. $\frac{42}{72}$	**17.** $\frac{144}{192}$	**21.** $\frac{125}{375}$	**25.** $\frac{450}{675}$	**29.** $\frac{375}{1000}$
14. $\frac{64}{80}$	**18.** $\frac{175}{210}$	**22.** $\frac{168}{192}$	**26.** $\frac{320}{768}$	**30.** $\frac{576}{1728}$
15. $\frac{45}{75}$	**19.** $\frac{108}{270}$	**23.** $\frac{270}{432}$	**27.** $\frac{392}{672}$	**31.** $\frac{560}{2240}$
16. $\frac{32}{96}$	**20.** $\frac{240}{640}$	**24.** $\frac{315}{378}$	**28.** $\frac{288}{864}$	**32.** $\frac{1760}{5280}$

138. Reduction of integers or mixed numbers to fractions.

1. Ellen has \$1, Bertha has \$$1\frac{1}{2}$, and Mary has \$2. To how many half dollars is Ellen's money equal? Bertha's? Mary's?

2. It takes George one hour to walk to school and Elmer $1\frac{1}{4}$ hours. How many quarter hours is each boy on the way?

3. Express as fifths: 1; $1\frac{4}{5}$; 2; $2\frac{3}{5}$; 3; $3\frac{2}{5}$.

4. How may an integer be reduced to a fraction having the denominator 2? 3? 4? 5?

5. How may a mixed number be reduced to a fraction?

EXERCISES

139. Reduce:

1. 6 to fifths	**3.** 4 to sixths	**5.** 9 to tenths
2. 8 to fourths	**4.** 6 to eighths	**6.** 7 to twelfths

7. Reduce to eighths: 2; $3\frac{1}{8}$; 6; $5\frac{3}{8}$; 9; $2\frac{5}{8}$; $6\frac{7}{8}$.

Read these mixed numbers rapidly as fractions:

8.	$4\frac{1}{2}$	$3\frac{3}{4}$	$4\frac{1}{3}$	$2\frac{1}{8}$	$3\frac{1}{6}$	$5\frac{1}{5}$	$3\frac{7}{8}$	$5\frac{3}{10}$
9.	$2\frac{3}{5}$	$7\frac{1}{4}$	$5\frac{3}{8}$	$3\frac{2}{5}$	$8\frac{3}{4}$	$6\frac{5}{6}$	$4\frac{3}{5}$	$4\frac{7}{12}$
10.	$4\frac{5}{8}$	$6\frac{4}{5}$	$8\frac{1}{3}$	$6\frac{7}{8}$	$3\frac{2}{3}$	$7\frac{4}{5}$	$5\frac{3}{4}$	$3\frac{4}{15}$
11.	$8\frac{2}{5}$	$9\frac{2}{3}$	$3\frac{5}{6}$	$2\frac{3}{8}$	$5\frac{2}{7}$	$9\frac{5}{8}$	$6\frac{4}{9}$	$2\frac{5}{16}$

WRITTEN EXERCISES

140. **1.** Reduce $48\frac{3}{4}$ to fourths.

$$48 = \frac{192}{4}$$

$$\frac{192}{4} + \frac{3}{4} = \frac{195}{4}$$

How many fourths are there in 1? in 48? How many fourths are there in 192 fourths and 3 fourths, that is, in $\frac{192}{4}$ and $\frac{3}{4}$?

Then how many fourths are there in $48\frac{3}{4}$?

Note. — When the denominator is small, we may do the work *mentally*, writing only the result.

Reduce to a fraction:

2. $28\frac{1}{4}$	**6.** $216\frac{2}{3}$	**10.** $42\frac{7}{10}$	**14.** $29\frac{17}{18}$
3. $32\frac{5}{8}$	**7.** $324\frac{3}{4}$	**11.** $68\frac{5}{12}$	**15.** $56\frac{19}{24}$
4. $84\frac{1}{3}$	**8.** $272\frac{5}{6}$	**12.** $36\frac{14}{15}$	**16.** $75\frac{23}{36}$
5. $96\frac{4}{5}$	**9.** $428\frac{7}{8}$	**13.** $89\frac{11}{16}$	**17.** $94\frac{25}{48}$

141. Reduction of improper fractions to integers or mixed numbers.

1. How many pints of ice cream will serve 2 children, if each eats a half pint? $\frac{2}{2} =$ ——.

To how many pints are 4 half pints equal? $\frac{4}{2} =$ ——.

2. How many ones and how many fourths over are $\frac{5}{4}$? $\frac{7}{4}$? $\frac{9}{4}$? $\frac{11}{4}$? $\frac{13}{4}$? $\frac{15}{4}$?

3. Reduce to a mixed number: $\frac{7}{2}$; $\frac{11}{3}$; $\frac{15}{4}$; $\frac{25}{6}$; $\frac{31}{8}$.

4. Find the value of: $\$\frac{8}{2}$; $\$\frac{5}{2}$; $\$\frac{12}{4}$; $\$\frac{17}{4}$; $\frac{20}{5}$; $\frac{24}{5}$; $\frac{32}{8}$; $\frac{35}{8}$.

142. A fraction *indicates division*, and its **value** is the quotient of the numerator divided by the denominator.

143. A fraction whose numerator is less than its denominator is called a **proper fraction.**

The value of a proper fraction is less than 1.

144. A fraction whose numerator equals or exceeds its denominator is called an **improper fraction.**

The value of an improper fraction is 1 or more than 1.

EXERCISES

145. **1.** Change to an integer: $\frac{8}{4}$; $\frac{9}{3}$; $\frac{10}{5}$; $\frac{12}{4}$; $\frac{18}{6}$; $\frac{20}{5}$; $\frac{24}{12}$.

2. Express as a mixed number: $\frac{7}{2}$; $\frac{9}{4}$; $\frac{11}{3}$; $\frac{14}{5}$; $\frac{13}{6}$; $\frac{16}{3}$; $\frac{19}{6}$; $\frac{21}{4}$; $\frac{23}{5}$; $\frac{25}{6}$; $\frac{35}{8}$.

3. Reduce to an integer or a mixed number: $\frac{12}{2}$; $\frac{13}{3}$; $\frac{16}{4}$; $\frac{18}{5}$; $\frac{24}{6}$; $\frac{25}{5}$; $\frac{29}{6}$; $\frac{32}{8}$; $\frac{35}{8}$; $\frac{40}{4}$; $\frac{43}{6}$; $\frac{45}{8}$.

Read rapidly as integers or mixed numbers:

4.	$\frac{32}{6}$	$\frac{29}{3}$	$\frac{57}{8}$	$\frac{22}{5}$	$\frac{41}{6}$	$\frac{35}{5}$	$\frac{32}{16}$	$\frac{35}{4}$	$\frac{48}{12}$
5.	$\frac{53}{6}$	$\frac{67}{8}$	$\frac{28}{2}$	$\frac{35}{6}$	$\frac{47}{5}$	$\frac{30}{6}$	$\frac{43}{8}$	$\frac{32}{5}$	$\frac{72}{8}$
6.	$\frac{37}{8}$	$\frac{51}{10}$	$\frac{47}{6}$	$\frac{29}{4}$	$\frac{69}{10}$	$\frac{53}{12}$	$\frac{45}{15}$	$\frac{37}{12}$	$\frac{79}{12}$

WRITTEN EXERCISES

146. **1.** Reduce $\frac{254}{8}$ to a mixed number.

$\frac{254}{8} = 254 \div 8 = 31\frac{6}{8} = 31\frac{3}{4}$ How many eighths make 1? How many times does 254 contain 8? How many ones, then, are there in 254 eighths? how many eighths over? Express $\frac{6}{8}$ in its lowest terms. Then $\frac{254}{8} = 31\frac{3}{4}$.

Reduce to an integer or a mixed number:

2. $\frac{87}{3}$	**5.** $\frac{196}{4}$	**8.** $\frac{531}{18}$	**11.** $\frac{515}{25}$	**14.** $\frac{1520}{20}$
3. $\frac{92}{4}$	**6.** $\frac{277}{8}$	**9.** $\frac{296}{24}$	**12.** $\frac{821}{12}$	**15.** $\frac{1360}{24}$
4. $\frac{99}{6}$	**7.** $\frac{650}{3}$	**10.** $\frac{672}{28}$	**13.** $\frac{956}{16}$	**16.** $\frac{3474}{36}$

147. Reduction of decimals to common fractions

1. How many halves are there in .5, that is, in $\frac{5}{10}$?

2. How many fifths are there in .4, that is, in $\frac{4}{10}$?

3. How many fourths are there in .75, that is, in $\frac{75}{100}$?

148. A fraction that expresses tenths, or hundredths, or thousandths, etc., is called a **decimal fraction**, or a **decimal**; all other fractions are called **common fractions**, or simply **fractions**.

.5, .72, .625, and .2683 are decimal fractions; $\frac{4}{5}$, $\frac{15}{16}$, and $\frac{75}{200}$ are common fractions.

WRITTEN EXERCISES

149. 1. Reduce .075 to a common fraction.

$.075 = \frac{75}{1000} = \frac{3}{40}$ Writing the decimal as a common fraction, we have $\frac{75}{1000}$, which being reduced to its lowest terms equals $\frac{3}{40}$.

Reduce each decimal to a common fraction in its lowest terms:

2. .44	8. .0625	14. .15625	20. 53.125
3. .85	9. .3125	15. .21875	21. 65.625
4. .025	10. 20.75	16. .28125	22. 7.8125
5. 4.25	11. .6875	17. .34375	23. 96.875
6. 8.75	12. .5625	18. .40625	24. .09375
7. .008	13. 9.325	19. .53125	25. .00625

150. Reduction of common fractions to decimals.

1. How many *tenths* are there in 1? in $\frac{1}{2}$? in $\frac{1}{5}$? in $\frac{2}{5}$? in $\frac{3}{5}$? in $\frac{4}{5}$? Express each result as a decimal.

2. Reduce to *hundredths*, as in § 137, and write each result as a decimal: $\frac{1}{4}$; $\frac{3}{4}$.

3. Reduce to *thousandths:* $\frac{1}{8}$; $\frac{3}{8}$; $\frac{5}{8}$; $\frac{7}{8}$. Express each result as a decimal.

WRITTEN EXERCISES

151. 1. Reduce $\frac{7}{8}$ to a decimal.

$$\begin{array}{r} 8\overline{)7.000} \\ \hline .875 \end{array}$$

Since the value of a fraction is the quotient of its numerator divided by its denominator, we *annex decimal 0's to the numerator and divide by the denominator*, as in the margin.

2. Reduce $\frac{5}{6}$ to a decimal.

$$\begin{array}{r} 6\overline{)5.00} \\ \hline .83\frac{1}{3} \end{array}$$

It is evident that the division is not exact, but such a result as $.83\frac{1}{3}$ is often a convenient one to use, when the terms of the fractional part are small.

3. Reduce $\frac{20}{27}$ to a five-place decimal.

$$\begin{array}{r} .74074^{+} \\ 27\overline{)20.0000} \\ 18\,9 \\ \hline 1\,10 \\ 1\,08 \\ \hline 200 \end{array}$$

After finding the first three figures of the quotient, it is seen that the new dividend is like the original dividend. Then the next three figures will be like the first three, and so on.

Hence, $\frac{20}{27}$ reduced to a five-place decimal is .74074+.

NOTE. — A common fraction in its *lowest terms* cannot be reduced to an exact decimal value, if its denominator contains any prime factor besides 2 and 5; for annexing a 0 to the numerator multiplies it by 10, introducing *only* the factors 2 and 5.

Reduce to decimals, expressing each in the form best adapted, and carrying none farther than six places:

4. $\frac{1}{8}$	11. $\frac{4}{7}$	18. $\frac{7}{40}$	25. $\frac{3}{32}$	32. $62\frac{1}{4}$
5. $\frac{3}{8}$	12. $\frac{5}{9}$	19. $\frac{84}{48}$	26. $\frac{5}{64}$	33. $3.4\frac{1}{2}$
6. $\frac{1}{6}$	13. $\frac{5}{12}$	20. $\frac{15}{16}$	27. $\frac{17}{40}$	34. $.85\frac{7}{11}$
7. $\frac{5}{8}$	14. $\frac{8}{15}$	21. $\frac{21}{25}$	28. $\frac{25}{36}$	35. $.00\frac{5}{16}$
8. $\frac{2}{3}$	15. $\frac{9}{16}$	22. $\frac{11}{24}$	29. $\frac{21}{80}$	36. $.084\frac{1}{8}$
9. $\frac{4}{3}$	16. $\frac{8}{11}$	23. $\frac{47}{30}$	30. $\frac{51}{42}$	37. $.046\frac{1}{4}$
10. $\frac{1}{9}$	17. $\frac{3}{14}$	24. $\frac{24}{25}$	31. $\frac{75}{36}$	38. $.176\frac{3}{4}$

152. Reduction to least common denominator.

1. Name some numbers that are divisible by 2; by 5.

2. What is the smallest number that is divisible by each of these numbers?

153. A number that is divisible by another number is a **multiple** of that number.

154. A number that is divisible by each of two or more numbers is a **common multiple** of those numbers.

155. The least number that is divisible by each of two or more numbers is their **least common multiple**.

156. 1. Change $\frac{1}{2}$ and $\frac{1}{5}$ each to tenths; to twentieths; to some other *common* denominator.

Which common denominator is the smallest, or *least?*

2. What is the least number that will exactly contain the denominator of each of the fractions $\frac{2}{3}$ and $\frac{1}{4}$?

Then what is the least common denominator to which these fractions can be reduced?

Reduce $\frac{2}{3}$ and $\frac{1}{4}$ each to twelfths.

157. Fractions that have the same denominator are said to have a **common denominator**, and are called **similar fractions**.

Those that do not have a common denominator are called **dissimilar fractions.**

158. Dissimilar fractions are reduced to similar fractions by changing them to fractions whose common denominator is *any* common multiple of the given denominators.

When the common denominator selected is the *least* common multiple of the given denominators, the fractions are said to be reduced to their **least common denominator**, provided each of the given dissimilar fractions was in its lowest terms.

EXERCISES

159. Reduce to similar fractions:

1. $\frac{1}{2}$ and $\frac{1}{4}$	**5.** $\frac{1}{2}$ and $\frac{3}{4}$	**9.** $\frac{2}{3}$ and $\frac{5}{6}$	**13.** $\frac{3}{4}$ and $\frac{2}{5}$
2. $\frac{1}{3}$ and $\frac{1}{6}$	**6.** $\frac{3}{4}$ and $\frac{1}{8}$	**10.** $\frac{1}{3}$ and $\frac{5}{9}$	**14.** $\frac{1}{6}$ and $\frac{1}{9}$
3. $\frac{1}{2}$ and $\frac{1}{6}$	**7.** $\frac{1}{2}$ and $\frac{7}{8}$	**11.** $\frac{3}{4}$ and $\frac{1}{6}$	**15.** $\frac{3}{5}$ and $\frac{3}{6}$
4. $\frac{1}{3}$ and $\frac{1}{9}$	**8.** $\frac{1}{3}$ and $\frac{5}{6}$	**12.** $\frac{2}{3}$ and $\frac{3}{8}$	**16.** $\frac{3}{8}$ and $\frac{1}{6}$

Reduce to fractions having the least common denominator:

17. $\frac{3}{4}, \frac{1}{2}, \frac{5}{8}$	**21.** $\frac{3}{4}, \frac{5}{6}, \frac{1}{12}$	**25.** $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$
18. $\frac{1}{2}, \frac{3}{5}, \frac{7}{10}$	**22.** $\frac{1}{3}, \frac{3}{5}, \frac{4}{15}$	**26.** $\frac{1}{3}, \frac{1}{2}, \frac{1}{6}$
19. $\frac{2}{3}, \frac{1}{4}, \frac{5}{12}$	**23.** $\frac{3}{4}, \frac{5}{8}, \frac{7}{16}$	**27.** $\frac{3}{4}, \frac{5}{6}, \frac{2}{3}$
20. $\frac{1}{2}, \frac{1}{8}, \frac{1}{16}$	**24.** $\frac{1}{2}, \frac{3}{4}, \frac{5}{16}$	**28.** $\frac{2}{3}, \frac{1}{4}, \frac{2}{5}$

WRITTEN EXERCISES

160. **1.** Reduce $\frac{5}{6}$, $\frac{3}{8}$, and $\frac{7}{12}$ to similar fractions.

$$\frac{5\times 4}{6\times 4}=\frac{20}{24}$$

$$\frac{3\times 3}{8\times 3}=\frac{9}{24}$$

$$\frac{7\times 2}{12\times 2}=\frac{14}{24}$$

By examining the denominators, 6, 8, and 12, we discover that 24 is a common multiple of them. Consequently, the fractions will be made similar, if we reduce them to fractions whose common denominator is 24.

Hence, by multiplying the terms of $\frac{5}{6}$ by 4, of $\frac{3}{8}$ by 3, and of $\frac{7}{12}$ by 2, we reduce the given fractions to the similar fractions $\frac{20}{24}$, $\frac{9}{24}$, and $\frac{14}{24}$, respectively.

NOTE. — It is desirable to select the *least* common multiple of the denominators for the common denominator.

Reduce to fractions having the least common denominator:

2. $\frac{3}{4}, \frac{7}{8}, \frac{15}{16}$	**6.** $\frac{5}{6}, \frac{5}{8}, \frac{5}{12}$	**10.** $\frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \frac{7}{16}$
3. $\frac{7}{8}, \frac{7}{12}, \frac{7}{24}$	**7.** $\frac{1}{6}, \frac{11}{12}, \frac{17}{24}$	**11.** $\frac{1}{4}, \frac{1}{5}, \frac{1}{10}, \frac{1}{20}$
4. $\frac{11}{12}, \frac{3}{4}, \frac{5}{36}$	**8.** $\frac{4}{5}, \frac{7}{12}, \frac{19}{20}$	**12.** $\frac{3}{4}, \frac{1}{2}, \frac{11}{18}, \frac{17}{36}$
5. $\frac{5}{16}, \frac{5}{8}, \frac{5}{32}$	**9.** $\frac{7}{10}, \frac{11}{20}, \frac{17}{30}$	**13.** $\frac{3}{5}, \frac{4}{15}, \frac{7}{30}, \frac{19}{20}$

161. It is sometimes difficult to discover by inspection the least common multiple of the denominators, that is, the *least common denominator* (l. c. d.).

In such cases the l. c. d. may be found by factoring the denominators, for it is the product of all the *different prime* factors, each factor used the greatest number of times that it occurs in any denominator.

Thus, if the given denominators are 8, 6, and 16, by factoring we find $8 = 2 \times 2 \times 2$; $6 = 2 \times 3$; and $16 = 2 \times 2 \times 2 \times 2$.

Then the factors of the l. c. d. are 2, 2, 2, 2 (the greatest number of 2's found in any denominator) and 3 (the only factor of any of them not already taken).

Hence the l. c. d. $= 2 \times 2 \times 2 \times 2 \times 3 = 48$.

WRITTEN EXERCISES

162. 1. Reduce $\frac{7}{8}$, $\frac{5}{12}$, and $\frac{19}{20}$ to fractions having the least common denominator.

Solution

Factoring the denominators we obtain $8 = 2 \times 2 \times 2$; $12 = 2 \times 2 \times 3$; and $20 = 2 \times 2 \times 5$.

The factors of the l. c. d., then, are 2, 2, 2, 3, and 5.

Consequently the l. c. d. $= 2 \times 2 \times 2 \times 3 \times 5 = 120$, and the fractions become $\frac{105}{120}$, $\frac{50}{120}$, and $\frac{114}{120}$.

Reduce to fractions having the l. c. d.:

2. $\frac{5}{12}$ and $\frac{7}{15}$	9. $\frac{4}{9}$, $\frac{13}{24}$, $\frac{23}{36}$	16. $\frac{5}{6}$, $\frac{14}{15}$, $\frac{9}{20}$, $\frac{29}{30}$
3. $\frac{9}{10}$ and $\frac{9}{16}$	10. $\frac{13}{15}$, $\frac{18}{25}$, $\frac{31}{45}$	17. $\frac{3}{8}$, $\frac{7}{12}$, $\frac{11}{36}$, $\frac{23}{24}$
4. $\frac{13}{18}$ and $\frac{17}{24}$	11. $\frac{27}{48}$, $\frac{8}{15}$, $\frac{17}{10}$	18. $\frac{8}{21}$, $\frac{10}{12}$, $\frac{4}{28}$, $\frac{13}{42}$
5. $\frac{11}{12}$ and $\frac{23}{30}$	12. $\frac{19}{20}$, $\frac{5}{18}$, $\frac{11}{24}$	19. $\frac{5}{12}$, $\frac{7}{16}$, $\frac{19}{24}$, $\frac{15}{6}$
6. $\frac{27}{40}$ and $\frac{14}{15}$	13. $\frac{9}{72}$, $\frac{24}{64}$, $\frac{17}{96}$	20. $\frac{11}{30}$, $\frac{24}{45}$, $\frac{49}{60}$, $\frac{23}{120}$
7. $\frac{5}{24}$ and $\frac{5}{16}$	14. $\frac{11}{12}$, $\frac{7}{15}$, $\frac{5}{18}$	21. $\frac{3}{20}$, $\frac{11}{60}$, $\frac{31}{30}$, $\frac{19}{150}$
8. $\frac{7}{32}$ and $\frac{7}{12}$	15. $\frac{4}{21}$, $\frac{11}{12}$, $\frac{13}{14}$	22. $\frac{17}{10}$, $\frac{9}{40}$, $\frac{11}{24}$, $\frac{13}{240}$

ADDITION AND SUBTRACTION

163. 1. How many fourths are 3 fourths and 1 fourth? 3 fourths less 1 fourth?

How many ones are 4 fourths? How many halves are 2 fourths?

$\frac{3}{4}+\frac{1}{4}=$ ——. $\frac{3}{4}-\frac{1}{4}=$ ——.

2. Can you add the fractions $\frac{3}{4}$ and $\frac{2}{3}$ without changing them? Can you subtract $\frac{2}{3}$ from $\frac{3}{4}$?

Reduce $\frac{3}{4}$ and $\frac{2}{3}$ to similar fractions.

$\frac{3}{4}+\frac{2}{3}=\frac{9}{12}+\frac{8}{12}=$ ——. $\frac{3}{4}-\frac{2}{3}=$ —— – —— = ——.

164. *Fractions must be made similar before they can be added or subtracted.*

EXERCISES

165. Add or subtract, as the signs indicate:

1. $\frac{2}{5}+\frac{2}{5}$	5. $\frac{3}{4}-\frac{3}{8}$	9. $\frac{3}{5}+\frac{1}{2}$	13. $\frac{1}{12}+\frac{2}{3}$
2. $\frac{7}{8}-\frac{3}{8}$	6. $\frac{5}{6}+\frac{5}{2}$	10. $\frac{3}{4}-\frac{2}{5}$	14. $\frac{5}{12}+\frac{1}{4}$
3. $\frac{5}{6}-\frac{1}{2}$	7. $\frac{7}{8}-\frac{1}{4}$	11. $\frac{1}{6}+\frac{1}{4}$	15. $\frac{7}{12}-\frac{1}{2}$
4. $\frac{1}{2}+\frac{3}{4}$	8. $\frac{3}{4}+\frac{5}{8}$	12. $\frac{5}{8}-\frac{1}{3}$	16. $\frac{11}{12}-\frac{5}{6}$

Add; then subtract:

17. $5\frac{1}{3}$	18. $9\frac{5}{8}$	19. $7\frac{3}{4}$	20. $4\frac{5}{6}$	21. $7\frac{3}{4}$
$\underline{2\frac{1}{6}}$	$\underline{4\frac{1}{2}}$	$\underline{3\frac{1}{2}}$	$\underline{2\frac{2}{3}}$	$\underline{4\frac{5}{12}}$
22. $6\frac{4}{5}$	23. $8\frac{1}{4}$	24. $5\frac{2}{3}$	25. $8\frac{1}{2}$	26. $9\frac{1}{2}$
$\underline{1\frac{1}{2}}$	$\underline{3\frac{1}{8}}$	$\underline{2\frac{1}{2}}$	$\underline{4\frac{1}{3}}$	$\underline{6\frac{3}{10}}$
27. $8\frac{5}{6}$	28. $5\frac{7}{8}$	29. $6\frac{3}{5}$	30. $7\frac{3}{4}$	31. $5\frac{2}{3}$
$\underline{6\frac{3}{4}}$	$\underline{4\frac{1}{2}}$	$\underline{3\frac{1}{4}}$	$\underline{5\frac{1}{6}}$	$\underline{2\frac{7}{12}}$

32. If high-grade olive oil made in France costs $\$1\frac{3}{5}$ per gallon to produce and sell in this country, find the profit on one gallon when it is sold at $\$2\frac{1}{10}$.

33. The United States military rifle weighs $9\frac{1}{2}$ pounds, the 100 cartridges accompanying it $6\frac{2}{5}$ pounds. When a soldier carries rifle and cartridges, how many pounds is he carrying?

34. If the price of pineapples per crate was $\$2\frac{1}{2}$ in season and $\$3\frac{1}{4}$ more at Christmas, what was the price at Christmas?

35. A steamship can cross the Atlantic in $5\frac{1}{2}$ days. The winning yacht in a race recently took $6\frac{5}{6}$ days longer to cover the same distance. How long a time did the yacht take?

36. How much does it cost to transport a bushel of wheat from Chicago to New York by water, if the charge to Buffalo is $1\frac{3}{4}$ ¢, the elevator charge $\frac{1}{2}$ ¢, and the charge from Buffalo to New York $2\frac{1}{2}$ ¢?

WRITTEN EXERCISES

166. **1.** Add $4\frac{2}{5}$, $5\frac{1}{2}$, and $9\frac{3}{4}$.

$$\begin{aligned} 4\tfrac{2}{5} &= 4\tfrac{8}{20} \\ 5\tfrac{1}{2} &= 5\tfrac{10}{20} \\ 9\tfrac{3}{4} &= 9\tfrac{15}{20} \\ \hline &\ \ 19\tfrac{13}{20} \end{aligned}$$

When the fractions are reduced to fractions having the l. c. d., they become $\frac{8}{20}$, $\frac{10}{20}$, and $\frac{15}{20}$, whose sum is $\frac{33}{20}$, or $1\frac{13}{20}$. Writing $\frac{13}{20}$ under the fractions and adding the 1 to the integers, we find that the whole sum is $19\frac{13}{20}$.

Find the sum of:

2. $\frac{3}{4}, \frac{5}{8}, \frac{1}{3}, \frac{11}{24}$	**5.** $\frac{7}{10}, \frac{4}{15}, \frac{23}{30}, \frac{17}{20}$	**8.** $5\frac{1}{2}, 7\frac{5}{6}, 9\frac{4}{15}$
3. $\frac{5}{6}, \frac{4}{9}, \frac{7}{18}, 1\frac{3}{4}$	**6.** $\frac{8}{15}, \frac{11}{12}, 4\frac{1}{3}, \frac{15}{36}$	**9.** $7\frac{19}{20}, 6\frac{14}{15}, 8\frac{7}{12}$
4. $\frac{3}{5}, \frac{7}{10}, \frac{1}{6}, 1\frac{2}{3}$	**7.** $\frac{19}{30}, \frac{14}{15}, 6\frac{3}{4}, \frac{17}{20}$	**10.** $4\frac{15}{24}, 5\frac{15}{16}, 2\frac{14}{32}$

Add:

11.	**12.**	**13.**	**14.**	**15.**
$24\frac{1}{3}$	$46\frac{3}{5}$	$20\frac{3}{4}$	$14\frac{7}{8}$	$84\frac{7}{10}$
$30\frac{1}{6}$	$38\frac{2}{3}$	$44\frac{5}{8}$	$21\frac{3}{4}$	$66\frac{4}{15}$
$48\frac{1}{2}$	$17\frac{11}{15}$	$19\frac{2}{3}$	$60\frac{7}{16}$	$42\frac{9}{20}$

16. From $9\frac{5}{8}$ subtract $3\frac{5}{6}$.

$$\begin{array}{rcrcr} 9\frac{5}{8} & = & 9\frac{15}{24} & = & 8\frac{39}{24} \\ \underline{3\frac{5}{6}} & = & 3\frac{20}{24} & = & \underline{3\frac{20}{24}} \\ & & & & 5\frac{19}{24} \end{array}$$

After reducing the fractions to similar fractions, it is seen that $\frac{20}{24}$ cannot be subtracted from $\frac{15}{24}$; hence 1 is taken from the 9, changed to twenty-fourths, and combined with $\frac{15}{24}$; then $\frac{20}{24}$ is subtracted from $\frac{39}{24}$ and 3 from 8, leaving $5\frac{19}{24}$.

Find the difference between :

17. $5\frac{3}{8}$ and $2\frac{3}{4}$

18. $4\frac{1}{2}$ and $1\frac{4}{5}$

19. $9\frac{2}{3}$ and $7\frac{1}{6}$

20. $\frac{7}{24}$ and $\frac{5}{48}$

21. $\frac{15}{40}$ and $\frac{3}{16}$

22. $\frac{12}{15}$ and $\frac{18}{75}$

23. 24 and $6\frac{1}{4}$

24. $3\frac{5}{6}$ and $1\frac{3}{8}$

25. $6\frac{4}{5}$ and $3\frac{2}{3}$

26. $\frac{14}{15}$ and $\frac{7}{12}$

27. $\frac{37}{48}$ and $\frac{17}{96}$

28. $\frac{63}{75}$ and $\frac{36}{50}$

Add; then subtract:

29. $\begin{array}{r} 42\frac{3}{4} \\ \underline{22\frac{1}{2}} \end{array}$ **30.** $\begin{array}{r} 28\frac{3}{8} \\ \underline{15\frac{2}{3}} \end{array}$ **31.** $\begin{array}{r} 6.4\frac{1}{3} \\ \underline{4.6\frac{2}{5}} \end{array}$ **32.** $\begin{array}{r} 14.9\frac{7}{8} \\ \underline{3.6\frac{1}{2}} \end{array}$ **33.** $\begin{array}{r} 26\frac{5}{8} \\ \underline{11\frac{7}{16}} \end{array}$

34. $\begin{array}{r} 91\frac{4}{5} \\ \underline{75\frac{1}{4}} \end{array}$ **35.** $\begin{array}{r} 89\frac{5}{8} \\ \underline{26\frac{5}{6}} \end{array}$ **36.** $\begin{array}{r} 3.4\frac{2}{3} \\ \underline{2.5\frac{3}{8}} \end{array}$ **37.** $\begin{array}{r} 4.67\frac{1}{4} \\ \underline{2.49\frac{1}{3}} \end{array}$ **38.** $\begin{array}{r} 46\frac{7}{12} \\ \underline{13\frac{5}{24}} \end{array}$

39. $\begin{array}{r} 44\frac{3}{9} \\ \underline{26\frac{1}{3}} \end{array}$ **40.** $\begin{array}{r} 92\frac{3}{4} \\ \underline{66\frac{2}{3}} \end{array}$ **41.** $\begin{array}{r} 8.4\frac{1}{4} \\ \underline{2.7\frac{5}{6}} \end{array}$ **42.** $\begin{array}{r} 8.90\frac{4}{5} \\ \underline{3.89\frac{5}{6}} \end{array}$ **43.** $\begin{array}{r} 77\frac{13}{25} \\ \underline{29\frac{11}{50}} \end{array}$

Add :

44. $\begin{array}{r} 29\frac{2}{3} \\ 18\frac{1}{4} \\ 72\frac{5}{6} \\ \underline{46\frac{1}{2}} \end{array}$ **45.** $\begin{array}{r} 42\frac{5}{8} \\ 63\frac{1}{3} \\ 24\frac{3}{4} \\ \underline{91\frac{1}{6}} \end{array}$ **46.** $\begin{array}{r} 1.4\frac{3}{5} \\ 7.6\frac{1}{6} \\ 9.4\frac{1}{3} \\ \underline{2.3\frac{1}{2}} \end{array}$ **47.** $\begin{array}{r} 3.33\frac{1}{3} \\ 5.87\frac{1}{2} \\ 7.83\frac{5}{6} \\ \underline{9.66\frac{2}{3}} \end{array}$ **48.** $\begin{array}{r} 34\frac{5}{12} \\ 69\frac{3}{10} \\ 57\frac{9}{20} \\ \underline{40\frac{11}{15}} \end{array}$

49. Add $6.12\frac{1}{2}$, $8.4\frac{3}{4}$, and $9.06\frac{1}{4}$.

Suggestion. — Write $6.12\frac{1}{2}$, $8.47\frac{1}{2}$, and $9.06\frac{1}{4}$, or 6.125, 8.475, and 9.0625.

50. From the sum of $6.6\frac{2}{3}$ and $.83\frac{5}{6}$ subtract $.333\frac{1}{3}$.

51. Add 2.7, $49\frac{1}{4}$, $.37\frac{1}{2}$, $2.5\frac{1}{4}$, 36.094, and $\frac{9}{25}$.

WRITTEN EXERCISES

167. 1. In a recent year the average load of freight per train was $140\frac{4}{5}$ tons in Great Britain and $287\frac{2}{5}$ tons in the United States. How much more did a freight-train load in this country weigh?

2. Find the difference in height between a locomotive wheel $6\frac{1}{6}$ feet high and a freight-car wheel $2\frac{3}{4}$ feet high.

3. It took a steam locomotive $3\frac{2}{5}$ minutes and an electric locomotive $2\frac{1}{8}$ minutes to attain a speed of fifty miles per hour. In how much shorter time did the electric locomotive attain that speed?

4. The cost per mile of running a suburban train by steam was: coal, $14\frac{1}{4}$¢; water, $\frac{1}{2}$¢; crew, $12\frac{1}{4}$¢; maintenance, $6\frac{1}{2}$¢; supplies, $\frac{5}{8}$¢. The cost with electrical equipment was: electric power, $10\frac{3}{8}$¢; crew, $6\frac{5}{8}$¢; maintenance, 4¢; supplies, $\frac{1}{4}$¢. Find the saving per mile with electricity.

5. Find the cost of a coal miner's outfit, as follows: drilling machine for coal, \$$8\frac{1}{4}$; drilling machine for rock, \$$8\frac{3}{5}$; pick, \$$\frac{3}{4}$; shovel, \$$\frac{1}{2}$; drill, \$$1\frac{2}{5}$; needle, \$$\frac{1}{4}$; scraper, \$$\frac{1}{4}$; ax, \$1; saw, \$$\frac{3}{4}$.

6. The United States 12-inch naval gun weighs $53\frac{3}{4}$ tons, the German gun of the same bore, $48\frac{7}{8}$ tons. How much less does the German gun weigh than the American?

7. The equipment of an American soldier in war time weighs $55\frac{1}{4}$ pounds. If the British soldier carries $3\frac{1}{8}$ pounds less and the German soldier $3\frac{1}{2}$ pounds more than the American, what is the weight of each one's equipment?

8. Following is the amount of the principal animal food annually consumed per person in a certain town: $19\frac{1}{2}$ pounds butter, $3\frac{1}{4}$ lb. ready-cooked meat, $34\frac{7}{8}$ lb. fish, $154\frac{3}{4}$ lb. beef, $25\frac{1}{2}$ lb. pork, $27\frac{5}{8}$ lb. fowl and game. Find the total amount.

MULTIPLICATION

168. Multiplication of integers by fractions.

1. $12 = 3 + 3 + 3 + 3$. $12 = 2 + 2 + 2 + 2 + 2 + 2$.

What is 1 of the 4 equal parts of 12, or $\frac{1}{4}$ of 12?

Find $\frac{3}{4}$ of 12. Find $\frac{1}{6}$ of 12, then $\frac{5}{6}$ of 12.

2. Find $\frac{2}{5}$ of 50; $\frac{1}{5}$ of 100. Compare the results.

How does 2 times $\frac{1}{5}$ of 50 compare with $\frac{1}{5}$ of 2 times 50?

Tell two ways of finding $\frac{2}{5}$ of 50, or of *multiplying* 50 by $\frac{2}{5}$.

3. Find $\frac{2}{5}$ of 8 by finding $\frac{1}{5}$ of 2 times 8. Find $\frac{2}{5}$ of 35 by finding $\frac{1}{5}$ of 2 times 35; also find $\frac{2}{5}$ of 35 *in a shorter way.*

4. Find $\frac{2}{3}$ of 11; $\frac{2}{3}$ of 21; $\frac{4}{5}$ of 7; $\frac{4}{5}$ of 45.

169. Finding a fractional part of a number is called **multiplying by a fraction.**

In using a fraction as a multiplier, we *multiply* by the *numerator* and divide by the *denominator*.

We may perform these two operations in either order; but it is better to divide first when the multiplicand exactly contains the denominator.

EXERCISES

170. Find:

1. $\frac{1}{3}$ of 12	**4.** $\frac{1}{4}$ of 20	**7.** $\frac{1}{5}$ of 30	**10.** $\frac{4}{5}$ of 15
2. $\frac{2}{3}$ of 12	**5.** $\frac{3}{4}$ of 20	**8.** $\frac{2}{5}$ of 30	**11.** $\frac{4}{5}$ of 11
3. $\frac{4}{3}$ of 12	**6.** $\frac{9}{4}$ of 20	**9.** $\frac{3}{5}$ of 12	**12.** $\frac{3}{4}$ of 13

Multiply:

13. 24 by $\frac{1}{6}$	**17.** 10 by $\frac{4}{7}$	**21.** 11 by $\frac{5}{8}$	**25.** 14 by $\frac{5}{2}$
14. 20 by $\frac{1}{6}$	**18.** 12 by $\frac{3}{8}$	**22.** 96 by $\frac{7}{8}$	**26.** 14 by $\frac{5}{3}$
15. 20 by $\frac{5}{6}$	**19.** 33 by $\frac{1}{8}$	**23.** 36 by $\frac{3}{4}$	**27.** 90 by $\frac{3}{10}$
16. 10 by $\frac{4}{5}$	**20.** 40 by $\frac{5}{8}$	**24.** 54 by $\frac{5}{6}$	**28.** 12 by $\frac{7}{10}$

29. If the average coal miner works $\frac{2}{3}$ of a month of 30 days, how many days during the month does he work?

30. A recipe for fudge calls for $\frac{1}{4}$ of a cake of chocolate. If a cake costs 20¢, find the cost of the chocolate called for by the recipe.

31. In target practice the battleship *Indiana* shot at a target 24 times. If $\frac{3}{4}$ of the shots hit, how many successful shots were fired?

32. A collection of mail that required just 6 hours for a postman to make with a horse and wagon was made in an automobile in $\frac{5}{12}$ the time. How long did the automobile take?

33. How many corks per day does a machine in Spain make from the bark of the cork tree, if it makes $\frac{1}{3}$ of a sack of 15,000 corks in that time?

WRITTEN EXERCISES

171. **1.** Find $\frac{5}{6}$ of 42; also $\frac{5}{6}$ of 22.

$\frac{5}{6}$ of $42 = 5$ times $\frac{1}{6}$ of 42

$$\begin{array}{r} 6\,)\,\underline{42} \\ 7 \\ \underline{5} \\ 35 \end{array}$$

Since 6 is exactly contained in 42, in finding $\frac{5}{6}$ of 42 it is easier and shorter to divide by the denominator before multiplying by the numerator, obtaining $\frac{1}{6}$ of 42, then $\frac{5}{6}$ of 42.

$\frac{5}{6}$ of $22 = \frac{1}{6}$ of 5 times 22

$$\begin{array}{r} 22 \\ \underline{5} \\ 6\,)\,\underline{110} \\ 18\frac{1}{3} \end{array} \quad \text{Or,} \quad \frac{5 \times \overset{11}{\cancel{22}}}{\underset{3}{\cancel{6}}} = \frac{55}{3} = 18\tfrac{1}{3}$$

Since 6 is not exactly contained in 22, it is easier and shorter to multiply before dividing. When the integer and the denominator contain a common factor, it may be canceled.

Find in the shortest way:

2. $\frac{2}{3}$ of 87	**5.** $\frac{3}{5}$ of 120	**8.** $\frac{5}{8}$ of 49	**11.** $\frac{5}{9}$ of 1080
3. $\frac{4}{5}$ of 65	**6.** $\frac{5}{6}$ of 100	**9.** $\frac{7}{8}$ of 75	**12.** $\frac{3}{7}$ of 2800
4. $\frac{3}{4}$ of 76	**7.** $\frac{3}{8}$ of 216	**10.** $\frac{4}{7}$ of 91	**13.** $\frac{7}{9}$ of 1000

Multiply, using cancellation when possible:

14. 54 by $\frac{5}{6}$	**18.** 132 by $\frac{3}{8}$	**22.** 17.6 by $\frac{7}{8}$
15. 72 by $\frac{3}{4}$	**19.** 244 by $\frac{4}{5}$	**23.** 4.96 by $\frac{3}{4}$
16. 96 by $\frac{5}{9}$	**20.** 100 by $\frac{5}{8}$	**24.** .625 by $\frac{3}{5}$
17. 75 by $\frac{5}{6}$	**21.** 333 by $\frac{2}{9}$	**25.** 9.94 by $\frac{3}{4}$

26. Multiply 462 by $34\frac{5}{8}$.

$$\begin{array}{rl} 462 & \\ \underline{34\frac{5}{8}} & \\ 8\overline{)2310} & \\ 288\frac{3}{4} & \text{, product by } \frac{5}{8} \\ 1848\phantom{\frac{3}{4}} & \text{, product by } 4 \\ \underline{1386\phantom{0\frac{3}{4}}} & \text{, product by } 30 \\ 15996\frac{3}{4} & \text{, product by } 34\frac{5}{8} \end{array} \qquad \begin{array}{rl} 462 & \\ \underline{34\frac{5}{8}} & \\ 231\phantom{\frac{3}{4}} & \text{, product by } \frac{1}{2}\text{, or } \frac{4}{8} \\ 57\frac{3}{4} & \text{, product by } \frac{1}{8} \\ 1848\phantom{\frac{3}{4}} & \text{, product by } 4 \\ \underline{1386\phantom{0\frac{3}{4}}} & \text{, product by } 30 \\ 15996\frac{3}{4} & \text{, product by } 34\frac{5}{8} \end{array}$$

Multiply:

27. 45 by $22\frac{3}{4}$	**32.** 18.6 by $22\frac{1}{2}$	**37.** 280 by $.45\frac{1}{2}$
28. 36 by $41\frac{2}{5}$	**33.** 43.5 by $16\frac{1}{5}$	**38.** 462 by $.66\frac{2}{3}$
29. 62 by $18\frac{2}{3}$	**34.** 7.38 by $24\frac{1}{3}$	**39.** 57.6 by $.48\frac{3}{4}$
30. 125 by $7\frac{5}{8}$	**35.** 6.45 by $51\frac{2}{3}$	**40.** \$8400 by $1.06\frac{1}{5}$
31. 485 by $6\frac{3}{8}$	**36.** 49.8 by $75\frac{3}{4}$	**41.** \$3750 by $2.11\frac{5}{6}$

42. If $\frac{5}{8}$ of a magazine of 312 pages consists of advertisements, how many pages are left for reading matter?

43. Of the 4475 million communications that passed through the British post offices one year, $\frac{3}{5}$ were letters. How many letters did the British post offices handle?

44. Railroad statistics one year showed that there were 225,000 passenger coaches in the world, $\frac{2}{3}$ as many locomotives, and $13\frac{1}{3}$ times as many freight cars. Find the number of each.

45. The United States in a recent year turned out 600,000 tons of tin plate. If the Welsh mills manufactured $1\frac{1}{24}$ times as much, find their output for the year.

172. Multiplication of fractions by integers.

1. How many tenths are $\frac{3}{10}+\frac{3}{10}+\frac{3}{10}$, or 3 times $\frac{3}{10}$?

In multiplying $\frac{3}{10}$ by 3 we do not change the denomination, tenths, but *multiply* the number of tenths, or the *numerator*, by 3.

2. Multiply $\frac{3}{10}$ by 5 without changing the denomination.

What additional step must be taken to express the product $\frac{15}{10}$ in the simplest form, $\frac{3}{2}$?

3. Since 5 times $\frac{1}{10}=\frac{5}{10}$, or $\frac{1}{2}$, how many *halves* are there in 5 times $\frac{3}{10}$?

In multiplying $\frac{3}{10}$ by 5 we do not change the number of parts, but increase each part by *dividing* the *denominator* by 5.

173. *A fraction may be multiplied by an integer by multiplying the numerator or by dividing the denominator by the integer.*

The second method should be used whenever the denominator exactly contains the multiplier.

EXERCISES

174. Multiply and explain each multiplication:

1. $\frac{2}{5}$ by 2	**4.** $\frac{7}{8}$ by 4	**7.** $\frac{7}{10}$ by 5	**10.** $\frac{11}{15}$ by 5
2. $\frac{3}{4}$ by 2	**5.** $\frac{2}{9}$ by 2	**8.** $\frac{5}{12}$ by 6	**11.** $\frac{7}{18}$ by 6
3. $\frac{5}{6}$ by 3	**6.** $\frac{3}{5}$ by 7	**9.** $\frac{3}{16}$ by 4	**12.** $\frac{5}{12}$ by 5

13. Find 2 times 3; 2 times $\frac{1}{4}$; 2 times $3\frac{1}{4}$.

14. Multiply $\frac{2}{3}$ by 3; by 6; by 9; by 12.

Multiply:

15. $4\frac{1}{2}$ by 2	**17.** $4\frac{3}{4}$ by 8	**19.** $8\frac{1}{3}$ by 3	**21.** $9\frac{1}{10}$ by 5
16. $3\frac{1}{2}$ by 4	**18.** $2\frac{2}{3}$ by 6	**20.** $6\frac{1}{4}$ by 8	**22.** $4\frac{5}{12}$ by 6

23. If shoes for a work horse weigh $1\frac{3}{4}$ pounds each, how many pounds does a set of 4 shoes weigh?

24. It costs about $1\frac{1}{3}$¢ to make a pound of paper out of cornstalks and 3 times as much to make a pound out of rags. Find the cost of making a pound of paper out of rags.

25. Find the cost of 2 armored cruisers for our navy at $6\frac{1}{2}$ million dollars each; of 3 scout cruisers at $2\frac{1}{5}$ million dollars.

26. A Vermont farmer had 4 sugar maple trees in his yard. Each yielded enough sap to make $2\frac{1}{2}$ pounds of sugar. How many pounds of maple sugar did he obtain?

27. The market price of sugar was $\$\frac{1}{8}$ per pound. Find the value of the farmer's sugar at the market price.

WRITTEN EXERCISES

175. 1. Multiply $\frac{8}{15}$ by 10.

$$10 \times \frac{8}{15} = \frac{\overset{2}{\cancel{10}} \times 8}{\underset{3}{\cancel{15}}} = \frac{16}{3} = 5\tfrac{1}{3}$$

Multiply:

2. $\frac{9}{16}$ by 12	**6.** $\frac{3}{28}$ by 21	**10.** $\frac{9}{20}$ by 36
3. $\frac{3}{20}$ by 15	**7.** $\frac{11}{36}$ by 27	**11.** $\frac{13}{50}$ by 20
4. $\frac{4}{15}$ by 18	**8.** $\frac{24}{35}$ by 42	**12.** $\frac{21}{100}$ by 75
5. $\frac{7}{24}$ by 32	**9.** $\frac{5}{72}$ by 27	**13.** $\frac{35}{144}$ by 84

14. Multiply $.32\frac{3}{4}$ by 18.

$$\begin{array}{r} .32\frac{3}{4} \\ 18 \\ \hline 13\frac{1}{2} \\ 256 \\ 32 \\ \hline 5.89\frac{1}{2} \end{array}$$

18 times $\frac{3}{4}$ of .01 = $.13\frac{1}{2}$.

18 times .32 is found in the usual way by multiplying by 8 and by 1 (ten).

Observe that since 18 times $.00\frac{3}{4}$ and 8 times .32 both give *hundredths*, the right-hand figure of 256 should stand under 3 in hundredths' column.

Multiply:

15. $44\frac{3}{4}$ by 84	**19.** $\$.37\frac{1}{2}$ by 26	**23.** $2.05\frac{1}{2}$ by 37
16. $62\frac{2}{5}$ by 92	**20.** $\$.18\frac{3}{4}$ by 70	**24.** $462\frac{2}{3}$ by 2.4
17. $50\frac{5}{8}$ by 60	**21.** $\$.26\frac{3}{8}$ by 44	**25.** $22.6\frac{2}{3}$ by 12.6
18. $87\frac{1}{2}$ by 50	**22.** $\$.33\frac{1}{3}$ by 54	**26.** $.312\frac{1}{2}$ by 2.56

27. A net for catching salmon has meshes $6\frac{1}{4}$ inches wide. If the net has 24 meshes in its width, how wide is it?

28. The pay of a private soldier in the army is $43\frac{1}{3}$¢ per day. How much does he receive per month of 30 days?

29. A railroad inspection car consumed $.03\frac{1}{3}$ gal. of gasoline per mile. How much did it consume on a trip of 1500 miles?

30. According to the *Scientific American*, a large railroad company imported hard-wood ties at a cost of $\$1\frac{3}{5}$ each. What was the cost of ties in one mile of track, if 2600 were used?

31. The average check for a meal sold in a penny lunch room was $3\frac{3}{4}$¢. Find 1 day's receipts, if 23,500 persons were fed.

32. How long did it take a woman to weave a Persian rug of 18 square feet, if she could weave 1 square foot in $3\frac{5}{6}$ weeks?

33. If the annual yield from one rubber tree is $7\frac{1}{2}$ pounds of rubber, what is the yield from 157 trees?

34. The Stephenson locomotive, one of the first locomotives built, weighed $4\frac{1}{2}$ tons. Find the weight of a modern locomotive that is 22 times as heavy.

35. A postman collected mail from 24 letter boxes. If the weight of mail per box averaged $4\frac{3}{4}$ pounds, how many pounds of mail did he collect?

36. If the average value of a load of diamond-bearing earth in a South African diamond mine is $\$6\frac{3}{4}$, find the value of the 424 loads put into one "washing pan" every twelve hours.

37. A factory is lighted by 50 electric incandescent lamps. If each lamp burns 720 hours per year at a cost of $\frac{13}{20}$¢ per hour, how much does it cost a year to light the factory?

38. The water supply of a town of 1200 inhabitants was pumped $21\frac{3}{4}$ hours every day during June. The consumption of water per inhabitant for every hour the pump worked was 16 gallons How much water was used during June?

176. Multiplication of fractions by fractions.

1. A boy who had $\frac{1}{2}$ of a dollar gave $\frac{1}{2}$ of his money to his sister. What part of a dollar did he give her?

How much is $\dfrac{1}{2}$ of $\dfrac{1}{2}$, or $\dfrac{1}{2} \times \dfrac{1}{2}$? Compare with $\dfrac{1 \times 1}{2 \times 2}$.

2. Find the value of $\frac{1}{3}$ of $\frac{1}{2}$; of $\frac{1}{3} \times \frac{1}{2}$.

Since $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$, how many times $\frac{1}{6}$ is $\frac{2}{3} \times \frac{1}{2}$? Then how many sixths are there in $\frac{2}{3}$ of $\frac{1}{2}$? how many thirds?

How does $\frac{2}{3} \times \frac{1}{2}$ compare in value with $\dfrac{2 \times 1}{3 \times 2}$?

3. How many twelfths are there in $\frac{1}{3}$ of $\frac{1}{4}$? in $\frac{2}{3}$ of $\frac{1}{4}$? How many times $\frac{2}{3}$ of $\frac{1}{4}$ is $\frac{2}{3}$ of $\frac{3}{4}$? Then how many twelfths are there in $\frac{2}{3}$ of $\frac{3}{4}$? in $\frac{2}{3} \times \frac{3}{4}$? how many halves?

How does $\frac{2}{3} \times \frac{3}{4}$ compare in value with $\dfrac{2 \times 3}{3 \times 4}$?

4. Since $\dfrac{1}{2} \times \dfrac{1}{2} = \dfrac{1 \times 1}{2 \times 2} = \dfrac{1}{4}$, $\dfrac{2}{3} \times \dfrac{1}{2} = \dfrac{2 \times 1}{3 \times 2} = \dfrac{1}{3}$, $\dfrac{2}{3} \times \dfrac{3}{4} = \dfrac{2 \times 3}{3 \times 4}$ $= \frac{1}{2}$, etc., how may a fraction be multiplied by a fraction?

177. Finding a fractional part of a fraction is called **multiplying a fraction by a fraction.**

The word "of" between two fractions signifies multiplication, and when the sign $\times$ is used between two fractions it may be read "of."

Such expressions are sometimes called **compound fractions.**

178. To multiply a fraction by a fraction, *multiply the numerators together for the numerator of the product and the denominators for the denominator of the product.*

EXERCISES

179. Multiply:

1. $\frac{1}{3}$ by $\frac{3}{8}$	**3.** $\frac{2}{3}$ by $\frac{3}{5}$	**5.** $\frac{3}{5}$ by $\frac{5}{6}$	**7.** $\frac{9}{10}$ by $\frac{2}{3}$
2. $\frac{3}{4}$ by $\frac{1}{2}$	**4.** $\frac{3}{8}$ by $\frac{2}{3}$	**6.** $\frac{3}{4}$ by $\frac{4}{5}$	**8.** $\frac{5}{12}$ by $\frac{3}{5}$

Find quickly:

9. $\frac{1}{2}$ of $\frac{4}{5}$
10. $\frac{2}{5}$ of $\frac{1}{6}$
11. $\frac{5}{8}$ of $\frac{3}{5}$
12. $\frac{3}{4} \times \frac{1}{3}$
13. $\frac{4}{5} \times \frac{3}{8}$
14. $\frac{5}{6} \times \frac{3}{4}$
15. $\frac{2}{3}$ of $\frac{5}{12}$
16. $\frac{5}{8}$ of $\frac{3}{10}$
17. $\frac{2}{3}$ of $\frac{9}{16}$
18. $\frac{9}{10} \times \frac{1}{6}$
19. $\frac{11}{12} \times \frac{3}{4}$
20. $\frac{15}{16} \times \frac{2}{3}$

21. A man dried $\frac{4}{5}$ of a ton of grapes and obtained $\frac{1}{4}$ that weight of raisins. What part of a ton of raisins did he obtain?

22. Robert had a $\frac{2}{3}$ interest in a boat and sold $\frac{1}{3}$ of his share. What part did he own then?

23. Grace bought $2\frac{1}{2}$ yards of ribbon, but used only $\frac{3}{4}$ of it. How many yards did she use?

24. If a British soldier in the field receives $1\frac{1}{4}$ lb. of bread per day, and $\frac{4}{5}$ as much meat, how much meat does he receive?

25. Find the cost of $\frac{5}{6}$ doz. cocoanuts at $\$\frac{3}{5}$ per dozen.

26. A man rode $\frac{5}{8}$ of the distance to the shop in an electric car and $\frac{1}{2}$ of the remaining distance on a ferry boat. What part of the distance did he ride on the boat?

WRITTEN EXERCISES

180. 1. Find the value of $\frac{5}{6} \times \frac{24}{25}$.

$$\frac{5}{6}\times\frac{24}{25}=\overset{4}{\underset{5}{\frac{\cancel{5}\times\cancel{24}}{\cancel{6}\times\cancel{25}}}}=\frac{4}{5}\qquad \text{Or, } \frac{\cancel{5}}{\cancel{6}}\times\frac{\overset{4}{\cancel{24}}}{\underset{5}{\cancel{25}}}=\frac{4}{5}$$

In practice it is not necessary to rewrite the fractions as in the first process. We may simply cancel as in the second process.

Find:

2. $\frac{3}{4} \times \frac{24}{30}$
3. $\frac{4}{5} \times \frac{13}{20}$
4. $\frac{2}{3} \times \frac{15}{24}$
5. $\frac{5}{12}$ of $\frac{8}{15}$
6. $\frac{7}{20}$ of $\frac{5}{21}$
7. $\frac{5}{18}$ of $\frac{27}{50}$

Multiply:

8. $\frac{15}{30}$ by $\frac{9}{20}$
9. $\frac{10}{28}$ by $\frac{7}{40}$
10. $\frac{25}{48}$ by $\frac{8}{10}$
11. $\frac{10}{21}$ by $\frac{14}{15}$
12. $\frac{33}{64}$ by $\frac{48}{55}$
13. $\frac{17}{36}$ by $\frac{45}{68}$

14. Find the value of $\frac{5}{8}$ of $10\frac{4}{5}$.

Suggestion. — Reduce the mixed number to an improper fraction.

Find the value of :

15. $\frac{3}{5}$ of $8\frac{1}{3}$

16. $\frac{7}{8}$ of $6\frac{2}{5}$

17. $10\frac{5}{6} \times \frac{4}{5}$

18. $12\frac{3}{4} \times \frac{2}{3}$

19. $9\frac{3}{4} \times 2\frac{2}{3}$

20. $8\frac{1}{6} \times 4\frac{3}{7}$

21. Find the value of $4\frac{1}{2} \times \frac{5}{12} \times 6 \times 1\frac{2}{3}$.

$$4\frac{1}{2}\times\frac{5}{12}\times 6\times 1\frac{2}{3}=\frac{\overset{3}{\cancel{9}}}{2}\times\frac{5}{\underset{2}{\cancel{12}}}\times\frac{\cancel{6}}{1}\times\frac{5}{\cancel{3}}=\frac{75}{4}=18\frac{3}{4}$$

Reducing the mixed numbers to improper fractions, regarding the integer 6 as $\frac{6}{1}$, and canceling, we find the product to be $\frac{75}{4}$, or $18\frac{3}{4}$.

Find the value of:

22. $6\frac{2}{5} \times 7\frac{1}{2} \times 8\frac{1}{4}$

23. $4\frac{1}{6} \times 9\frac{7}{8} \times 5\frac{2}{5}$

24. $\frac{14}{15} \times 6\frac{1}{4} \times 21$

25. $1\frac{7}{8} \times 2\frac{1}{4} \times 8 \times 2\frac{3}{5}$

26. $2\frac{11}{12} \times \frac{9}{14} \times 6\frac{2}{5} \times \frac{2}{3} \times 4$

27. $4\frac{3}{4} \times 6\frac{7}{8} \times \frac{4}{7} \times \frac{8}{11} \times 33$

28. A California vineyard of $11\frac{1}{4}$ acres yielded $4\frac{4}{5}$ tons of grapes per acre. Find the total yield.

29. During a recent balloon contest, a balloon remained in the air $35\frac{3}{4}$ hours and traveled $33\frac{1}{3}$ miles per hour. How far did it travel?

30. If an electric plow travels $353\frac{1}{4}$ feet per minute, how far will it travel in $12\frac{1}{2}$ minutes?

31. The Simplon tunnel between Switzerland and Italy is $12\frac{1}{4}$ miles long. If $\frac{4}{7}$ of its length is in Italian territory, how many miles of the tunnel are in each country?

32. Find the value of cocoa at $\$\frac{1}{4}$ per pound obtained from $\frac{2}{3}$ of an acre of cacao trees, planted 408 trees to the acre, if $1\frac{1}{2}$ pounds of marketable cocoa are obtained per tree.

DIVISION

181. Division of fractions by integers.

1. If $\frac{5}{6}$ of a cantaloupe is divided equally among 5 girls, what part of the cantaloupe will each receive? $\frac{5}{6} \div 5 = ?$

In dividing $\frac{5}{6}$ by 5, which term of the fraction is divided?

2. Multiply the denominator of $\frac{5}{6}$ by 5 and express the result in its lowest terms. Compare your answer with $\frac{5}{6} \div 5$

3. In what two ways, then, may $\frac{5}{6}$ be divided by 5?

4. Which of these two ways should you use to divide $\frac{4}{5}$ by 2? $\frac{3}{5}$ by 2? $\frac{9}{10}$ by 3? $\frac{7}{10}$ by 3?

182. *A fraction may be divided by an integer by dividing the numerator or by multiplying the denominator by the integer.*

The first method should be used whenever the numerator exactly contains the divisor.

EXERCISES

183. Divide:

1. $\frac{3}{5}$ by 3	**4.** $\frac{2}{3}$ by 7	**7.** $\frac{12}{25}$ by 6	**10.** $\frac{24}{11}$ by 12
2. $\frac{3}{5}$ by 4	**5.** $\frac{5}{6}$ by 2	**8.** $\frac{16}{3}$ by 8	**11.** $\frac{13}{2}$ by 15
3. $\frac{5}{8}$ by 4	**6.** $\frac{9}{8}$ by 3	**9.** $\frac{14}{5}$ by 7	**12.** $\frac{80}{21}$ by 20

13. Divide 4 by 2; $\frac{2}{3}$ by 2; $4\frac{2}{3}$ by 2.

14. It is not so easy to divide $3\frac{2}{3}$ by 2 as to divide $4\frac{2}{3}$ by 2. How many thirds are there in $3\frac{2}{3}$? Then $3\frac{2}{3} \div 2 = ?$

Divide:

15. $6\frac{3}{5}$ by 3	**17.** $25\frac{5}{8}$ by 5	**19.** $1\frac{4}{5}$ by 3	**21.** $5\frac{1}{4}$ by 7
16. $8\frac{4}{7}$ by 4	**18.** $49\frac{7}{8}$ by 7	**20.** $3\frac{1}{3}$ by 2	**22.** $3\frac{3}{8}$ by 9

23. If it is $16\frac{4}{5}$ feet around a square, how long is each side?

24. A boy sawed a board $13\frac{1}{2}$ feet long into 3 equal pieces to make shelves for a cupboard. How long was each shelf?

25. Three boys weighed $210\frac{3}{8}$ lb. Find their average weight.

26. A man did $\frac{2}{3}$ of a piece of work in 6 days. What part of the work did he do in 1 day?

27. A fly wheel made $12\frac{1}{2}$ revolutions in 5 seconds. How many revolutions per second did it make?

28. Four cars contained $168\frac{1}{2}$ tons of coal. Find the average weight of coal per car.

29. A retail meat dealer used $3\frac{3}{4}$ tons of ice in his refrigerators in 5 days. Find the daily consumption of ice.

30. A boy earned $\$3\frac{3}{5}$ in 9 days. What part of a dollar did he earn per day?

WRITTEN EXERCISES

184. **1.** Divide $\frac{14}{15}$ by 7; divide $\frac{5}{9}$ by 2; divide $\frac{15}{7}$ by 6.

$$\frac{14}{15} \div 7 = \frac{14 \div 7}{15} = \frac{2}{15}$$

In the first process the division is performed by *dividing the numerator* by the integer (§ 182).

$$\frac{5}{9} \div 2 = \frac{5}{2 \times 9} = \frac{5}{18}$$

In the second process the division is performed by *multiplying the denominator* by the integer (§ 182).

$$\frac{15}{7} \div 6 = \frac{\overset{5}{\cancel{15}}}{\underset{2}{\cancel{6}} \times 7} = \frac{5}{14}$$

In the third process the work is indicated and the result simplified by cancellation.

Divide:

2. $\frac{21}{5}$ by 7	**6.** $\frac{7}{16}$ by 2	**10.** $\frac{4}{11}$ by 12	**14.** $\frac{25}{6}$ by 10
3. $\frac{18}{11}$ by 3	**7.** $\frac{15}{16}$ by 6	**11.** $\frac{24}{5}$ by 16	**15.** $\frac{27}{2}$ by 15
4. $\frac{42}{25}$ by 4	**8.** $\frac{9}{10}$ by 12	**12.** $\frac{28}{11}$ by 14	**16.** $5\frac{1}{3}$ by 24
5. $\frac{9}{20}$ by 6	**9.** $\frac{44}{3}$ by 33	**13.** $\frac{45}{4}$ by 30	**17.** $6\frac{1}{4}$ by 35

18. Divide $323\frac{1}{2}$ by 4.

$$\begin{array}{r} 4\underline{)323\frac{1}{2}} \\ 80\frac{7}{8} \end{array}$$

The integral part of the dividend is large and the divisor is small. In such cases short division may be used.

Thus 4 is contained in $323\frac{1}{2}$, 80 times with $3\frac{1}{2}$, or $\frac{7}{2}$, undivided; $\frac{7}{2} \div 4 = \frac{7}{8}$. The entire quotient, then, is $80\frac{7}{8}$.

Divide:

19. $14\frac{2}{3}$ by 4	**22.** $169\frac{1}{9}$ by 2	**25.** $321\frac{1}{3}$ by 2
20. $16\frac{4}{5}$ by 6	**23.** $264\frac{4}{5}$ by 4	**26.** $283\frac{1}{2}$ by 7
21. $17\frac{2}{5}$ by 3	**24.** $426\frac{1}{2}$ by 8	**27.** $146\frac{5}{8}$ by 5

28. Divide $1006\frac{1}{2}$ by 22.

$$\begin{array}{l} 22)1006\frac{1}{2} \\ \underline{\ 2 \qquad\ \ 2\ \ } \\ 44)2013(45\frac{3}{4} \\ \quad \underline{176} \\ \quad\ \ 253 \\ \quad\ \ \underline{220} \\ \qquad \frac{33}{44} = \frac{3}{4} \end{array}$$

Here it is necessary to use long division. The work may be simplified by changing both divisor and dividend to halves, obtaining 44 (halves) and 2013 (halves), respectively.

Dividing as in integers, the quotient is found to be $45\frac{33}{44}$, or $45\frac{3}{4}$.

Divide:

29. $760\frac{1}{2}$ by 18	**33.** $899\frac{1}{2}$ by 21	**37.** $5215\frac{7}{8}$ by 42
30. $751\frac{1}{2}$ by 24	**34.** $616\frac{4}{5}$ by 36	**38.** $7321\frac{1}{2}$ by 45
31. $773\frac{1}{3}$ by 32	**35.** $657\frac{1}{2}$ by 25	**39.** $5466\frac{2}{3}$ by 64
32. $990\frac{1}{2}$ by 28	**36.** $635\frac{3}{5}$ by 35	**40.** $5156\frac{2}{3}$ by 56

41. Express trains in France cover the $184\frac{1}{2}$ miles between Paris and Calais in 3 hours. Find their rate per hour.

42. If $38\frac{1}{4}$ ounces of gold were extracted from 68 tons of ore, what was the amount of gold per ton of ore?

43. From 50,000 pounds of cod $68\frac{3}{4}$ gallons of cod-liver oil were obtained. Find the yield of oil per 1000 pounds of fish.

44. It cost $\$7\frac{1}{2}$ to store 30 cases of eggs in Philadelphia. Find the storage rate per case.

45. It cost $\$206\frac{1}{4}$ to remove 330 cubic yards of clay in digging a reservoir. What was the expense per cubic yard?

46. If $\frac{10}{11}$ of this clay was carted away at a cost of $\$\frac{1}{5}$ per cubic yard and sold for $\$112\frac{1}{2}$, how much was gained per cubic yard over not selling it at all?

185. Division of integers by fractions.

1. Divide 40 by 8; by 4; by 2; by 1.

When the divisor is halved, how is the quotient affected?

2. Since 1 is contained 40 times in 40, how many times is $\frac{1}{2}$ of 1 contained in 40?

$$40 \div 1 = \_\_\_\_; \quad 40 \div \tfrac{1}{2} = \_\_\_\_ \text{ times } 40 = \_\_\_\_.$$

3. In the same way divide 40 by $\frac{1}{3}$; by $\frac{1}{4}$; by $\frac{1}{5}$.

Compare $40 \div \frac{1}{3}$ with 3×40; $40 \div \frac{1}{4}$ with 4×40; $40 \div \frac{1}{5}$ with 5×40.

4. Divide 20 by 1; by 2; by 4; by 8.

How does multiplying the divisor by 2 affect the quotient?

5. Divide 20 by $\frac{1}{3}$, then by 2 times $\frac{1}{3}$. What part of $20 \div \frac{1}{3}$ is $20 \div \frac{2}{3}$?

Since $20 \div \frac{1}{3} = 3 \times 20$, $20 \div \frac{2}{3} = \_\_\_\_$ of $3 \times 20 = \_\_\_\_$ of 20.

6. Divide 8 by $\frac{1}{5}$. By what number must you divide this result to find the value of $8 \div \frac{4}{5}$?

186. *An integer may be divided by a fraction by multiplying the integer by the denominator of the fraction and dividing the product by the numerator.*

EXERCISES

187. Divide:

1. $5 \div \frac{1}{2}$	4. $8 \div \frac{1}{5}$	7. $6 \div \frac{1}{4}$	10. $9 \div \frac{3}{8}$
2. $4 \div \frac{1}{3}$	5. $7 \div \frac{1}{8}$	8. $6 \div \frac{3}{4}$	11. $8 \div \frac{4}{5}$
3. $4 \div \frac{2}{3}$	6. $7 \div \frac{7}{8}$	9. $9 \div \frac{1}{8}$	12. $8 \div \frac{2}{7}$

13. Robert paid \$2 for phonograph records. If the cost of each was \$$\frac{1}{4}$, how many did he purchase?

14. If it costs \$$\frac{4}{5}$ to store one ton of goods, how many tons can be stored for \$4 for the same length of time?

15. If a laborer on a sugar-cane plantation earns \$$\frac{3}{4}$ per day, in how many days will he earn \$6?

188. Divide 1 by 4; by 2; by 1; by $\frac{1}{3}$; by $\frac{2}{3}$; by $\frac{4}{5}$.

1 divided by any number is the **reciprocal** of the number.

189. 1. What is the reciprocal of 5? of 1? of $\frac{1}{4}$? of $\frac{3}{4}$?

2. How does the reciprocal of $\frac{1}{3}$ compare with $\frac{3}{1}$? the reciprocal of $\frac{2}{3}$ with $\frac{3}{2}$? the reciprocal of $\frac{4}{5}$ with $\frac{5}{4}$?

The reciprocal of a fraction is the fraction inverted.

190. Compare $6 \div \frac{2}{5}$ with 6 multiplied by $\frac{5}{2}$; $6 \div \frac{3}{4}$ with $6 \times \frac{4}{3}$; $10 \div \frac{5}{6}$ with $10 \times \frac{6}{5}$.

To divide by a fraction we may multiply by the reciprocal of the fraction or by the fraction inverted.

WRITTEN EXERCISES

191. 1. Divide 15 by $1\frac{5}{7}$.

$$15 \div 1\tfrac{5}{7} = 15 \div \frac{12}{7} = 15 \times \frac{7}{12} = \frac{\overset{5}{\cancel{15}} \times 7}{\underset{4}{\cancel{12}}} = \frac{35}{4} = 8\tfrac{3}{4}$$

Find quotients:

2. $25 \div \frac{5}{3}$	**7.** $42 \div 2\frac{2}{5}$	**12.** $40 \div 6\frac{2}{5}$
3. $20 \div \frac{8}{5}$	**8.** $30 \div 2\frac{1}{2}$	**13.** $35 \div 3\frac{4}{7}$
4. $16 \div \frac{2}{3}$	**9.** $45 \div 7\frac{1}{2}$	**14.** $64 \div 2\frac{2}{11}$
5. $27 \div \frac{6}{7}$	**10.** $32 \div 5\frac{1}{3}$	**15.** $24 \div 1\frac{13}{15}$
6. $28 \div \frac{7}{4}$	**11.** $44 \div 3\frac{1}{5}$	**16.** $60 \div 1\frac{7}{11}$

17. Divide 1000 by $66\frac{2}{3}$ by reducing both numbers to thirds.

Find quotients:

18. $540 \div 16\frac{1}{5}$	**24.** $2730 \div 24\frac{3}{8}$	**30.** $2646 \div 24\frac{1}{2}$
19. $840 \div 12\frac{5}{6}$	**25.** $2216 \div 69\frac{1}{4}$	**31.** $9185 \div 41\frac{3}{4}$
20. $954 \div 19\frac{7}{8}$	**26.** $4112 \div 42\frac{5}{6}$	**32.** $8160 \div 16\frac{7}{8}$
21. $340 \div 21\frac{1}{4}$	**27.** $2800 \div 33\frac{1}{3}$	**33.** $7854 \div 25\frac{2}{3}$
22. $728 \div 29\frac{1}{3}$	**28.** $7200 \div 37\frac{1}{2}$	**34.** $1728 \div 28\frac{4}{5}$
23. $180 \div 19\frac{1}{5}$	**29.** $4710 \div 62\frac{4}{5}$	**35.** $5134 \div 37\frac{3}{4}$

36. The steel rails for $2\frac{3}{4}$ miles of railroad track cost $5170. What was the cost per mile?

37. The expense for labor in laying $24\frac{7}{10}$ miles of railroad track was $12,350. Find the expense per mile.

38. If $4\frac{12}{25}$ tons of coal evaporate 76,900 pounds of water, how many pounds of water will 1 ton of coal evaporate?

39. If the weight of 1 gallon of kerosene oil is $6\frac{2}{5}$ pounds, how many gallons does a tank car contain whose capacity is 41,600 pounds?

40. How long will it take a wind storm, moving at the rate of $37\frac{1}{2}$ miles per hour, to travel from Bismarck, N.D., to Chicago, about 750 miles?

41. If it takes a match factory $8\frac{1}{3}$ days to manufacture 150 million matches, what is the output per day?

42. In a good year Bulgaria produced 13,770 pounds of attar of roses. How many acres of rose bushes were under cultivation, if the flowers from one acre produced $\frac{15}{16}$ of a pound of attar of roses?

43. A fan blower supplies a hall with 2760 cubic feet of air per minute. Find the weight of the air forced into the room in an hour, if 1 pound of it occupies $13\frac{1}{7}$ cubic feet.

44. A film for a moving picture consisted of a series of separate pictures, each $\frac{3}{4}$ of an inch long. If the film was 150 feet long, of how many individual pictures was it made up?

45. If a woman can reel $5\frac{5}{8}$ pounds of silk from cocoons in 6 days of 10 hours each, how many hours does it take her to reel 1 pound of silk?

46. How long a time is consumed by a train in traveling from San Francisco to New York, if the average rate per hour for the 2565 miles to Chicago is $33\frac{3}{4}$ miles, and $54\frac{1}{6}$ miles per hour for the remaining distance of 975 miles?

192. Division of fractions by fractions.

1. How many times are 2 fifths contained in 4 fifths?

2. Divide 6 eighths by 2 eighths; by 3 eighths. Divide 15 sixteenths by 3 sixteenths; by 5 sixteenths.

$$\frac{6}{8} \div \frac{2}{8} = ? \quad \frac{6}{8} \div \frac{3}{8} = ? \quad \frac{15}{16} \div \frac{3}{16} = ? \quad \frac{15}{16} \div \frac{5}{16} = ?$$

3. When two fractions are similar, how may one be divided by the other?

4. How many times is $\frac{1}{4}$ contained in 1? in $\frac{1}{2}$ of 1, that is, in $\frac{1}{2}$? $\frac{1}{2} \div \frac{1}{4} = ?$

How does $\frac{1}{2} \div \frac{1}{4}$ compare with $\frac{1}{2}$ multiplied by $\frac{4}{1}$?

5. How many times is $\frac{3}{8}$ contained in 1? in 3? in $\frac{1}{4}$ of 3, that is, in $\frac{3}{4}$? $\frac{3}{4} \div \frac{3}{8} = ?$

How does $\frac{3}{4} \div \frac{3}{8}$ compare with $\frac{3}{4} \times \frac{8}{3}$?

6. How many times is $\frac{2}{3}$ contained in 1? in 5? in $\frac{1}{6}$ of 5, that is, in $\frac{5}{6}$? $\frac{5}{6} \div \frac{2}{3} = ?$

How does $\frac{5}{6}$ divided by $\frac{2}{3}$ compare with $\frac{5}{6}$ multiplied by the *reciprocal* of $\frac{2}{3}$?

193. To divide a fraction by a fraction, *change to similar fractions and divide the numerator of the dividend by the numerator of the divisor.* Or,

Multiply the dividend by the reciprocal of the divisor.

EXERCISES

194. Divide:

1. $\frac{1}{2}$ by $\frac{4}{5}$	**3.** $\frac{7}{8}$ by $\frac{1}{4}$	**5.** $\frac{3}{10}$ by $\frac{1}{8}$	**7.** $\frac{2}{5}$ by $\frac{7}{10}$
2. $\frac{1}{3}$ by $\frac{5}{6}$	**4.** $\frac{2}{5}$ by $\frac{4}{5}$	**6.** $\frac{11}{12}$ by $\frac{1}{6}$	**8.** $\frac{2}{3}$ by $\frac{5}{12}$

Give quotients quickly:

9. $\frac{3}{8} \div \frac{1}{2}$	**12.** $\frac{5}{6} \div \frac{1}{2}$	**15.** $\frac{2}{3} \div \frac{3}{4}$	**18.** $\frac{3}{10} \div \frac{1}{2}$
10. $\frac{3}{4} \div \frac{1}{8}$	**13.** $\frac{3}{8} \div \frac{3}{4}$	**16.** $\frac{3}{4} \div \frac{5}{6}$	**19.** $\frac{7}{10} \div \frac{2}{3}$
11. $\frac{3}{8} \div \frac{1}{4}$	**14.** $\frac{4}{5} \div \frac{1}{2}$	**17.** $\frac{3}{4} \div \frac{2}{3}$	**20.** $\frac{11}{12} \div \frac{3}{4}$

WRITTEN EXERCISES

195. **1.** Divide $\frac{25}{32}$ by $\frac{5}{8}$.

$$\frac{25}{32}\div\frac{5}{8}=\frac{\overset{5}{\cancel{25}}}{\underset{4}{\cancel{32}}}\times\frac{\cancel{8}}{\cancel{5}}=\frac{5}{4}=1\tfrac{1}{4}$$

Since $1\div\frac{5}{8}=\frac{8}{5}$, $\frac{25}{32}\div\frac{5}{8}$ is $\frac{25}{32}$ of $\frac{8}{5}$, or $\frac{25}{32}\times\frac{8}{5}$. Canceling, we find the result to be $\frac{5}{4}$, or $1\frac{1}{4}$.

Divide:

2. $\frac{7}{8}$ by $\frac{7}{16}$ **4.** $\frac{3}{4}$ by $\frac{24}{30}$ **6.** $\frac{15}{24}$ by $\frac{2}{3}$ **8.** $\frac{25}{48}$ by $\frac{5}{16}$

3. $\frac{5}{6}$ by $\frac{25}{36}$ **5.** $\frac{7}{8}$ by $\frac{21}{48}$ **7.** $\frac{27}{36}$ by $\frac{3}{10}$ **9.** $\frac{13}{40}$ by $\frac{39}{70}$

10. Divide $9\frac{4}{5}$ by $\frac{7}{8}$.

Suggestion. — Reduce the mixed number to an improper fraction.

Divide:

11. $3\frac{9}{10}$ by $\frac{4}{5}$ **14.** $25\frac{1}{2}$ by $2\frac{1}{4}$ **17.** $116\frac{3}{8}$ by $36\frac{3}{4}$

12. $8\frac{3}{4}$ by $\frac{7}{12}$ **15.** $40\frac{7}{8}$ by $2\frac{1}{2}$ **18.** $342\frac{9}{10}$ by $48\frac{3}{5}$

13. $6\frac{3}{5}$ by $24\frac{3}{4}$ **16.** $64\frac{3}{5}$ by $1\frac{3}{4}$ **19.** $639\frac{7}{12}$ by $145\frac{5}{6}$

20. Find the value of $\frac{3}{4}\div 6\div\frac{3}{10}\times 6\frac{3}{8}\div\frac{5}{6}$.

$$\frac{3}{4}\div 6\div\frac{3}{10}\times 6\tfrac{3}{8}\div\frac{5}{6}=\frac{\cancel{3}}{\underset{2}{\cancel{4}}}\times\frac{1}{\cancel{6}}\times\frac{\overset{2}{\cancel{10}}}{\cancel{3}}\times\frac{51}{8}\times\frac{\cancel{6}}{\cancel{5}}=\frac{51}{16}=3\tfrac{3}{16}$$

After reducing all integers and mixed numbers to improper fractions, we take the reciprocals of the fractions that are divisors and write them as multipliers. Canceling and reducing, we find the result to be $3\frac{3}{16}$.

Find the value of:

21. $\frac{4}{5}\times\frac{7}{8}\div\frac{14}{15}\times 2\frac{1}{4}$

22. $\frac{5}{8}\times 18\div\frac{3}{4}\div 15$

23. $\frac{5}{6}\div 2\frac{2}{3}\times 14\div\frac{5}{9}$

24. $\frac{4}{5}\div\frac{3}{10}\div 1\frac{3}{5}\times 3\frac{1}{2}$

25. $\frac{2}{3}\times 2\frac{5}{8}\div 1\frac{1}{6}\div 1\frac{11}{16}$

26. $1\frac{3}{4}\div 6\frac{1}{2}\times 9\times\frac{39}{5}\div\frac{7}{12}$

27. $42\div\frac{2}{3}\times 2\frac{4}{5}\div 7\div\frac{15}{16}$

28. $\frac{24}{25}\div 16\div\frac{8}{15}\times 1\frac{5}{6}\times 80$

29. $12\times 2\frac{2}{3}\div 3\frac{3}{5}\times\frac{5}{14}\times 2\frac{1}{10}$

30. $16\div\frac{4}{15}\div 2\frac{2}{9}\times 1\frac{1}{5}\div 1\frac{4}{5}$

31. Find the value of $\frac{2}{3}$ of $6 \div \frac{3}{4}$ of 8; of $\frac{2}{3} \times 6 \div \frac{3}{4} \times 8$.

SUGGESTION. — The word "of" indicates a closer connection than the signs $\times$ and $\div$; thus, $\frac{2}{3}$ of $6 \div \frac{3}{4}$ of $8 = (\frac{2}{3} \times 6) \div (\frac{3}{4} \times 8) = \frac{2}{3} \times 6 \times \frac{4}{3} \times \frac{1}{8}$, while $\frac{2}{3} \times 6 \div \frac{3}{4} \times 8 = \frac{2}{3} \times 6 \times \frac{4}{3} \times 8$.

Find the value of:

32. $\frac{4}{5}$ of $20 \div \frac{5}{6}$ of $30 \times \frac{7}{8}$ of 40

33. $\frac{2}{3}$ of $14 \times \frac{3}{4}$ of $18 \div \frac{1}{6}$ of 42

34. $\frac{5}{12}$ of $\frac{4}{5}$ of $\frac{7}{8} \div \frac{1}{2}$ of $\frac{14}{15}$ of $2\frac{1}{4}$

35. $\frac{8}{15}$ of $\frac{5}{16}$ of $\frac{11}{12}$ of $7 \div \frac{2}{3}$ of $\frac{3}{4}$ of 11

36. $2\frac{1}{4} \times \frac{4}{5} \times 8\frac{5}{8} \times 9\frac{7}{12} \div (9 \times 3\frac{5}{6} \times 6\frac{3}{4} \times \frac{2}{3})$

37. $6\frac{3}{8} \times 15 \times \frac{9}{10} \times 4\frac{2}{3} \div (1\frac{1}{16} \times 5\frac{1}{4} \times \frac{5}{8} \times 21\frac{1}{3})$

38. $\frac{15}{16} \div 2\frac{5}{8} \times \frac{2}{5} \div \frac{19}{20} \div \frac{7}{8} \times 12\frac{2}{3} \div 3\frac{1}{5} \times 4\frac{1}{12} \times 2$

39. $\frac{16}{25} \div 13\frac{1}{2} \times \frac{9}{16} \times 9\frac{1}{6} \div \frac{27}{48} \div 1\frac{1}{10} \times 26\frac{1}{4} \times \frac{5}{8} \div \frac{45}{72} \times \frac{9}{10}$

40. $\frac{11}{12} \times \frac{3}{14} \div 6\frac{3}{5} \times \frac{18}{25} \times 8\frac{3}{4} \div \frac{26}{36} \div 12\frac{5}{6} \times \frac{13}{15} \times 7\frac{1}{3} \div \frac{3}{25} \times 1\frac{2}{5}$

41. A steam shovel used in excavating canals handled $45\frac{1}{2}$ cubic yards of earth in $4\frac{1}{3}$ minutes. What was its rate per minute?

42. In $2\frac{1}{2}$ days a machine in a Maine paper mill turned out $43\frac{3}{4}$ tons of paper. How many tons did it make per day?

43. A gallon of naphtha weighs $5\frac{3}{4}$ pounds. If the naphtha contained in a barrel weighs $287\frac{1}{2}$ pounds, how many gallons of naphtha does this barrel contain?

44. If $10\frac{5}{8}$ pounds of amber brought $\$123\frac{1}{4}$, what was the selling price of 1 pound?

45. A maple-sugar manufacturer hired 3 men to gather sap from the maple trees and paid them $\$87\frac{3}{4}$ for their work. If they worked $22\frac{1}{2}$ days, how much did each receive per day?

46. The average expenditure for the public schools of the United States in a recent year was $\$21.16\frac{4}{5}$ per pupil. Find the average number of school days, if the daily expenditure per pupil was $14\frac{7}{10}$ cents.

196. Simplifying complex fractions.

Since division may be indicated by a fraction, the numerator being the dividend and the denominator the divisor, we may indicate the division of a fraction by an integer, of an integer by a fraction, or of a fraction by a fraction, in *fractional form*, by writing the dividend above a line and the divisor below.

We may write $\frac{5}{8} \div 4$ like this, $\dfrac{\frac{5}{8}}{4}$; $9 \div \frac{3}{5}$ like this, $\dfrac{9}{\frac{3}{5}}$; and $\frac{2}{3} \div \frac{3}{4}$ like this, $\dfrac{\frac{2}{3}}{\frac{3}{4}}$.

Such indicated expressions of division are sometimes called **complex fractions**.

When the indicated division is performed, the complex fraction is said to be **simplified**.

WRITTEN EXERCISES

197. 1. Simplify $\dfrac{\frac{3}{4}}{\frac{5}{16}}$.

SOLUTION. $$\dfrac{\frac{3}{4}}{\frac{5}{16}} = \frac{3}{4} \div \frac{5}{16} = \frac{3}{\cancel{4}} \times \frac{\overset{4}{\cancel{16}}}{5} = \frac{12}{5} = 2\tfrac{2}{5}.$$

Simplify:

2. $\dfrac{\frac{3}{8}}{\frac{9}{10}}$ 7. $\dfrac{12}{\frac{5}{6}}$ 12. $\dfrac{33\frac{1}{3}}{100}$ 17. $\dfrac{\frac{4}{5} \div \frac{2}{3}}{\frac{3}{4} \text{ of } 5\frac{1}{3}}$

3. $\dfrac{\frac{5}{8}}{\frac{5}{12}}$ 8. $\dfrac{\frac{24}{25}}{8}$ 13. $\dfrac{87\frac{1}{2}}{100}$ 18. $\dfrac{\frac{2}{3} \text{ of } 4\frac{3}{8}}{4\frac{1}{3} + \frac{11}{12}}$

4. $\dfrac{\frac{5}{6}}{\frac{15}{16}}$ 9. $\dfrac{4\frac{7}{8}}{\frac{5}{6}}$ 14. $\dfrac{.6\frac{1}{4}}{3.7\frac{1}{2}}$ 19. $\dfrac{4\frac{3}{8} - 2\frac{1}{2}}{2\frac{1}{4} + 1\frac{4}{5}}$

5. $\dfrac{\frac{7}{8}}{\frac{7}{24}}$ 10. $\dfrac{6\frac{3}{5}}{\frac{3}{4}}$ 15. $\dfrac{.62\frac{1}{2}}{.66\frac{2}{3}}$ 20. $\dfrac{6\frac{2}{3} \div 3\frac{1}{8}}{\frac{4}{5} \div 3\frac{5}{6}}$

6. $\dfrac{\frac{14}{15}}{\frac{21}{25}}$ 11. $\dfrac{4\frac{1}{6}}{4\frac{2}{3}}$ 16. $\dfrac{4.3\frac{1}{4}}{25.9\frac{1}{2}}$ 21. $\dfrac{\frac{19}{4} \text{ of } 7\frac{7}{10}}{2\frac{3}{8} \times 1\frac{1}{6}}$

FRACTIONAL RELATIONS

198. Finding what part one number is of another.

EXERCISES

1. What part of 6 is 4?

SOLUTION. — Since 1 is $\frac{1}{6}$ of 6, 4 is 4 times $\frac{1}{6}$ of 6, or $\frac{4}{6}$ of 6; that is, 4 is $\frac{2}{3}$ of 6.

What part of

2. 5 is 3?	5. 12 is 4?	8. 7 is 21?	11. 24 is 10?
3. 8 is 5?	6. 15 is 6?	9. 6 is 18?	12. 48 is 21?
4. 9 is 3?	7. 20 is 5?	10. 8 is 24?	13. 36 is 27?

14. Of 12 boys camping at Moose Lake, 4 went fishing and the rest hunting. What part of the party went fishing? what part went hunting?

15. One boy caught 12 fish, another 8, and another 4. The fourth boy caught none. The hunters shot nothing. What part of the fish did each of the first three boys catch? what part of the whole party was unsuccessful?

16. Three boys went home after one week, the rest stayed two weeks. What part stayed one week? two weeks?

17. A grocer's boy delivered 48 packages — 8 at one house, 4 at another, 6 at another, and the rest at the Home Restaurant. What part did he deliver at each place?

18. He solicited orders at 30 houses and received 18 orders. What part of his visits resulted in orders? what part did not?

19. Out of a class of 50 girls and 30 boys graduating from a grammar school, 44 girls and 20 boys entered the high school. What part of the girls entered the high school? what part of the boys? what part of the class?

WRITTEN EXERCISES

199. **1.** What part of $\frac{5}{6}$ is $\frac{3}{4}$?

Solution. — Since $\frac{5}{6} = \frac{10}{12}$ and $\frac{3}{4} = \frac{9}{12}$, $\frac{3}{4}$ is the same part of $\frac{5}{6}$ that $\frac{9}{12}$ is of $\frac{10}{12}$; but $\frac{9}{12}$ is the same part of $\frac{10}{12}$ that the numerator 9 is of the numerator 10.

Since 9 is $\frac{9}{10}$ of 10, $\frac{3}{4}$ is $\frac{9}{10}$ of $\frac{5}{6}$.

Observe that the same result may be obtained by dividing $\frac{3}{4}$ by $\frac{5}{6}$.

What part of

2. $\frac{3}{4}$ is $\frac{1}{2}$? **4.** $\frac{1}{3}$ is $\frac{1}{6}$? **6.** $\frac{3}{8}$ is $\frac{9}{10}$? **8.** $\frac{4}{15}$ is $\frac{1}{10}$?

3. $\frac{5}{8}$ is $\frac{1}{4}$? **5.** $\frac{4}{5}$ is $\frac{3}{4}$? **7.** $\frac{3}{4}$ is $\frac{5}{16}$? **9.** $\frac{18}{25}$ is $\frac{27}{40}$?

10. What part of $6\frac{1}{4}$ is 5?

Suggestion. — Reduce both numbers to fourths, or divide 5 by $6\frac{1}{4}$.

What part of

11. $4\frac{4}{5}$ is 4? **14.** 10 is $3\frac{1}{3}$? **17.** 8 is $\frac{4}{5}$? **20.** $3\frac{1}{4}$ is $2\frac{3}{5}$?

12. $9\frac{1}{3}$ is 7? **15.** 12 is $6\frac{1}{2}$? **18.** 12 is $\frac{3}{5}$? **21.** $8\frac{2}{3}$ is $6\frac{1}{2}$?

13. $3\frac{1}{8}$ is 2? **16.** 14 is $4\frac{2}{3}$? **19.** 5 is $6\frac{2}{3}$? **22.** $9\frac{3}{4}$ is $4\frac{7}{8}$?

23. A train of 88 coal cars was $\frac{2}{3}$ of a mile long. How many cars would there have been if it had been $\frac{1}{2}$ of a mile long?

Suggestion. — What part of $\frac{2}{3}$ is $\frac{1}{2}$?

24. The winner of a 5-mile automobile race covered the distance in $2\frac{4}{5}$ minutes. How far did he go in $1\frac{2}{5}$ minutes?

25. If one square foot of sheet iron $\frac{1}{2}$ of an inch thick weighs 20.141 pounds, find the weight, to the nearest thousandth of a pound, of a square foot $\frac{3}{16}$ of an inch in thickness.

26. If a machine makes 350 grape baskets in $\frac{5}{6}$ of an hour, how many baskets does it make in $4\frac{1}{4}$ hours?

27. If 220 trees in the Philippine Islands yield 180 pounds of hemp fiber for cordage, how many trees are necessary to provide fiber for a bale of 270 pounds?

28. The average time of transmitting a 20-word cable mes-

sage on the first cable was $6\frac{2}{3}$ minutes. Such a message can now be transmitted in $\frac{2}{5}$ of a minute. What part of the time originally required for transmission is the present time?

200. Finding the whole when a fractional part of it is given.

1. If $\frac{2}{3}$ of a number is 20, what is $\frac{1}{3}$ of it? $\frac{3}{3}$ of it?

2. If $\frac{4}{5}$ of a number is 16, what is $\frac{1}{5}$ of the number? What is the number?

EXERCISES

201. 1. Our football team won $\frac{3}{4}$ of the games that it played. It won 12 games. How many games did it play?

SOLUTION. $\frac{3}{4}$ of the number of games played = 12.
$\frac{1}{4}$ of the number of games played = $\frac{1}{3}$ of 12, or 4.
The number of games played = 4×4, or 16.

2. One fourth of the domestic reindeer of Alaska one year were fawns. If 2000 were fawns, how many reindeer were there?

3. The balcony of a music hall holds 400 people. If this is $\frac{1}{4}$ of the hall's capacity, how many people will the hall hold?

4. A certain steamship can accommodate 300 first-class passengers. This is $\frac{1}{5}$ of the total number that can be carried. How many passengers can the vessel accommodate?

5. In a test for silver $\frac{3}{5}$ of a ton of ore yielded 6 pounds of silver. What was the yield of silver per ton?

6. A ball, or "loaf," of raw rubber when cleansed of impurities for manufacture weighed 9 pounds, or $\frac{2}{3}$ of its original weight. How much did it weigh at first?

7. When a balloon had traveled 800 miles it had completed $\frac{2}{3}$ of its journey. How far did it travel?

8. Of the people who lost their lives in one year in vessels wrecked on our shores $\frac{5}{6}$, or 20, perished on the Atlantic coast. How many lost their lives in all?

WRITTEN EXERCISES

202. Find the number of which

1. 84 is $\frac{2}{3}$ **3.** 224 is $\frac{4}{5}$ **5.** 756 is $\frac{9}{10}$ **7.** 2684 is $\frac{5}{4}$

2. 91 is $\frac{3}{4}$ **4.** 375 is $\frac{5}{6}$ **6.** 924 is $\frac{11}{12}$ **8.** 4725 is $\frac{7}{8}$

9. If $\frac{5}{6}$ is $\frac{2}{3}$ of a number, what is the number?

SOLUTION. $\frac{2}{3}$ of the number $= \frac{5}{6}$.
$\frac{1}{3}$ of the number $= \frac{1}{2}$ of $\frac{5}{6}$, or $\frac{5}{12}$.
The number $= 3 \times \frac{5}{12} = \frac{5}{4}$, or $1\frac{1}{4}$.

Find the number of which

10. $\frac{5}{8}$ is $\frac{3}{4}$ **12.** $\frac{7}{12}$ is $\frac{5}{6}$ **14.** $3\frac{3}{5}$ is $\frac{3}{10}$ **16.** $5\frac{1}{2}$ is $\frac{11}{4}$

11. $\frac{2}{3}$ is $\frac{2}{5}$ **13.** $\frac{9}{10}$ is $\frac{3}{8}$ **15.** $6\frac{5}{6}$ is $\frac{5}{12}$ **17.** $4\frac{1}{4}$ is $\frac{34}{5}$

18. If Holland has 1500 miles of railroad, and only $\frac{2}{3}$ as many miles of railroad as of canals, what length of canals has she?

19. Find the cost of the German North Sea Canal, if $\frac{7}{16}$ of the cost, or $17\frac{1}{2}$ million dollars, was paid for excavating.

20. The buildings of the great Krupp steel works in Germany cover 150 acres. If this is $\frac{1}{6}$ of the land owned by the company, how many acres do they own?

21. The cost of a British third-class cruiser was \$675,000. This was \$5000 more than $\frac{4}{5}$ of the cost of a German warship of the same type. How much did the German cruiser cost?

22. One section of a 50-ton anchor chain is 180 feet long. If this section is $\frac{2}{11}$ of the chain, how long is the chain?

23. Ordinarily the cruiser *Charleston* has a coal supply of 650 tons, but this is only $\frac{13}{30}$ of her capacity. How many tons of coal can she carry?

24. In the menagerie Harold saw a test in which a horse pulled a weight of 1875 pounds, or $\frac{3}{14}$ of the weight pulled by an elephant. A camel pulled $\frac{11}{15}$ as much as the horse. What was the pulling strength of each animal?

ALIQUOT PARTS

203. 1. How many times is \$.25 contained in \$1? .25 lb. in 1 lb.? .25 in 1? 25 in 100? 250 in 1000?

2. Mention several parts of \$1 that are exactly contained in \$1; parts of 1 that are exactly contained in 1, that is, **aliquot parts** of 1; aliquot parts of 100; aliquot parts of 1000.

3. What part of \$1 is \$.12½? Beginning with \$.12½, add \$.12½ successively until you reach \$1.25.

Which of these *multiples* of \$.12½ are aliquot parts of \$1?

4. What part of \$.25 must be added to \$.25 to give \$.37½?

5. What part of \$.50 added to \$.50 will give \$.62½?

6. What part of \$1 subtracted from \$1 will give \$.87½? \$.75? \$.90? \$.66⅔? \$.83⅓?

7. What part of \$1 added to \$1 will give \$1.10? \$1.12½? \$1.20? \$1.25? \$1.37½? \$1.50?

8. What is the cost of 44 articles at \$1 each? at \$.50, or \$½, each? at \$.25, or \$¼, each? at \$1.25, or \$1 + \$¼, each?

9. From the cost of any number of articles at \$1 each, how may the cost at \$.50 each be found? at \$.25? at \$.12½?

EXERCISES

204. Find the cost of 240 articles:

1. At 10¢ each
2. At 90¢ each
3. At 20¢ each
4. At 80¢ each
5. At 40¢ each
6. At 60¢ each
7. At 25¢ each
8. At 50¢ each
9. At 12½¢ each
10. At 87½¢ each
11. At 33⅓¢ each
12. At 66⅔¢ each
13. At 16⅔¢ each
14. At 83⅓¢ each
15. At 37½¢ each
16. At 62½¢ each
17. At \$.75 each
18. At \$1.05 each
19. At \$1.10 each
20. At \$1.20 each
21. At \$1.25 each
22. At \$1.50 each
23. At \$1.75 each
24. At \$1.90 each

205. The following aliquot parts of 1 and multiples of such aliquot parts should be committed to memory:

$.10 = \frac{1}{10}$	$.12\frac{1}{2} = \frac{1}{8}$	$.75 = \frac{3}{4}$	$.83\frac{1}{3} = 1 - \frac{1}{6}$
$.20 = \frac{1}{5}$	$.25 = \frac{1}{4}$	$.87\frac{1}{2} = \frac{7}{8}$	$.90 = 1 - \frac{1}{10}$
$.40 = \frac{2}{5}$	$.37\frac{1}{2} = \frac{3}{8}$	$.16\frac{2}{3} = \frac{1}{6}$	$1.25 = 1 + \frac{1}{4}$
$.60 = \frac{3}{5}$	$.50 = \frac{1}{2}$	$.33\frac{1}{3} = \frac{1}{3}$	$1.50 = 1 + \frac{1}{2}$
$.80 = \frac{4}{5}$	$.62\frac{1}{2} = \frac{5}{8}$	$.66\frac{2}{3} = \frac{2}{3}$	$1.75 = 2 - \frac{1}{4}$

We may now construct the parts of 10, 100, 1000, thus:

Parts	Of 1	Of 10	Of 100	Of 1000
$\frac{1}{8}$	.125	1.25	12.5	125
$\frac{1}{4}$	.25	2.5	25	250
$\frac{3}{8}$	.375	3.75	37.5	375

and so on.

WRITTEN EXERCISES

206. 1. Find the cost of 27 buggies at $75 each.

$$\begin{array}{r} \$2700 \\ 675 \\ \hline \$2025 \end{array}$$

At $100 each, 27 buggies would cost $2700.
At $25 each, they would cost $\frac{1}{4}$ of $2700, or $675.
At $75 each, then, they cost $2700 – $675, or $2025.
The business man writes as few figures as possible.

In the following, count any fraction of a cent in results as an extra cent.

Find the cost of 85 articles:

2. At 20¢ each	7. At $16\frac{2}{3}$¢ each	12. At $1.25 each
3. At 90¢ each	8. At $33\frac{1}{3}$¢ each	13. At $1.50 each
4. At 25¢ each	9. At $66\frac{2}{3}$¢ each	14. At $1.75 each
5. At 50¢ each	10. At $83\frac{1}{3}$¢ each	15. At $2.50 each
6. At 75¢ each	11. At $37\frac{1}{2}$¢ each	16. At $2.25 each

Find the cost of 34 articles at:

17. $25 each	20. $250 each	23. $12.50 each
18. $75 each	21. $225 each	24. $37.50 each
19. $45 each	22. $500 each	25. $62.50 each

Find products:

26. $27 \times .125$	**31.** 250×67	**36.** $25 \times 44 \times 19$
27. 48×12.5	**32.** 500×99	**37.** $50 \times 24 \times 77$
28. 71×750	**33.** 875×48	**38.** $32 \times 75 \times 22$
29. 62×37.5	**34.** $900 \times .81$	**39.** $75 \times 71 \times 72$
30. $94 \times .625$	**35.** 125×3.9	**40.** $66 \times 16 \times 1.25$

41. How many 50-cent articles can be bought for $1? $5? $75? $250? $105? $225?

42. How many 25-cent articles can be bought for the amounts mentioned in exercise 41? $12\frac{1}{2}$-cent articles? 20-cent articles?

43. How many yards of cloth costing $66\frac{2}{3}$¢ per yard can be bought for $144?

$$\begin{array}{r} 144 \text{ yd.} \\ \underline{72 \text{ yd.}} \\ 216 \text{ yd.} \end{array}$$

At $1 per yard, 144 yards could be bought for $144. But since the price is only $\$\frac{2}{3}$ per yard, $1\frac{1}{2}$ times as many yards can be bought; that is, $1\frac{1}{2} \times 144$ yards can be bought for $144.

How many yards of cloth can be bought for $120 at

44. $66\frac{2}{3}$¢ per yd.?	**47.** $1.25 per yd.?
45. 75¢ per yd.?	**48.** $1.50 per yd.?
46. $83\frac{1}{3}$¢ per yd.?	**49.** $2.50 per yd.?

Divide:

50. 47 by .25	**55.** 316 by 500
51. 64 by 2.5	**56.** 428 by 250
52. 48 by 7.5	**57.** 715 by 625
53. 111 by 37.5	**58.** 240 by 12.5
54. 34.3 by 87.5	**59.** .425 by .125

60. Find the cost of 288 articles @ 50¢; @ 51¢; @ 49¢.

61. Find the cost of 432 articles @ $33\frac{1}{3}$¢; @ 33¢; @ $33\frac{1}{2}$¢.

62. A dealer bought 1024 pounds of tea at $47\frac{1}{2}$¢ per pound and sold it at 61¢ per pound. Find his gain.

REVIEW PROBLEMS IN INDUSTRIES

207. 1. A coffee planter had 85.8 acres of land on which were planted 500 coffee trees per acre. How many trees did his plantation contain?

2. The average yield of raw coffee was 1.6 lb. per tree. Find the yield from the 42,900 trees.

3. The planter sold his coffee on the plantation to a coffee buyer, who paid him $4719 for the entire crop of 68,640 lb. How much was received per pound?

4. How much did he gain, if the whole cost of production was $2951.52?

5. How many cents per pound did it cost him to raise the coffee? How much was his gain on each pound?

6. How much did he gain on each acre?

7. The man who bought the coffee transported it on beasts of burden to the nearest shipping port at a cost of $274.56, and sold it at $7\frac{1}{2}$¢ per pound. Find his gain.

8. The coffee was shipped to the New York market in bags averaging 132 pounds each. How many bags were required?

9. To send the coffee by ship to New York cost $37\frac{1}{5}$¢ per bag. How much were the freight charges on the 520 bags?

10. The coffee was imported by a coffee roaster. If coffee loses .15 of its weight in the process of roasting, how much did this lot of coffee weigh when roasted?

11. One year 11.75 pounds of coffee was the average amount used by each person in the United States, while in Great Britain the average per person was only .67 of a pound. At this rate how much more coffee would be used per year by a city of 75,000 inhabitants in the United States than by a city of the same population in Great Britain?

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12. On a tea plantation of 240 acres in India there were 512,640 tea plants. Find the number of plants per acre.

13. At the age of three years the plants were $4\frac{1}{2}$ feet high. If $2\frac{5}{6}$ feet were then pruned from each plant, how high was each?

14. A woman could pick only $7\frac{3}{4}$ pounds of leaves per day from plants of this age, but twice as much from plants eight years old. Find the earnings of a woman in 6 days, picking leaves from the 8-year-old plants at $\frac{3}{4}$¢ per pound.

15. If each plant furnished $1\frac{1}{8}$ pounds of green leaves during the year, what was the average yield of fresh leaves in pounds from an acre of 2136 plants?

16. It took $4\frac{1}{2}$ pounds of green leaves to make 1 pound of marketable tea. How much finished tea was produced per plant?

17. How many pounds of marketable tea were made from the green leaves (2403 lb.) of one acre? of the entire plantation?

18. In the first step of preparing the tea for market, the green leaves were spread to wither on trays. If 1890 pounds of fresh leaves were withered in one day and each pound required 10 square feet of space, how many trays of 6 square feet each were used?

19. In rolling the withered tea to break the oil cells, a machine rolled as much in 1 hour as a man did by hand in $4\frac{1}{3}$ days. If a good day's work by hand was 75 pounds, how much did the machine roll in a day of 9 hours?

20. When dried and graded, the tea was packed in chests holding 40 pounds each. How many chests were required for the 128,160 lb. of tea?

21. If the actual time consumed in packing one grade of the tea, or $\frac{1}{6}$ of the 3204 chests, was $22\frac{1}{4}$ hours, how many chests were packed per hour?

22. The sum received in Calcutta for the tea of this grade was $2990.40. Find the price received per pound.

23. Shipments of tea to the United States one year were as follows: from China, 53,157,000 pounds; from Japan, 42,700,000 pounds; from India, 7,679,000 pounds; from Great Britain, 6,647,000 pounds; and from other countries, 2,723,000 pounds. Find the total amount.

24. Of the tea shipped to Great Britain that year, 155,196,000 pounds were shipped from India and 11,048,000 pounds from China. How much more or less than the United States did Great Britain receive from each?

25. The average amount of tea consumed during a recent year by every 50 persons in each of the following countries was: Great Britain, 301.5 pounds; Russia, 47.5 pounds; Germany, 6 pounds; France, 3 pounds; United States, 65 pounds. Find the average consumption of tea per person in each country named.

India rubber is obtained by cutting or tapping the bark of certain tropical trees, which give out a milky fluid containing the rubber. The rubber is then hardened from the milk.

26. In the Amazon valley a native tapped 144 trees and secured $\frac{3}{8}$ gal. of milk per tree. How much milk did he secure?

27. Each of the 54 gallons of milk yielded $2\frac{1}{4}$ lb. of rubber. Find the yield from this tapping.

28. From June to December the trees were tapped 16 times. If each tapping yielded as much milk as the first, how much rubber did the season's work produce?

29. How many pounds of rubber were obtained per tree, the total yield being 1944 lb.?

30. The rubber was obtained from the liquid by dipping a paddle into it and then holding the paddle in smoke. If it took 27 hours to obtain the $121\frac{1}{2}$ lb. of rubber from the milk of one tapping, how much rubber was obtained in 1 hour?

31. If at the rate of $4\frac{1}{2}$ lb. per hour a loaf of rubber was made in $2\frac{2}{3}$ hr., how many loaves were made that season?

32. How many chests of 300 pounds capacity were used to pack them in, and how many loaves were left for a smaller package?

33. This rubber was sold in New York as *Fine Para* and brought \$2410.56. What was the price per pound?

34. At this price find the value of the yield from one tree.

35. The Amazon district one year furnished $\frac{5}{12}$ of the world's supply of rubber, or 25,000 tons. Find the world's production.

36. The United States that year purchased 24,675 tons of rubber, of which $\frac{9}{25}$ was used for rubber boots and shoes. How many tons were thus used?

37. Of the rubber boots and shoes made, 24,686,643 pairs were men's, 18,847,865 women's, and 6,445,231 children's. Find the total number of pairs manufactured.

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38. A man engaged in silk culture bought 13 ounces of silkworm eggs at $1.75 per ounce. Find the cost of the eggs.

39. Each ounce contained 36,000 eggs. If $\frac{1}{2}$ of an ounce failed to hatch, how many silkworms were actually obtained?

40. After they began to hatch, the 450,000 silkworms hatched in five days. $\frac{1}{20}$ hatched the first day, $\frac{3}{8}$ the second, $\frac{2}{5}$ the third, $\frac{1}{8}$ the fourth, and $\frac{1}{20}$ the fifth. How many silkworms hatched each day?

41. The silkworms from each ounce of eggs consumed 1800 lb. of mulberry leaves during their feeding period. How many mulberry trees, bearing 125 lb. of leaves each, were needed to provide leaves for the silkworms from $12\frac{1}{2}$ ounces of eggs?

42. If $\frac{1}{4}$ of the worms died during this period, how many lived to spin cocoons?

43. When every silkworm had spun its cocoon it was found that 250 fresh cocoons weighed one pound. What was the total weight of fresh cocoons spun by the 337,500 worms?

44. To prevent the insects emerging from the cocoons as moths, thus breaking the silk thread, they were killed or "choked" in ovens. If the 1350 lb. of cocoons lost .68 of their weight by this process, how much did the choked cocoons weigh?

45. At $\$\frac{7}{8}$ per pound, find the value of these cocoons, 432 lb.

46. Find the weight of fresh cocoons from the silkworms of 1 ounce of eggs; the weight of these cocoons when choked; and the value of the choked cocoons.

47. The amount of raw silk obtained from the choked cocoons was .275 of their weight. How many pounds of silk did the whole number of cocoons furnish?

48. If it took 6 women 26.4 days to reel the 118.8 lb. of silk from the cocoons, how much silk did each woman reel per day?

49. Find the cost of the raw silk to a manufacturer who bought it at $4.15 per pound.

50. A man starting to raise silkworm eggs to sell, first bought 2 ounces, 40,000 eggs to the ounce. $\frac{3}{4}$ of the eggs hatched and the insects grew, spun cocoons, and emerged as moths. If $\frac{1}{2}$ of the moths laid eggs, 300 apiece, and $\frac{2}{3}$ of the eggs were marketable, how many ounces did the man have to sell?

51. Find the value of the 150 ounces of eggs @ $\$2\frac{1}{4}$.

52. The world's production of raw silk one year was: France, 1,256,625 pounds; Italy, 9,803,855 pounds; China, 12,288,565 pounds; Japan, 10,515,940 pounds; and other countries, 8,502,130 pounds. Find the total supply.

53. The price of raw silk per pound in New York was as follows, for the best grade from each place named: Italy, $4.40; Japan, $3.95; Shanghai, China, $4.70; Canton, $3.25. Find the cost of 240 pounds from each of these places.

METHODS OF SOLVING PROBLEMS

208. There is often more than one method of arriving at the answer of a problem, however simple it may be. Suppose that 5000 sheets of paper cost $8. How much will 10,000 sheets cost at the same price?

The answer may be found as follows:

1. **Indirectly**, by finding the cost of 1 sheet, then of 10,000 sheets. Find the answer by this method, called **unitary analysis**.

2. **Indirectly**, by finding the cost of 1 thousand sheets, then of 10 thousand sheets. Find the answer by this method.

3. **Directly**, by comparing the cost of 10,000 sheets with that of 5000 sheets. Find the answer by this method, called **analysis by comparison**. Which is the best method for this problem?

ORAL ANALYSIS

209. In the following problems try to discover the best method of solution. Give the answer first, then the steps in the solution, or the *analysis*, in few words.

One pupil may declare the answer and others give analyses. If more than one analysis is given, the best should be chosen after a comparison.

1. It requires 7 nails to fasten a shoe on a horse's foot. How many are required for 4 shoes?

Answer	Analysis
28 nails.	1 shoe requires 7 nails.
	4 shoes require 4×7 nails.

2. Find the cost of 18 oranges at 3 for 10 cents.

Answer	Analysis
60 cents.	3 oranges cost 10 ¢.
	1 orange costs $\frac{1}{3}$ of 10 ¢.
	18 oranges cost $\frac{18}{3}$ of 10 ¢.

This is not the best analysis. Give a better one.

3. A boy had \$5. After buying a hat he had \$3 remaining. How much did he pay for the hat?

Analysis

All his money = \$5; one part = \$3; other part paid for hat = \$5 − \$3.

4. A man bought 60 pears at 3 ¢ each and sold them at 5 ¢ each. How much did he gain?

Analyze by finding the amounts paid and received; again by finding first the gain on 1 pear. Compare analyses.

5. Find the cost of 30 towels at 3 for \$1.

6. How far is it around a city block each of whose sides is 300 feet long?

7. How far will a motor car go in 8 hours at the rate of 25 miles an hour?

8. Mary has 40 stamps and Luther has 4 times as many. How many stamps have both?

9. A grocer bought 80 quarts of berries at 8 ¢ a quart and sold them at 10 ¢ a quart. Find his gain.

10. If a man smokes 2 10-cent cigars a day, how much will cigars cost him per month of 30 days?

11. Charles bought 8 bananas at 2 ¢ each and had 14 ¢ left. How much money had he at first?

12. Clarence had \$2 and bought 2 chickens at 75 ¢ each. How much money had he left?

13. Edwin bought 2 sailboats at 80 ¢ each and handed the dealer \$2. How much change was due him?

14. If 1 yard of ribbon costs 9¢, how many yards can be bought for $1.80?

ANALYSIS

1 yard costs 9¢; the number of yards that can be bought for $1.80 is the same as the number of times $1.80 contains $.09.

15. At a fire-engine house the time from 9 P.M. to 6 A.M. is divided into watches of 3 hours each. How many watches are there?

16. For this work 9 men are detailed every night. How many are there on each watch?

17. The hours of exercise for the horses are from 7 A.M. to 5 P.M. Only 2 of the 10 teams in a precinct can be exercised at a time. How many hours may each team be exercised?

18. The chemical wagon carries 750 feet of engine hose and 250 feet of chemical hose. What part is engine hose?

ANALYSES

1. 750 ft. $= 3 \times 250$ ft.; 750 ft. + 250 ft. $= 4 \times 250$ ft.; 3 of the 4 equal parts of 1000 ft., or $\frac{3}{4}$ of the whole, is engine hose.

2. 750 ft. + 250 ft. = 1000 ft.; 750 ft. $= \frac{750}{1000}$, or $\frac{3}{4}$ of 1000 ft.

19. The chemical wagon carries also a 60-gallon extinguisher and 2 3-gallon extinguishers. What part of the extinguishing liquid is carried in the large extinguisher?

20. The city paid $16,800 for 8 chemical wagons. Find the cost of each.

21. The hook-and-ladder men can erect their ladder in 52 seconds, or in —— of a minute.

22. At a zoölogical garden the animals eat 250 pounds of beef per day. Find the cost at $6 per 100 pounds.

23. The daily ration of bread is 100 pounds and of fish 30 pounds. Find the amount of each consumed in 30 days.

24. The weekly ration of vegetables is 8 barrels. How many barrels is that per year of 52 weeks?

25. The animals eat 280 bananas per week. How many dozen bananas do they eat per day?

26. The hippopotamus Caliph weighs about 6000 pounds. How many tons (2000 pounds) does he weigh?

27. Caliph eats 50 pounds of hay every day. How long does a ton of hay last him?

28. A baby elephant weighs about 200 pounds, an adult elephant about 10,000 pounds. Compare in two ways the weight at birth with the weight when full grown.

29. The boundary line between Massachusetts and New York is marked by 121 monuments, 83 of which are of granite and the rest of iron. How many more granite monuments are there than iron monuments?

30. I can buy ready-made screens for my 12 windows at 50 ¢ each, but better ones made to order will cost me $1.50 each. How much more will the better ones cost?

31. How many times as much will the better screens cost?

32. If the better screens will last 5 times as long as the poorer ones, how many times as expensive are the latter as the former?

33. If $12\frac{1}{2}$ yards of silk cost $15, how much will 25 yards cost? How much silk can be bought for $60?

34. If 1200 oysters cost $6, how much will 1000 cost?

35. If 100 pounds of meat cost $8, how much will 225 pounds cost? $50 will buy —— pounds.

36. If 1000 sheets of paper cost $3.20, how much will 750 sheets cost? $16 will buy —— sheets.

37. If $37\frac{1}{2}$ pounds of butter cost $7.50, how much will 100 pounds cost? $15 will buy —— pounds.

38. If 1000 bricks cost $10, how much will 2500 cost? How many can be bought for $125?

39. If $12\frac{1}{2}$ yards of matting cost \$5, how much will $62\frac{1}{2}$ yards cost? \$20 will buy —— yards.

40. If $2\frac{1}{2}$ pounds of peanut candy cost 50¢, how much will $6\frac{1}{2}$ pounds cost?

ANALYSES

1. $\frac{5}{2}$ lb. cost 50¢; $\frac{1}{2}$ lb. costs $\frac{1}{5}$ of 50¢; $\frac{13}{2}$ lb. cost 13 times $\frac{1}{5}$ of 50¢.

2. $\frac{5}{2}$ lb. cost 50¢; $\frac{2}{5}$ of $\frac{5}{2}$ lb., or 1 lb., costs $\frac{2}{5}$ of 50¢; $\frac{13}{2}$ lb. cost $\frac{13}{2}$ of $\frac{2}{5}$ of 50¢.

41. If $1\frac{1}{2}$ pounds of sirloin steak cost 30 ¢, how much will 5 pounds cost?

42. Mr. Jones bought $12\frac{1}{2}$ pounds of lamb for \$1.50 and sold me $3\frac{1}{2}$ pounds at cost. How much did I pay?

43. If $\frac{2}{5}$ of the value of a store is \$4000, how much is the store worth?

44. If 2 men can do a piece of work in 12 days, how long will it take 4 men to do it?

ANALYSES

1. The amount of work to be done is 2×12 days' work, or 24 days' work. In 1 day 4 men do 4 days' work. To do 24 days' work 4 men will require $\frac{24}{4}$ days, or 6 days.

2. 4 men can do twice as much work as 2 men in the same time, or the same work in half the time. It will take 4 men $\frac{1}{2}$ of 12 days.

45. If 5 men can paint a bridge in 10 days, how long will it take 10 men?

46. How long would it take 2 men to paint the bridge?

47. If 40 men can dig a ditch in 5 days, how long will it take 20 men?

48. A contractor has 30 days in which to do a piece of work. His present force of 60 men would require 50 days to do the work. How many more men must he employ?

49. A contractor who has 80 laborers in his employ can complete a piece of work in 30 days. If he discharges 20 laborers, how much longer will it take to complete the work?

WRITTEN ANALYSIS

210. Under modern business conditions problems must be solved with the utmost rapidity and with absolute accuracy. An inaccurate result is worthless — it may be disastrous.

No answer may be regarded as accurate unless it has been *checked*, preferably by solving the problem or parts of it in two different ways. Knowing this, yet keeping in mind the value of time, the business man sets down his work, briefly but in a neat form, so that he or any other person may review it at a glance with complete understanding.

211. 1. At a manufacturing plant, 9600 men are employed 55 hours a week. The weekly pay roll is $138,500. Find the average wage per hour to the nearest tenth of a cent.

Mental Estimate

10 thousand men working 50 hours each would do 500 thousand hours' work. 138 thousand dollars for 500 thousand hours' work is about $\$\frac{1}{4}$, or 25¢ per hour.

Written Analysis

1 man does 55 hours' work.
9600 men do 9600 × 55 hours' work.
9600 × 55 hours' work earns $138,500 in wages.

1 hour's work earns $\frac{\$138{,}500}{9600 \times 55}$ in wages.

Business Process

12	\$1385~~00~~
8	115.417
11	14.427
5	1.312
	.262

After arriving mentally at the result given in the last line of the written analysis, the business man would employ this process instead of the cancellation process, because it is neater, more easily reviewed, and less liable to error.

Checks. — 1. The result agrees well with the estimate, consequently no large errors (such as pointing off wrong) have been made.

2. Reviewing the process rapidly, the work is found to be correct.

In solving the following problems, estimate the result in advance when you can; write brief analyses; arrange computations in a businesslike manner; check results.

2. A telephone company owned 32,425 telephones worth \$12.50 each. How much were all worth?

SUGGESTION. \$12.50 is $\frac{1}{8}$ of \$100.

3. A merchant bought 96 suits of clothes at \$12 each and sold them at \$19.50 each. How much did he gain?

4. He bought 125 overcoats at \$35 each, and sold them at \$60 each. Of this money \$425 was lost in bad debts. Find the average gain on each overcoat.

5. Of the pears picked from an orchard, 65 bushels were sold at the home canning factory at 95¢ per bushel. How much did they bring?

6. Also 284 bushels were shipped to Boston, kept in cold storage for a month, and sold at \$2.25 per bushel. Freight charges were \$32, storage 15¢ per bushel, and charges for selling 5¢ per bushel. Find the proceeds of this sale.

7. The medium-sized late pears were packed in boxes and sold at Christmas time at \$1.10 per box. There were 432 boxes. Find the proceeds, deducting \$77.50 for freight, etc.

8. The largest late pears were shipped in boxes to Liverpool for the holiday trade. There were 360 boxes, and they brought $6\frac{1}{2}$ shillings (1 shilling = $24\frac{1}{3}$¢) per box. Transportation and other charges amounted to $\frac{3}{4}$ shillings per box. Find the proceeds.

9. Find the net income from the pear orchard, deducting 45¢ per bushel and 25¢ per box, as the cost of production.

10. It required 1,836,000 bricks to build an office building. Find the cost of the bricks at \$9.25 per M.

11. A stove manufacturer bought $22\frac{1}{2}$ long tons (1 long ton = 2240 lb.) of pig iron at \$16.80 per ton. How much did it cost him? How much per pound did he pay?

Find the cost of:

12. 1650 pounds of lard at \$9.75 per 100 pounds.

13. 7425 bushels of corn at $49\frac{7}{8}$¢ per bushel.

14. 1460 yards of flannel at \$.75 per yard.

15. 2025 baskets of peaches at \$1.25 per basket.

16. 6847 pounds of sugar at \$4.25 per 100 pounds.

17. A car load of wheat, 1075 bushels @ $77\frac{3}{8}$¢.

18. A dealer sold a lot of rugs for \$9360, which was $\frac{1}{3}$ more than they cost him. How much did they cost him?

19. The Maynard Co. bought 12 pianos for \$3300, and sold each at \$25 more than $\frac{7}{5}$ of the cost. Find the selling price of each piano and the total gain.

20. In the settlement of an estate worth \$62,500 there were lawsuits that cost the estate \$43,750. What part of the estate was left to divide among the heirs?

21. The senior partner of a firm owned $\frac{4}{5}$ of the business, and sold $\frac{1}{3}$ of his share for \$15,120. At that rate how much was the whole business worth?

22. What is the cost of 10 pieces of cloth containing $42\frac{1}{2}$, $45\frac{3}{4}$, 46, 44, $47\frac{3}{4}$, $48\frac{1}{2}$, $48\frac{1}{2}$, $45\frac{3}{4}$, 47, and $43\frac{3}{4}$ yards, respectively, at $4\frac{3}{4}$¢ per yard?

23. A speculator bought 6000 bales of cotton averaging 500 pounds per bale, at 9.12¢ per pound. The market declined 4 points, or .04 of a cent, and he sold $\frac{1}{3}$ of the cotton; it then advanced 6 points, and he sold the rest. Did he gain or lose, and how much?

24. A man bought 4 car loads of corn, net weights 47,600 lb., 50,400 lb., 49,000 lb., and 53,200 lb., at $42\frac{3}{4}$¢ per bushel of 56 pounds. He sold the first car load at $42\frac{7}{8}$¢ per bushel, the second at $43\frac{5}{8}$¢, the third at $43\frac{3}{8}$¢, and the fourth at $44\frac{1}{4}$¢. How much did he gain?

ANALYSIS BY EQUATIONS

212. How many pounds added to 25 pounds will give 30 pounds?

The statement of the problem may be condensed to

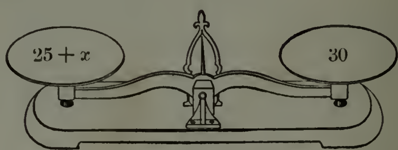
$$\begin{array}{r} 25 \text{ pounds} \\ +\underline{\ ?\ } \text{ pounds} \\ 30 \text{ pounds} \end{array} \quad \text{or} \quad \begin{array}{r} 25 \text{ pounds} \\ +\underline{\ x\ } \text{ pounds} \\ 30 \text{ pounds} \end{array} \quad \text{or} \quad 25 + x = 30$$

The letter x is only a convenient symbol for the **unknown number** (of pounds), or the number (of pounds) to be found. 25 and 30, on the other hand, are **known numbers.**

$25 + x = 30$, the briefest possible statement of the relation between the known and unknown numbers in the problem, is an **equation.** Finding the value of x is called **solving** the equation, $25 + x$ is the **first member** of the equation, and 30 is the **second member.**

Solution of Equations in x

213. 1. If 25 lb. is subtracted from the weight in each scale pan, the balance will be preserved.



In the same way, if 25 is subtracted from each member of the equation $25 + x = 30$, the equality will be preserved.

$$\begin{array}{rcr} 25 + x & = & 30 \\ \underline{25} & & \underline{25} \\ x & = & 5 \end{array}$$

2. What number subtracted from $x + 10$ will give x?

If the first member of $x + 10 = 12$ is decreased to x by subtracting 10, what must be done to the second member to preserve the equality?

Tell how the equation $x + 10 = 12$ may be solved.

3. Suppose that $x - 4 = 3$ and we wish to find the value of x. How much greater is x than $x - 4$?

If the first member of $x - 4 = 3$ is increased to x by adding 4, what must be done to the second member to preserve the equality? Tell how the equation may be solved.

The same number may be added to both members of an equation, or subtracted from both, without destroying the equality.

EXERCISES

214. First state what must be done to both members to change one member to x without destroying the equality; then solve:

1. $x + 6 = 8$	9. $x + 2 = 10$	17. $12 = 10 + x$
2. $x - 3 = 2$	10. $x - 5 = 11$	18. $15 = 11 + x$
3. $x - 4 = 5$	11. $x + 1 = 12$	19. $30 = 20 + x$
4. $x + 7 = 9$	12. $x - 7 = 10$	20. $14 = x + 10$
5. $x - 3 = 8$	13. $x + 9 = 12$	21. $22 = x + 20$
6. $x - 5 = 1$	14. $9 + x = 12$	22. $16 + x = 25$
7. $x + 5 = 7$	15. $5 + x = 15$	23. $20 + x = 24$
8. $x + 3 = 9$	16. $3 + x = 17$	24. $30 + x = 50$

215. 1. Just as $3 + 3 + 3 + 3 = 4 \times 3$, so $x + x + x + x = 4 \times x$. It is written $4\,x$, however, *without a multiplication sign between the figure and the letter.* What does $5\,x$ mean? $6\,x$?

2. If $x = 8$, what is the value of $2\,x$? of $3\,x$? of $\frac{5}{2}\,x$?

3. If $6\,x = 12$, what is the value of $1\,x$, or of x?

4. If $\frac{1}{3}\,x = 10$, what is the value of 3 times $\frac{1}{3}\,x$, or of x?

5. What must be done to both members of each of the following equations to give an equation whose first member is x?

$\frac{1}{2}\,x = 3$ $\qquad$ $\frac{1}{3}\,x = 5$ $\qquad$ $4\,x = 12$ $\qquad$ $5\,x = 35$

Both members of an equation may be multiplied or divided by the same number without destroying the equality.

EXERCISES

216. First state what must be done to both members to change one member to x without destroying the equality; then solve:

1. $2x=6$	7. $\frac{1}{2}x=5$	13. $\frac{1}{2}x=1.5$
2. $5x=5$	8. $\frac{1}{3}x=2$	14. $\frac{1}{2}x=4.2$
3. $4x=8$	9. $\frac{1}{4}x=3$	15. $\frac{1}{4}x=.25$
4. $3x=15$	10. $\frac{1}{5}x=7$	16. $5x=12$
5. $8x=24$	11. $\frac{1}{6}x=5$	17. $2x=17$
6. $9x=18$	12. $\frac{1}{8}x=4$	18. $4x=4.4$

WRITTEN EXERCISES

217. 1. What number increased by 6 is equal to 44?

SOLUTION

Let x = the number.

Then, $x + 6 = 44$

Subtracting 6 from both members. $x = 38$

Test. $38 + 6 = 44$.

2. What number increased by 15 is equal to 51?

3. What number decreased by 32 is equal to 60?

4. What number multiplied by 3 is equal to 78?

5. What number divided by 8 is equal to 62?

6. A certain number added to 87.5 gives 100. Find it.

7. Twelve times a certain number is 15. Find the number.

8. If $\frac{1}{6}$ of the number of books I have is 18, how many books have I?

9. If 20 is added to a certain number and 14 is subtracted from the sum, the result is 19. Find the number.

10. One half of a number, and 11 more, is equal to 37. Find $\frac{1}{2}$ of the number, then find the number.

WRITTEN EXERCISES

218. **1.** Solve the equation $\frac{3}{2}x = 15$.

FIRST SOLUTION

$$\frac{3}{2}x = 15$$

Dividing both members by 3, $\frac{1}{2}x = 5$

Multiplying both members by 2, $x = 10$

SECOND SOLUTION

By multiplying by 2 *before* dividing by 3, fractions may be avoided.

$$\frac{3}{2}x = 15$$

Multiplying both members by 2, $3x = 30$

Dividing both members by 3, $x = 10$

Test. $\frac{3}{2}$ of $10 = 15$.

Solve:

2. $\frac{3}{2}x = 9$

3. $\frac{4}{3}x = 8$

4. $\frac{5}{2}x = 10$

5. $\frac{2}{3}x = 14$

6. $\frac{3}{5}x = 21$

7. $\frac{2}{3}x = 30$

8. $\frac{4}{5}x = 28$

9. $\frac{5}{6}x = 20$

10. $\frac{5}{8}x = 15$

11. $\frac{7}{3}x = 21$

12. $\frac{7}{8}x = 63$

13. $\frac{6}{7}x = 48$

14. If $\frac{3}{4}$ of a certain number is 18, what is the number?

15. If $\frac{5}{8}$ of the number of pupils in a school is 250, what is the whole number of pupils?

16. A basket-ball team won 16 games, or $\frac{2}{3}$ of the games it played. Find the number of games it played.

17. If $\frac{3}{8}$ of the number of persons who went on an excursion to Niagara Falls were teachers, and 240 teachers went, find the whole number of persons who went.

18. Find the number of feet in the width of a street, if $\frac{3}{5}$ of the width, or 48 feet, lies between the curbstones.

19. On an elevated belt-line railway, $9\frac{3}{5}$ minutes, or $\frac{4}{15}$ of the time required to make a round trip, was consumed in stops. Find the number of minutes required to make a round trip.

219. **1.** How does the value of $6+4-4$ compare with 6? with $(6-4)+4$? with $(4-4)+6$?

2. Compare $5+7-2-3$ with $12-5$; with $5-(2+3)+7$; with $7-2-3+5$; with $7+(5-2-3)$; with $(-2-3+5)+7$.

3. 5, or $+5$, means $1+1+1+1+1$, that is, 5 units taken *additively*, or 5 **positive units**; -2 means $-1-1$, that is, 2 units taken *subtractively*, or 2 **negative units**. What does $+7$ mean? -3?

4. The expression $5+7-2-3$ is composed of 12 positive units and 5 negative units, and in whatever order or in whatever groups we unite the units, the 5 negative units will finally cancel 5 of the positive units, leaving 7 positive units, or $+7$.

5. Unite **terms** as indicated by their signs:

20	2 tens	2×10	$2t$	$2x$	$4x$
$+40$	$+$ 4 tens	$+4\times 10$	$+4t$	$+4x$	$-3x$
$-\underline{30}$	$-$ <u>3 tens</u>	$-\underline{3\times 10}$	$-\underline{3t}$	$-\underline{3x}$	$+\underline{2x}$

Such terms as $+4x$, $-3x$, and $+2x$ are called **like terms** because they have the same unit, x.

The multipliers, $+4$, -3, $+2$, are called **coefficients** of x.

In adding like terms, any number of negative units cancel an equal number of positive units.

EXERCISES

220. Add downward, then upward:

1.	2.	3.	4.	5.
$+5x$	$+7x$	$+2x$	$+8x$	$+5x$
$+3x$	$-6x$	$-2x$	$-4x$	$+2x$
$-\underline{4x}$	$+\underline{5x}$	$+\underline{\ \ x}$	$+\underline{9x}$	$-\underline{7x}$

Unite the terms of the following in various orders and by grouping in different ways, to find their sum:

6. $2x+5x-2x$

7. $7x-6x+4x$

8. $3x+8x-9x$

9. $6x-4x-2x+3x$

10. $8x-5x+4x-3x$

11. $9x-7x-3x+4x$

221. 1. Adding 7 to both members of the equation

$$x - 7 = 3,$$

we obtain $x = 3 + 7$, or 10.

-7 has been removed from the first member, but reappears in the second member with the opposite sign.

2. Subtracting 5 from both members of the equation

$$x + 5 = 9,$$

we obtain $x = 9 - 5$, or 4.

When $+5$ is removed, or **transposed**, from the first member to the second, what must be done to its sign?

3. Explain transposition of terms in the following:

$2x - 1 = 5;$	$3x + 2 = 11;$	$4x = 14 - 3x;$
$2x = 5 + 1.$	$3x = 11 - 2.$	$4x + 3x = 14.$

Any term may be transposed from one member of an equation to the other, provided its sign is changed.

WRITTEN EXERCISES

222. 1. Solve the equation $2x + 20 = 80 - 4x$.

First Solution	Second Solution
$2x + 20 = +80 - 4x$	$2x + 20 = 80 - 4x$
$4x, -20, \quad -20, +4x$	$2x + 4x = 80 - 20$
$6x \quad = +60$	$6x = 60$
$x = +10$	$x = 10$

The first step in solving an equation is to get the unknown terms into one member (usually the first member) and the known terms into the other member.

Adding $4x$ to both members (as in the first solution), or transposing $-4x$ from the second member to the first, and changing its sign (as in the second solution), places all unknown terms in the first member.

Subtracting 20 from both members, or transposing $+20$ from the first member to the second and changing its sign, places all known terms in the second member.

2. Solve $2x + 2x - 4x + 3x + 8x = 3x + 40$.

SOLUTION. — Since $2x + 2x - 4x = 0$, these terms may be omitted, or *canceled*, from the first member.

If the term $3x$ in the second member were transposed to the first member, it would become $-3x$ and so cancel $+3x$ in the first member. Consequently $+3x$ may be canceled from both members without transposing.

The equation becomes $8x = 40$

Dividing both members by 8, $x = 5$

Test. — We should always test the answer by finding whether the value obtained for x is such as to make the members of the original equation equal. Thus, if $x = 5$, the first member becomes

$$10 + 10 - 20 + 15 + 40, \text{ or } 55,$$

and the second $15 + 40$, or 55.

Solve and test:

3. $7x + 12 = 5x + 16$

4. $5x - 20 = 2x + 13$

5. $9x - 17 = 23 + x$

6. $22 - 6x = 40 - 8x$

7. $2x + 3x - 2x = 21$

8. $9x - 4x + 2x = 14$

9. $8x + 5x - 5x = 48$

10. $6x - 2x - x = 45$

11. $5x + 8x + 9x = 44$

12. $7x + 6x - 7x = 42$

13. $4x - 11 + 2x = 2x - 5$

14. $3x + 14 + 7x = 78 + 2x$

15. $44 - 3x - 2x = 79 - 12x$

16. $62 - 2x - 12 = 75 - 27x$

17. $14 - 21 - 30 = x - 2x - 21$

18. $22 - 15 + 21 = 2x - 4 + 6x$

19. $46 + 3x - 60 = 5x - 10 - 4x$

20. $8x - 20 - 2x = 30 - 2x + 14$

21. $5x + 16 - 6x = 16 + 24 - 6x$

22. $6x + 5x - 70 = 5x + 54 - 70$

23. $10x - 39 - 4x - 9x + 42 + 12x = 30 + 12 - 4x$

24. $16x + 12 - 75 + 2x - 12 - 110 = 8x - 50 - 25$

25. $11x - 60 + 5x + 17 - 2x - 3x + 41 = 106 + 2x$

26. $18x + 16 = 8 + 12x + 8 - 13 + 25x - 9 + 100 - 25x$

27. $14x - 35 = 9 - 11x + 4 + 16 - 10x + x + 136 - 16x$

28. Solve $\frac{1}{2}x - \frac{1}{3}x = 10 - \frac{1}{4}x$.

FIRST SOLUTION

	$\frac{1}{2}x - \frac{1}{3}x = 10 - \frac{1}{4}x$
Transposing,	$\frac{1}{2}x - \frac{1}{3}x + \frac{1}{4}x = 10$
Uniting terms,	$\frac{5}{12}x = 10$
Multiplying by 12,	$5x = 120$
Dividing by 5,	$x = 24$

If the steps of the first solution are taken in a different order, by multiplying by the l. c. d. 12 *before* transposing and uniting terms, the equation will be freed, or *cleared, of fractions*, thus:

SECOND SOLUTION

	$\frac{1}{2}x - \frac{1}{3}x = 10 - \frac{1}{4}x$
Multiplying by 12,	$6x - 4x = 120 - 3x$
Transposing,	$6x - 4x + 3x = 120$
Uniting terms,	$5x = 120$
Dividing by 5,	$x = 24$

Solve and test:

29. $\frac{1}{4}x + \frac{1}{6}x = 10$

30. $\frac{3}{4}x - \frac{1}{9}x = 23$

31. $\frac{9}{10}x - 10 = \frac{2}{5}x$

32. $\frac{1}{2}x + \frac{7}{8}x = 440$

33. $\frac{2}{3}x = \frac{1}{3}x - \frac{7}{6}x + 27$

34. $\frac{4}{5}x + 12 = \frac{3}{8}x + 29$

35. $\frac{9}{4}x - 50 = 15 - \frac{1}{14}x$

36. $\frac{1}{2}x + \frac{1}{3}x + \frac{3}{4}x + \frac{4}{5}x = 143$

WRITTEN EXERCISES

223. 1. The sum of two numbers is 55 and the larger is 4 times the smaller. What are the numbers?

SOLUTION. — Let x = smaller number.
Then $4x$ = larger number, and $x + 4x$, or $5x$, = sum.

Therefore,	$5x = 55$
	$x = 11$, smaller number
	$4x = 44$, larger number

2. Separate 116 into two parts, one of which shall be 3 times the other.

3. Two boys dug 160 clams. If one dug 3 times as many as the other, how many did each dig?

4. The water and steam in a boiler occupied 120 cubic feet of space, and the water occupied twice as much space as the steam. How many cubic feet of space did each occupy?

5. What number added to 5 times itself equals 12?

6. Two boys bought a boat for $45. One furnished 4 times as much money as the other. How much did each furnish?

7. During the summer they earned $52.50 by renting the boat. Find the amount of rent due each boy.

8. Separate 72 into two parts one of which shall be $\frac{1}{3}$ of the other.

SUGGESTION. — Let $x =$ the smaller part.

9. Separate 78 into two parts one of which shall be $\frac{1}{5}$ of the other.

10. The sum of a number and .04 of itself is 46.8. What is the number?

11. What number decreased by .35 of itself equals 52?

12. Messrs. Jones, Hollis & Frye invested $225,000 in a line of steamboats. Mr. Hollis invested 3 times as much as Mr. Jones, and Mr. Frye 5 times as much as Mr. Jones. How much did each invest?

13. At the end of a season they divided $2340 in profits according to their respective investments. How much did each receive?

14. It cost a mine owner $1.90 per ton to mine soft coal and ship it to market. The cost of shipping the coal was $.10 more per ton than the cost of mining it. Find the cost of mining it.

15. A wagon loaded with coal weighed 4200 pounds. The coal weighed 1800 pounds more than the wagon. How much did the wagon weigh? the coal?

16. The total height of a certain brick chimney in St. Louis is 172 feet. The height above ground is 2 feet more than 16 times the depth below. How high is the part above ground?

17. At the waterworks 2 large pumps and 4 small ones delivered 4800 gallons of water per minute. Each of the large pumps delivered 4 times as much water as each small pump. How many gallons per minute did each small pump deliver? each large pump?

18. One year 1500 violins were made in the United States. Twice as many were made in New York as in Massachusetts, and these two states made half of all that were made. How many violins were made in Massachusetts? in New York?

19. In lighting a hall a certain number of 16-candle power electric lamps and twice as many 20-candle power lamps were used. The total illumination amounted to 224 candle power. Find the number of lamps of each kind used.

20. It cost Mary 60¢ to send a telegram, at "30–2," or 30¢ for the first 10 words and 2¢ for each additional word. How many words did her message contain?

SOLUTION. — Let x = number of words in message.

Then, $x - 10$ = number of words in excess of 10 words.

Therefore, $$30 + 2(x - 10) = 60$$

Since $x - 10$ is 10 less than x, $2(x-10)$ is 20 less than $2x$.

$$30 + 2x - 20 = 60$$

$$x = 25$$

Test. — She paid 30¢ for 10 words and 30¢ for the 15 additional words.

Find the number of words in each of the following:

	RATE	COST		RATE	COST		RATE	COST
21.	20–1	30¢	**24.**	30–2	66¢	**27.**	50–3	83¢
22.	25–2	39¢	**25.**	25–1	52¢	**28.**	60–4	96¢
23.	25–2	61¢	**26.**	40–3	55¢	**29.**	75–5	95¢

30. If you had 50¢, how many minutes could you converse over the telephone with a friend in a distant city, at 25¢ for the first three minutes and 5¢ for each additional minute?

31. Recently there were 435 piano factories in Germany. This was 15 more than 3 times the number in Berlin. How many piano factories were there in Berlin?

32. The total amount of plastering in the Agricultural Building at St. Louis was 64,000 square yards. There was 4000 square yards more than 4 times as much outside plastering as inside plastering. Find the amount of each.

33. What number increased by $\frac{1}{2}$ of itself equals 54?

34. If $\frac{7}{8}$ of a number is 112, what is the number?

35. What number decreased by $\frac{1}{5}$ of itself equals 84?

36. At one time $\frac{3}{19}$ of the miners in South Africa were Chinese. How many miners were there, if 21,000 were Chinese?

37. How many pounds of green hides are required to make 240 lb. of leather, if the leather weighs $\frac{3}{4}$ as much as the hides?

38. During a mild February, coke declined $\frac{3}{14}$ in market price. The price at the end of the month was $2.20 per ton. What was the price at the beginning of the month?

39. The width of the St. Lawrence River at Quebec at a point where it is spanned by a bridge is 1800 feet. This is $\frac{6}{11}$ of the length of the bridge. How long is the bridge?

40. The distance between two cities is 35 miles by rail. This is $\frac{5}{6}$ of the distance by boat. Find the distance by boat.

41. Of the steam vessels built on the Great Lakes one year, 21, or 5 less than $\frac{1}{3}$ of all, were of steel. How many steam vessels were built on the Lakes that year?

42. The Canadian, or Horseshoe Falls, in the Niagara River are 158 feet high. This is 8 feet more than $\frac{10}{11}$ of the height of the American Falls. Find the height of the American Falls.

43. A girl spent $\frac{1}{3}$ of her money for ice cream and $\frac{1}{4}$ of it for cake. She then had left 5 cents less than $\frac{1}{2}$ of her money. How much money had she at first?

Solution. — Let x = number of cents she had at first.

Then she spent $\frac{1}{3}x$ cents for ice cream and $\frac{1}{4}x$ cents for cake, and had left $\frac{1}{2}x - 5$ cents.

Amount spent for ice cream + amount spent for cake + amount left = all her money.

Therefore,
$$\frac{1}{3}x + \frac{1}{4}x + \frac{1}{2}x - 5 = x$$
$$4x + 3x + 6x - 60 = 12x$$
$$x = 60$$

Test. — She had 60¢ at first, spent 20¢ for ice cream, 15¢ for cake, and had left 25¢, or 5¢ less than $\frac{1}{2}$ of 60¢.

44. After spending $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{5}$ of my money, I had 80 cents left. How much had I at first?

45. A man spent \$700 more than $\frac{3}{5}$ of his income and then had \$2200 left. What was his income?

46. A man deposited $\frac{2}{3}$ of his month's salary in a bank and spent \$4 less than $\frac{4}{5}$ of the remainder. He then had \$14 left. How much salary did he receive per month?

47. What is the number whose double, half, and third added to 33 will give 152?

48. Of the passengers on board an ocean liner $\frac{1}{5}$ were first-cabin passengers, $\frac{1}{3}$ were second-cabin passengers, and the rest, 805 persons, were steerage passengers. Find the whole number of passengers.

49. At a recent date Germany had 1025 ships of over 1000 tons capacity. There were 25 more than $\frac{1}{4}$ as many sailing vessels as steamers. Find the number of each.

50. If $\frac{2}{3}$ of the members of a boys' club live in the fifth ward, $\frac{1}{4}$ in the sixth, and the rest, 20 boys, in the seventh, how many boys belong to the club?

51. During March the A class deposited \$6 more in the school savings bank than the B class. $\frac{2}{3}$ of the A class deposits were equal to $\frac{4}{5}$ of the B class deposits. How much did each class deposit?

SOLUTION. — Let $x =$ number of dollars deposited by B class.

Then, $x + 6 =$ number of dollars deposited by A class.

Therefore $\frac{2}{3}(x + 6) = \frac{4}{5}x$.

We find $\frac{2}{3}$ of $(x + 6)$ by finding $\frac{2}{3}$ of x and $\frac{2}{3}$ of 6, and adding the results. Therefore $\frac{2}{3}(x + 6) = \frac{2}{3}x + 4$.

Hence,

$$\frac{2}{3}x + 4 = \frac{4}{5}x$$
$$10x + 60 = 12x$$
$$60 = 2x$$
$$x = 30,\ x + 6 = 36.$$

Test. — The A class deposited \$36; the B class, \$30. $\frac{2}{3}$ of \$36 = $\frac{4}{5}$ of \$30.

52. Separate 84 into two parts such that $\frac{1}{2}$ of the greater is equal to $\frac{5}{6}$ of the less.

SUGGESTION. — Let $x =$ greater part, $84 - x =$ less part.

53. Separate 100 into two parts such that $\frac{1}{2}$ of the greater is equal to $\frac{3}{4}$ of the less.

54. The Massachusetts school term recently was 189 days. If $\frac{1}{3}$ of this term was $\frac{3}{5}$ of the school term in Alabama, what was the length of the school term in Alabama?

55. The weight of a locomotive, baggage car, and 7 coaches was 502.4 tons. The baggage car weighed $\frac{1}{4}$ as much as the locomotive, and each coach .27 as much as the locomotive. Find the weight of the locomotive; of the baggage car; of each coach.

56. The width of a room is $\frac{3}{4}$ of its length, and the distance around the room is 70 feet. Find the length and the width.

57. For every car load of iron ore dumped into a furnace, $\frac{7}{8}$ of a car load of coke was used for fuel and $\frac{3}{8}$ of a car load of limestone was used for a flux. In all 450 car loads of ore, coke, and limestone were used per day. How much of each was used per day in the furnace?

Solution of Equations in x and y

224. If 4 bananas and 9 oranges cost 35¢, and 4 bananas and 6 oranges cost 26¢, and it is required to find the cost of 1 of each, we may simplify the problem thus:

	4 bananas and 9 oranges cost 35¢	(1)
	4 bananas and 6 oranges cost 26¢	(2)
Subtracting,	3 oranges cost 9¢	(3)

By thus *eliminating* the cost of the bananas, we have obtained a relation, (3), more simple than either of the two given relations, (1) and (2), for it involves only one unknown cost.

Or, let x be the number of cents 1 banana costs, and y the number of cents 1 orange costs.

Then 4 bananas will cost $4x$ cents, 9 oranges $9y$ cents, etc.

$$4x + 9y = 35 \qquad (1)$$
$$4x + 6y = 26 \qquad (2)$$

Eliminating the x's,

$$3y = 9 \qquad (3)$$

$y = 3$, or 1 orange costs 3¢.

Since $y = 3$, $9y$ in the first equation is equal to 27.

Substituting 3 for y in the first equation,

$$4x + 27 = 35 \qquad (4)$$

$x = 2$, or 1 banana costs 2¢.

To **test** the answers we substitute 2 for x and 3 for y in the given equations.

Equation (1) becomes $8 + 27 = 35$, or $35 = 35$;
Equation (2) becomes $8 + 18 = 26$, or $26 = 26$.

Therefore the values obtained for x and y *satisfy* both equations, and the answers are correct.

This method of elimination is called **elimination by subtraction.**

In eliminating the x's on page 129, equal numbers, $4x + 6y$ and 26, are subtracted from both members of (1). Therefore the results are equal, giving a true equation, $3y = 9$.

If equals are subtracted from equals, the results are equal.

225. How must the equations $2x + 3y = 16$ and $5x - 3y = 19$ be combined to eliminate the y's?

$$\begin{array}{r} 2x + 3y = 16 \\ 5x - 3y = 19 \\ \hline 7x \qquad = 35 \end{array}$$

This method of elimination is called **elimination by addition.**

If equals are added to equals, the results are equal.

WRITTEN EXERCISES

226. **1.** If $2x + 3y = 18$ and $2x + y = 10$, what is the value of each unknown number?

SOLUTION

$$2x + 3y = 18 \qquad (1)$$

$$2x + y = 10 \qquad (2)$$

Subtracting, $2y = 8$; $\therefore y = 4$

Substituting 4 for y in (2), $2x + 4 = 10$; $\therefore x = 3$

Test. — Substituting 3 for x and 4 for y in (1) and (2),

(1) becomes $6 + 12 = 18$, or $18 = 18$;

(2) becomes $6 + 4 = 10$, or $10 = 10$.

NOTES. — 1. The sign $\therefore$ means "*therefore*."

2. The value of y may be substituted in *either* of the given equations.

Solve the following and test results:

2. $\begin{cases} 5x + 2y = 22 \\ 5x + y = 21 \end{cases}$

3. $\begin{cases} 3x + 4y = 23 \\ 3x + 2y = 19 \end{cases}$

4. $\begin{cases} 3x - 4y = 16 \\ 5x + 4y = 48 \end{cases}$

5. $\begin{cases} 6x + 5y = 70 \\ x - 5y = 0 \end{cases}$

Solve and test:

6. $\begin{cases}4x+5y=32\\2x+5y=26\end{cases}$

7. $\begin{cases}7x-2y=22\\3x+2y=18\end{cases}$

8. $\begin{cases}6x+7y=13\\6x+y=7\end{cases}$

9. $\begin{cases}9x-2y=41\\7x-2y=31\end{cases}$

10. $\begin{cases}5x-4y=8\\3x-4y=0\end{cases}$

11. $\begin{cases}8x+5y=18\\8x+3y=14\end{cases}$

12. $\begin{cases}7x-3y=39\\5x-3y=27\end{cases}$

13. $\begin{cases}5y-4x=9\\6y+4x=46\end{cases}$

14. If $2x+3y=16$ and $5x+4y=33$, find x and y.

SOLUTION

$$2x+3y=16 \qquad (1)$$

$$5x+4y=33 \qquad (2)$$

We may eliminate either x or y. If we choose to eliminate x, we must first prepare the equations, so that x may have the same coefficient in each. Multiplying both members of (1) by 5, and both members of (2) by 2,

$$10x+15y=80 \qquad (3)$$

and $$\underline{10x+8y=66} \qquad (4)$$

Subtracting (4) from (3), $7y=14$; $\therefore y=2$

Substituting 2 for y in (1), $2x+6=16$; $\therefore x=5$

Test. — These values give $10+6=16$ and $25+8=33$ in (1) and (2).

NOTE. — To eliminate y instead of x, proceed as follows:

Multiplying (1) by 4, $8x+12y=64$

Multiplying (2) by 3, $15x+12y=99$

Subtracting the *upper equation from the lower*, thus avoiding negative coefficients,

$$7x=35; \therefore x=5$$

Substituting 5 for x in (1), $10+3y=16$; $\therefore y=2$

Solve and test:

15. $\begin{cases} 9x + 2y = 20 \\ 3x + y = 7 \end{cases}$

16. $\begin{cases} 6x + 5y = 28 \\ 2x + 3y = 12 \end{cases}$

17. $\begin{cases} 7x + 4y = 40 \\ 3x + 2y = 18 \end{cases}$

18. $\begin{cases} 10x + 3y = 62 \\ 6x + 4y = 46 \end{cases}$

19. $\begin{cases} 11x + 8y = 37 \\ 5x + 6y = 18 \end{cases}$

20. $\begin{cases} 18x + 2y = 9\frac{2}{3} \\ 4x + 3y = 3 \end{cases}$

Equations in this book are intended to give aid in solving the more difficult problems of arithmetic, and are not given as exercises for their own sake. To keep within the limits of arithmetic such arithmetical problems or equations as,

"Find two numbers whose sum is 10 and whose difference is 2," or $\begin{cases} x + y = 10 \\ x - y = 2 \end{cases}$

should be solved by eliminating by addition or subtraction the unknown number found in the *negative* term or terms, if there are any.

Solve and test:

21. $\begin{cases} x + y = 10 \\ x - y = 2 \end{cases}$

22. $\begin{cases} 5x + 2y = 49 \\ 3x - 2y = 23 \end{cases}$

23. $\begin{cases} 4x - y = 27 \\ x - y = 3 \end{cases}$

24. $\begin{cases} y + 2x = 18 \\ y - 2x = 2 \end{cases}$

25. $\begin{cases} 2y - 3x = 5 \\ 5y + 4x = 93 \end{cases}$

26. $\begin{cases} 4x - 7y = 12 \\ 3x + 5y = 50 \end{cases}$

27. Solve the equations $\frac{3x}{2} - \frac{y}{3} = 1$, and $\frac{x}{4} + \frac{y}{5} = 4$.

Suggestion. — Multiplying the members of each equation by the l. c. m. of the denominators in that equation will *clear the equation of fractions.*

Thus, 6 times $\frac{3x}{2} = 9x$; 6 times $\frac{y}{3} = 2y$; 6 times $1 = 6$. Multiplying the members of the first equation by 6, then, changes it to $9x - 2y = 6$.

Solve and test:

28. $\begin{cases} \dfrac{x}{2}+\dfrac{y}{8}=7 \\ \dfrac{x}{3}+\dfrac{y}{4}=6 \end{cases}$

29. $\begin{cases} \dfrac{x}{4}+\dfrac{y}{2}=13 \\ \dfrac{x}{5}-\dfrac{y}{8}=2 \end{cases}$

30. $\begin{cases} \dfrac{x}{3}-\dfrac{y}{9}=2 \\ \dfrac{x}{6}+\dfrac{y}{6}=5 \end{cases}$

31. $\begin{cases} \dfrac{2x}{3}-\dfrac{3y}{4}=6 \\ \dfrac{3x}{4}-\dfrac{5y}{8}=12 \end{cases}$

Find two numbers related to each other as follows:

32. Sum $=14$; difference $=8$.

33. Sum of 2 times the first and 3 times the second $=34$; sum of 2 times the first and 5 times the second $=50$.

34. Sum $=18$; sum of the first and 2 times the second $=20$.

35. A grocer sold 2 boxes of raspberries and 3 of cherries to one customer for 54¢, and 3 boxes of raspberries and 2 of cherries to another for 56¢. Find the price of each per box.

36. A druggist wishes to put 500 grains of quinine into 3-grain and 2-grain capsules. He has 220 capsules. How many capsules of each size can he fill?

37. On the Fourth of July, 850 glasses of soda water were sold at a fountain, some at 5¢ each, the others at 10¢ each. The receipts were $55. How many were sold at each price?

38. A fruit dealer bought 36 pineapples for $2.50. He sold some at 12¢ each and the rest at 10¢ each, thereby gaining $1.50. How many did he sell at each price?

39. An errand boy went to the bank to deposit some bills for his employer. Some of them were 1-dollar bills, and the rest 2-dollar bills. The number of bills was 38 and their value was $50. Find the number of each.

40. A man noticed that a 15-word day message by telegraph cost him 40¢ and a 22-word day message 54¢, between the same two cities. Find the charge for the first ten words and for each additional word.

41. When 2 baskets of Delaware peaches cost 15¢ more than 3 of Whitestone peaches, and 3 baskets of Delaware peaches cost 45¢ less than 6 of Whitestone peaches, what is the price of each per basket?

42. At a factory where 1000 men and women were employed, the average daily wage was $2.50 for a man and $1.50 for a woman. If labor cost $2340 per day, how many men were employed and how many women?

43. The receipts from 300 tickets for a musical recital were $100. Adults were charged 50¢ and children 25¢, each. How many tickets of each kind were sold?

44. It required 60 inches of tape to bind the four edges of a card on which a photograph was mounted. The length of the card was 6 inches greater than the width. How many inches long was the card? how many inches wide?

45. A lieutenant of the U. S. navy received $150 per month while on sea duty and $127.50 per month while on shore duty. A lieutenant's salary for a year amounted to $1620. How many months was he on sea duty? on shore duty?

46. The great columns of Bedford stone in the Indianapolis postoffice building weigh 94 tons each, including the shafts and the capitals resting on them. Each shaft weighs 74 tons more than its capital. Find the weight of a shaft; of a capital.

47. The receipts from a football game were $700. Admission tickets to the grounds cost 50¢, and to the grand stand, 25¢ in addition. If twice as many persons had purchased tickets for the grand stand, the receipts would have been $800. How many tickets of each kind were sold?

DENOMINATE NUMBERS

227. To ascertain the quantity of anything, or to *measure* it, is to find how many times it contains some established unit called the **unit of measure.**

Thus, to measure the water in a tank is to find how many times the whole quantity of water in the tank contains some unit of measure, as 1 gallon or 1 barrel, or perhaps 1 pound or 1 ton.

228. A concrete number in which the unit of measure is established by law or custom is called a **denominate number.**

10 feet is a denominate number; also 10 feet 7 inches.

229. A denominate number is **simple**, if it is composed of units of one denomination; **compound**, if it is composed of units of two or more related denominations.

10 feet is a simple denominate number; 10 feet 7 inches is a compound denominate number.

NOTE. — Tables of denominate numbers are found at the end of the book, preceding the Index.

REDUCTION

230. 1. How many quarts are there in 1 gallon? in 2 gal.? in $2\frac{1}{2}$ gal.? in 2.5 gal.?

2. In 10 quarts, how many gallons are there and how many quarts over? how many gallons and what part of a gallon?

$$10 \text{ qt.} = 2 \text{ gal. } 2 \text{ qt.} = 2\tfrac{1}{2} \text{ gal.} = 2.5 \text{ gal.}$$

231. Changing the form of a number without changing its value is called **reduction**.

Reduction to a *lower* denomination is called **reduction descending**; to a *higher* denomination, **reduction ascending**.

EXERCISES

232. **1.** Give these tables: liquid measures; dry measures; avoirdupois weight; troy weight; time measures; circular measures.

2. How many feet are there in 220 yards? in 96 inches?

3. What part of a gallon is 3 quarts? 5 pints?

4. How many quarts are there in 2 bushels? in $2\frac{1}{2}$ pecks?

5. Reduce 4 T. 5 cwt. to tons; to pounds; to hundredweight.

6. What part of an ounce is 10 pennyweights? Find the weight of a dozen silver spoons, each weighing 10 pwt.

7. Reduce 12 minutes to seconds; to a decimal of an hour.

8. Reduce $3° 45'$ to degrees; to minutes; $150''$ to minutes.

9. What part of a right angle is 45°? $22° 30'$? 30°? 60°?

WRITTEN EXERCISES

233. **1.** A dealer has 31 gal. 2 qt. of olive oil that he wishes to put into pint bottles. How many bottles does he need?

$$\begin{array}{l} 31, \text{ number of gal.} \\ \underline{\quad 4} \\ 124 \\ \underline{+2} \\ 126, \text{ number of qt.} \\ \underline{\quad 2} \\ 252, \text{ number of pt.} \end{array}$$

Since there are 4 quarts in 1 gallon, in 31 gallons there are 31 times 4 quarts (§ 68), or 124 quarts; and in 31 gal. 2 qt. there are 124 quarts + 2 quarts, or 126 quarts.

Since there are 2 pints in 1 quart, in 31 gal. 2 qt., or in 126 quarts, there are 126 times 2 pints, or 252 pints.

Hence 252 pint bottles will be needed to hold the oil.

2. A train made a run of $\frac{5}{8}$ of a mile in 55 seconds. How many feet did it run per second?

$$\tfrac{5}{8} \text{ mi.} = \tfrac{5}{8} \text{ of } 320 \text{ rd.} = 200 \text{ rd.}$$
$$200 \text{ rd.} = 200 \times 16\tfrac{1}{2} \text{ ft.} = 3300 \text{ ft.}$$
$$3300 \text{ ft.} \div 55 = 60 \text{ ft., the rate per second.}$$

Reduce:

3. 220 yd. to inches

4. $184\frac{3}{4}$ tons to pounds

5. 12 bu. 3 pk. to pecks

6. 1 day to minutes

7. 5.8 ft. to inches

8. $2° \ 24' \ 18''$ to seconds

9. A peanut vender at a fair sold 1276 pints of peanuts. Express his sales in bushels, pecks, etc.

$$\begin{array}{r|l} 2 & 1276 \\ \hline 8 & \ \ 638 \\ \hline 4 & \ \ \ \ 79, + 6 \text{ qt.} \\ \hline & \ \ \ \ 19, + 3 \text{ pk.} \end{array}$$

19 bu. 3 pk. 6 qt.

Since 2 pt. = 1 qt., 1276 pt. = 638 qt.
Since 8 qt. = 1 pk., 638 qt. = 79 pk. and 6 qt. over.
Since 4 pk. = 1 bu., 79 pk. = 19 bu. and 3 pk. over.
Hence the vender's sales were 19 bu. 3 pk. 6 qt.

10. An athlete ran a 440-yard race. What part of a mile did he run?

$$1 \text{ mi.} = 320 \text{ rd.} = 320 \times 5\tfrac{1}{2} \text{ yd.} = 1760 \text{ yd.}$$

Then, $$440 \text{ yd.} = \frac{440}{1760} \text{ mi.} = \tfrac{1}{4} \text{ mi.}$$

11. Reduce 14,420 pounds to tons and pounds.

12. Reduce 30,000 seconds to hours and minutes.

13. What part of 360° is $67° \ 30'$? $86° \ 24'$? $7° \ 12'$?

14. What part of a ton is 4 cwt. 80 lb.? 16 cwt. 25 lb.?

Reduce:

15. 6 lb. 3 oz. (av.) to ounces.

16. 150 qt. 1 pt. to pints.

17. 15,136 pounds to tons.

18. 275 ft. 10 in. to inches.

19. 14 yr. 5 mo. to months.

20. 2000 min. to hours and minutes.

21. $365\frac{1}{4}$ days to weeks, days, and hours.

22. $\frac{1}{8}$ of a right angle to degrees and minutes.

23. 11,225 feet (height of Mt. Hood) to miles and feet.

24. My watch runs 36 hours after being wound. How many times must I wind it during April?

25. The balance wheel of a watch should make 5 oscillations per second. How many is that per hour?

26. If the balance wheel makes 17,985 oscillations per hour, how many minutes and seconds will the watch lose in a week?

27. A woman in Gloversville made 100 pairs of gloves in a week. How much did she earn at 66¢ per dozen pairs?

28. If one pound of corn yields 4 ounces of starch, how much starch can be obtained from a bushel of corn weighing 56 pounds?

29. An Indiana factory, running continuously, made 11,250 dozen fruit jars in 5 da. 15 hr. How many single jars did it make per day?

30. Find the value of the 312,500 great gross of brass buttons made in this country one year, at 20¢ per gross.

31. How many bushels of charcoal weighing 15 pounds per bushel are required to make a ton of gunpowder that is $\frac{3}{4}$ saltpeter, $\frac{1}{10}$ sulphur, and the remaining part charcoal?

32. One year 8028 gross of fountain pens were manufactured in this country. If the average wholesale value of each fountain pen was 78.9¢, find the value of all.

33. The tidal wave travels about 700 miles per hour. How many feet does it travel per second?

34. A stamp-canceling machine, working 1 hr. 15 min. 12 sec., canceled 13,912 stamps. Find its capacity per minute.

35. A vessel under its own steam passed through the Suez Canal in 15 hr. 45 min. The canal is 90 miles long. What was its rate of passage per hour to the nearest tenth of a mile?

36. How long will it take a torpedo to reach its mark 1870 yards away, if it travels $\frac{1}{2}$ of a mile per minute?

37. In the West Indies 120 barrels of limes were obtained from an acre of land. Each barrel yielded 8 gallons of juice, and each gallon of juice yielded $\frac{3}{4}$ of a pint of concentrated juice. How many gallons of concentrated juice were obtained from an acre of land?

Foreign Money

234. **Canada** has a decimal currency, the table being the same as that for United States money.

235. The unit of **English** or **sterling money** is the **pound**, or **sovereign**.

The system is not a decimal system.

4 farthings (far.) = 1 penny (*d.*)
12 pence = 1 shilling (*s.*)
20 shillings = 1 pound (£)

Farthings (not coined) are usually written as fractions of a penny.
The sign £ precedes the number of pounds.

236. The unit of **French money** is the **franc**.

The system is a decimal system.

100 centimes (c.) = 1 franc (fr.)

Centimes is pronounced *sän′-tēms′*.

The monetary unit of Belgium and Switzerland is the same as that of France. The following units are identical in weight and fineness with the franc: the *peseta* (Spain), the *lira* (Italy), the *drachma* (Greece), and the *bolivar* (Venezuela).

237. The unit of **German money** is the **mark**.

100 pfennigs (pf.) = 1 mark (M.)

The German money system is a decimal system.

238. These official **equivalents** may be learned.

1 pound = $4.8665
1 franc = $0.193
1 mark = $0.238

In estimates we think of the pound as $5, the franc as 20¢, and the mark as 25¢.

EXERCISES

239. Reduce:

1. £10 to shillings.
2. £1 10*s*. to pence.
3. $\frac{1}{2}$ sovereign to pence.
4. 18.40 M. to pfennigs.
5. 450 c. to francs.
6. 6.75 fr. to centimes.
7. How many articles costing 6*d*. each can be bought for £1?
8. A man changed a 5-pound note and 3 half sovereigns for shillings. How many shillings did he receive?
9. How many toys at 60 pf. each can be bought for 12 M.?
10. How many handkerchiefs costing 75c. each can be bought for 1.50 fr.? for 3 fr.? for 7.5 fr.?

Find the cost of the following in the foreign money mentioned; then estimate the cost in United States money:

11. 4 Swiss statuettes purchased in Geneva, at 2.50 fr. each.
12. 30 bouquets purchased in Florence, at 50 centesimi ($\frac{1}{2}$ lira) each.

WRITTEN EXERCISES

240. Find, to the nearest cent, the value of:

1. £420
2. £24.75
3. £56.125
4. 1800 fr.
5. 144.50 fr.
6. 850.75 M.
7. 36.25 lire
8. 49.80 pesetas
9. 2000 drachmas
10. Find the value in United States money of a Bank of England note for £500; of a Bank of Naples note for 1000 lire.

11. Reduce £24 8*s.* 4*d.* to United States money.

$$\begin{array}{r|l} & \pounds \quad s. \quad d. \\ & 24 \quad 8 \quad 4 \\ 12 & \qquad\quad\ \ 4 \\ \hline 20 & \qquad 8.33+ \\ \hline & 24.4\,17^{-} \end{array}$$

Since $1d. = \frac{1}{12}s.$, $4d. = \frac{4}{12}s. = .33+s.$ Bringing down the 8*s.*, we have $8.33+s.$ Since $1s. = £\frac{1}{20}$, $8.33+s. = £\frac{8.33+}{20} = £.417-$. Bringing down the £24, we have £24.417−. Then we multiply \$4.8665 by 24.417, obtaining \$118.83−.

NOTE. — When pounds are expressed to the nearest third decimal place, the error is within $\frac{1}{2}$ of £.001, equal to about $\frac{1}{4}$ cent. Hence *pounds to the nearest thousandth give United States money to the nearest cent.*

Reduce to United States money to the nearest cent:

12. £8 3*s.* 4*d.* **15.** £12 7*s.* 7*d.* **18.** £100 8*s.* $1\frac{1}{2}d.$

13. £5 2*s.* 6*d.* **16.** £25 4*s.* 3*d.* **19.** £125 6*s.* 10*d.*

14. £7 6*s.* 9*d.* **17.** £66 8*s.* 5*d.* **20.** £440 18*s.* $9\frac{1}{2}d.$

21. Reduce \$225.50 to English money.

$\$225.50 \div \$4.8665 = 46.337^{+}$, the number of pounds
$.337 \times 20s. = 6.74s.$, and $.74 \times 12d. = 8.88d.$

Hence, to the nearest penny, \$225.50 = £46 6*s.* 9*d.*

NOTE. — To obtain the result correct to the nearest penny, the division need be carried only to the nearest third decimal place.

Reduce to English money, to the nearest penny:

22. \$1000 **25.** \$48.25 **28.** \$77.14 **31.** \$122.43

23. \$3200 **26.** \$62.75 **29.** \$84.40 **32.** \$760.50

24. \$5500 **27.** \$90.10 **30.** \$50.07 **33.** \$325.85

34. Two dredging machines built in Glasgow for Russia cost £38,000 apiece. Find the cost of both in American money.

35. At 10*d.* per hour, how much does a British bricklayer earn in 6 days, working 9 hours per day?

36. In a London fruit store I purchased 15 pounds of Baldwin apples @ 3*d.* and 20 pounds of California Newtowns @ 5*d.* How much change did I receive out of a sovereign?

37. Find the cost of 12 tons of house coal in London at £1 6*s.* per ton. Find the cost per ton in our money.

38. The cost of bread in some English cities is a penny per pound. How much does a baker receive for 85 1-lb loaves of bread, 42 2-lb. loaves, and 64 $1\frac{1}{2}$-lb. loaves?

39. A party of three traveled by train from Bristol to London, 194 miles, at 2*d.* each per mile. Find the fare for all in English money, and the fare for each in U. S. money.

40. A cloth weaver in Germany received 20 pfennigs per hour. He worked 10 hours a day and 6 days a week. How much did he earn per week in U. S. money?

41. By some German railways, tea and coffee are sold to employees at 2 pfennigs per cup. If an employee buys 8 cups per week, how much will he expend per year of 52 weeks?

42. The toll on loaded vessels passing through the Suez Canal is 8 francs 50 centimes per ton of the vessel's capacity. Find the toll on a loaded vessel whose capacity is 3964 tons.

43. The receipts of the Suez Canal Company for tolls, one year, were 103,120,268 francs. Find the receipts in U. S. money.

44. An American tobacco company was fined, in Germany, 5000 marks for violating the laws of competition. Compute the fine in U. S. money.

45. The London Metropolitan Railway cost £500 per yard. A New York street railway cost $214.13 per yard. Find the difference in cost per yard in U. S. money.

46. Great Britain built the New Zealand Government Railway at a cost of £7779 per mile. The length was 2212 miles. Find the cost in sterling money, and in U. S. money.

Area and Volume

ANGLE

241. The difference in the direction of two lines that meet is called an **angle.**

RECTANGLE

242. A figure that is bounded by four straight lines and has four *equal* angles is called a **rectangle.**

The angles of a rectangle are called **right angles**.

243. A rectangle whose sides are equal is called a **square.**

A square, each of whose sides is *one inch*, is called a **square inch.** Describe a square foot; a square yard; other **square units**.

244. If each square in this rectangle represents some square unit, each row represents 5 of those units, and since there are 3 rows, the rectangle contains 3×5 square units, or 15 square units.

The number of square units that any surface contains is called its **area**.

245. If the length and width of a rectangle are expressed in *inches*, the area is found by multiplying the *number* of inches in the length by the *number* of inches in the width and calling the result *square inches*. If both are in *feet*, the area is found by multiplying one dimension by the other and calling the result *square feet*.

This may be stated briefly as follows:

The area of a rectangle is equal to the product of its length and width, expressed in like units.

For brevity we speak of the *product of lines* when we mean the product of the *numbers* that represent them.

A rectangle that is 5 feet long and 3 feet wide, or more briefly 5 ft. by 3 ft., has an area of 15 sq. ft.

WRITTEN EXERCISES

246. 1. Find the area of a rectangle 16 ft. by 5 ft. 3 in.

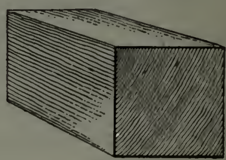
SOLUTION

$$\text{Length} = 16 \text{ ft.}; \text{ width} = 5\tfrac{1}{4} \text{ ft.}$$
$$\text{Area} = (5\tfrac{1}{4} \times 16) \text{ sq. ft.} = 84 \text{ sq. ft.}$$

Since the length and width must be expressed in *like units*, 5 ft. 3 in. is first changed to *feet*. Then the *number* of *square feet* in the area is found by multiplying 16 by $5\frac{1}{4}$.

Find the area of rectangles of the following dimensions:

2. 16 ft. by 7 ft.	5. 110 ft. by 4 yd.	8. 24 ft. by 3 ft. 6 in.
3. 28 ft. by $4\frac{1}{2}$ ft.	6. 120 ft. by 40 in.	9. 54 ft. by 7 ft. 2 in.
4. 39 ft. by $6\frac{1}{3}$ ft.	7. 280 ft. by 50 in.	10. 32 ft. by 4 ft. 9 in.

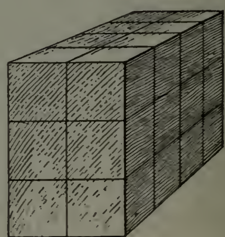


247. A solid having six rectangular faces is called a **rectangular solid.**

248. A rectangular solid whose faces are equal squares is called a **cube.**

A cube each of whose faces is a square inch is called a **cubic inch.** Describe a cubic foot; other **cubic units.**

249. If each cube in this rectangular solid represents some cubic unit, each row represents 4 of those units, each layer 2×4 of them, and since there are 3 layers, the rectangular solid contains $3 \times 2 \times 4$ cubic units, or 24 cubic units.



The number of cubic units that any solid contains is called its **volume.**

250. If the length, width, and thickness of a rectangular solid are expressed in *inches*, the volume is found by multiplying the *number* of inches in the length by the *number* of inches in the width and that product by the

number of inches in the thickness, calling the result *cubic inches.* If all the dimensions are in feet, the volume is found by finding their product and calling the result *cubic feet.*

Stated more briefly:

The volume of a rectangular solid is equal to the product of its length, width, and thickness, all expressed in like units.

A rectangular solid 4 ft. by 2 ft. by 3 ft. has a volume of 24 cu. ft.

WRITTEN EXERCISES

251. **1.** Find the volume of a rectangular solid 12 ft. by 8 ft. 6 in. by 4 ft. 4 in.

SOLUTION

Length = 12 ft.; width = $8\frac{1}{2}$ ft.; thickness = $4\frac{1}{3}$ ft.
Volume = $(4\frac{1}{3} \times 8\frac{1}{2} \times 12)$ cu. ft. = 442 cu. ft.

Since the dimensions must be expressed in *like units*, 8 ft. 6 in. and 4 ft. 4 in. are first changed to *feet.* Then the *number* of *cubic feet* in the volume is obtained by finding the product of 12, $8\frac{1}{2}$, and $4\frac{1}{3}$.

Find the volume of these rectangular solids:

2. 8 ft. by 4 ft. by 3 ft. 3 in.
3. 9 ft. by 5 ft. by 2 ft. 4 in.
4. 6 ft. by 6 ft. by 4 ft. 8 in.
5. 10 ft. by 5 ft. by 30 in.
6. 14 ft. by 3 ft. 6 in. by 3 ft.
7. 24 ft. by 8 ft. 4 in. by 5 ft.

8. A rectangular solid is 14 ft. 5 in. by 6 ft. 7 in. by 9 ft. 11 in. Find its volume to the nearest .001 of a cubic foot.

SUGGESTION. — The dimensions may be reduced to inches. Then the volume in cubic inches may be changed to cubic feet.

Find the volume of each of the following rectangular solids to the nearest .001 of a cubic foot:

9. 23 ft. 8 in. long, 11 ft. 5 in. wide, 5 ft. thick.

10. 21 ft. 7 in. long, 16 ft. 1 in. wide, 4 ft. thick.

11. 35 ft. 10 in. long, 13 ft. 4 in. wide, 6 ft. 5 in. thick.

12. Some of the land near the Bank of England is valued at £75 per square foot. Find the value of a piece of this ground 42 feet by 60 feet.

13. Air is about $\frac{1}{5}$ oxygen. How many cubic feet of oxygen are there in a room 16 ft. by $13\frac{1}{2}$ ft. by 10 ft.?

14. How many pressed bricks $8\frac{1}{4}$ in. long and 2 in. thick, laid on edge, are required to pave a street 2310 ft. by 30 ft.?

15. Find the cost, at 55¢ per square yard, of constructing a stone road in New Jersey, 15 ft. wide and 6 mi. 1980 ft. long.

16. How many pounds of blasting powder are required to blast out a tunnel 9 ft. square and 440 yd. long through solid rock, if 1 pound will loosen 3 cu. yd. of rock?

17. Find the weight of a box of dynamite whose contents measure 4 ft. by $2\frac{1}{2}$ ft. by 3 ft., if 1 cubic foot weighs $103\frac{1}{8}$ lb.

18. The floors of a warehouse were designed to carry 250 pounds per square foot as a maximum load. Find the maximum load for a floor 62 ft. by 36 ft.

19. How many rubber erasers 2 in. long, $1\frac{1}{4}$ in. wide, and $\frac{1}{2}$ in. thick can be cut from a block of India rubber 1 ft. 6 in. by 1 ft. 3 in. by 9 in.?

20. Find the area of a large illustration on the front page of a periodical $11\frac{1}{4}$ in. wide and $16\frac{1}{4}$ in. long, if the side margins of the page are $\frac{7}{8}$ in. each, the bottom $1\frac{1}{4}$ in., and the top $3\frac{1}{4}$ in.

21. Find the number of cubic inches of space in a chest made of wood $\frac{3}{4}$ in. thick, if the outside dimensions are 22 in., 19 in., and 11 in.

22. A drill for sowing wheat covered 24 acres of ground per day. How long did it take to sow a field 216 rd. by 160 rd.?

23. A reaper used in a Minnesota wheat field cuts a swath 18 feet wide. How far must it go in a straight line to reap an acre of wheat?

OPERATIONS WITH DENOMINATE NUMBERS

252. Addition, subtraction, multiplication, and division with denominate numbers are performed just as with other numbers. It is necessary, however, to bear in mind the number of units of any denomination required to make a unit of the next higher denomination.

WRITTEN EXERCISES

253. 1. Add:

ft.	in.	
14	11	
9	6	
24	5,	sum

6 in. + 11 in. = 17 in. = 1 ft. 5 in.; 1 ft. + 9 ft. + 14 ft. = 24 ft.

2. Subtract:

gal.	qt.	
31	2	
13	3	
17	3,	difference

3 qt. cannot be taken from 2 qt.; change 31 gal. 2 qt. to 30 gal. 6 qt.

3. Multiply:

bu.	pk.	
8	3	
	9	
78	3,	product

9×3 pk. = 27 pk. = 6 bu. 3 pk.; 9×8 bu. = 72 bu.; 72 bu. + 6 bu. = 78 bu.

4. Divide:

	mi.	rd.	
4)	33	40	
	8	90,	quotient

33 mi. ÷ 4 = 8 mi. and 1 mi. over; 1 mi. = 320 rd.; 320 rd. + 40 rd. = 360 rd.; 360 rd. ÷ 4 = 90 rd.

5. Add 2 T. 5 cwt.; 1 T. 12 cwt.; 4 T. 15 cwt.; 3 T. 8 cwt.

6. Subtract 3 T. 17 cwt. from 10 T.; 22 T. 14 cwt. from 25 T.; 17 lb. 11 oz. from 42 lb.

7. Multiply 12 ft. 8 in. by 6; 2 yd. 27 in. by 4.

8. Divide 85 ft. 8 in. by 4; 181 ft. $8\frac{1}{2}$ in. by 7.

9. How much less than a right angle is $77° 42'$?

10. How much less than 2 right angles is $154° 28' 33''$?

Add:

11. 6 lb. 402.15 pwt. and 2 lb. 7 oz. 9 pwt.

12. $14° 17' 15''$; $62° 40' 32''$; $24° 0' 54''$; $41° 50' 19''$.

13. £865 4*s.* 8*d.*; £82 9*s.* 10$\frac{1}{2}$*d.*; £48 18*s.* 5$\frac{1}{2}$*d.*

14. £642 3*s.* 2*d.*; £66 0*s.* 6$\frac{1}{2}$*d.*; £34 15*s.* 3*d.*

15. Subtract each amount of money in exercise 14 from the amount written above it in exercise 13.

16. Find $\frac{1}{4}$ of $16° 32' 30''$; $\frac{3}{4}$ of $16° 32' 30''$; $\frac{5}{8}$ of 125 ft. 4 in.

17. Find 5 times 11 lb. 10 oz. (av.); $\frac{5}{6}$ of 11 lb. 10 oz. (av.).

18. How long was it from June 26, 1893, to May 15, 1906?

yr.	mo.	da.
1906	5	15
1893	6	26
12	10	19

The later date is written as the minuend and the earlier date as the subtrahend, the number of the month being used instead of its name.

Subtract as in denominate numbers, considering 30 days as a month and 12 months as a year. The remainder is the difference in time as accurately as it can be expressed in years, months, and days.

Find the time from to-day to the following dates:

19. Jan. 1, 1920	**21.** Sept. 21, 1925	**23.** Oct. 28, 1918
20. Feb. 8, 1915	**22.** Apr. 18, 1930	**24.** Aug. 11, 1922

25. Find the difference in elevation between Lake Huron, 576 ft. 9 in. above sea level, and Lake Superior, 602 ft. 11 in. above sea level.

26. The leakage from a 63-gallon hogshead of rock candy sirup was 2 qt. 1 pt. How much sirup remained?

27. If a passenger locomotive consumes 86 lb. 9 oz. of coal per mile, how much coal will be consumed on a trip of 95 miles?

28. A man picked 94 bu. 2 pk. of cranberries in 12 days. What quantity did he pick per day, on the average?

29. If a skillful operator stamps the name on 24 gross 4 dozen pens per hour, how many will he stamp in 9 hours?

30. Find by this table the total time required to make 100 medium-grade coats by hand; by machinery.

Operations	Time for each Operation, per 100 Coats			
	By Hand		By Machinery	
Shrinking cloth	11 hr.	40 min.	1 hr.	48 min.
Cutting cloth	33	20	4	32.5
Sewing seams	1000		66	40
Cutting buttonholes	3	20		17.5
Working buttonholes	275		6	17.8
All others	1978	23	1296	20.5

31. Find the difference in time for hand and machine work for each operation; for all.

32. A merchant vessel passed through the Suez Canal four times one year, the time for each passage being, respectively, 15 hr. 32 min., 19 hr. 58 min., 17 hr. 15 min., and 18 hr. 12 min. Find the average time of passage.

33. A shipment of candles to Calcutta consisted of 124 cases, each containing 21 lb. 14 oz. of candles. How much did the shipment of candles weigh?

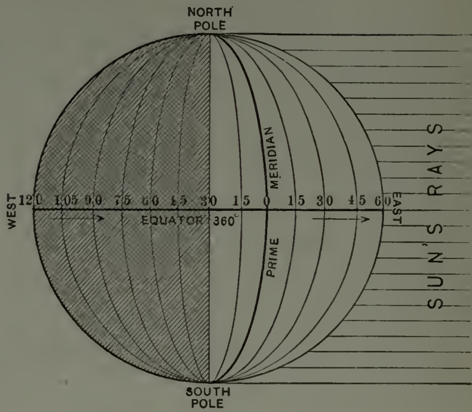
34. Each case held 25 packets of 6 candles each. Find the weight of a packet; of a candle.

35. A man owned 84 A. 82.5 sq. rd. of land and bought 16 A. 147.5 sq. rd. Afterward he sold $\frac{1}{3}$ of his land. How much land did he then own?

36. The number and length of the spans of the bridge across the Firth of Tay, Scotland, the longest bridge in the world, are as follows: 28 spans, each 3 rd. $10\frac{1}{2}$ ft.; 2 spans, each 4 rd. 14 ft.; 21 spans, each 7 rd. $4\frac{1}{2}$ ft.; 12 spans, each 8 rd. 4 ft.: 13 spans, each 13 rd. $15\frac{1}{2}$ ft.; 1 span, 9 rd. $1\frac{1}{2}$ ft.; 1 span, 9 rd. $6\frac{1}{2}$ ft.; 6 spans, each $10\frac{1}{2}$ ft. Find the length of the bridge in miles, rods, and feet.

LONGITUDE AND TIME

254. An imaginary line passing north and south from one pole of the earth to the other is called a **meridian**.



255. The imaginary line around the earth running east and west halfway between the poles is called the **equator**.

Since the equator is the circumference of a circle, distances along it are measured in *degrees*.

256. Distances east and west are measured from some selected meridian, called the **prime meridian**.

The prime meridian commonly used is that passing through the Royal Observatory at Greenwich, England.

257. Distance east or west of the prime meridian, measured in degrees along the equator, is called **longitude**.

East longitude is the distance *east* of the prime meridian; *west longitude* is the distance *west* of it.

258. 1. Since the earth rotates on its axis once in 24 hours, any meridian passes through 360° in that time.

Then how many degrees of longitude pass under the sun's rays during 24 hours? during 1 hour?

2. Since 15° of longitude pass under the sun's rays in 1 hour, what part of a degree passes in 1 minute? How many minutes (′) of longitude pass in 1 minute of time?

3. Since 15′ of longitude pass under the sun's rays in 1 min., how many seconds (″) of longitude pass in 1 sec. of time?

259. As developed in the foregoing, the relation existing between *longitude* and *time* may be expressed briefly as follows:

360° of longitude correspond to 24 hours of time.
15° of longitude correspond to 1 hour of time.
15′ of longitude correspond to 1 minute of time.
15″ of longitude correspond to 1 second of time.

260. When the sun's rays are vertical at any point on a meridian, it is noon at all places on that meridian.

Since the earth turns from west to east, the sun *appears* to move from east to west. Therefore when it is noon at any place it is *before* noon, or **earlier**, at all places **west**, because the sun has not yet reached the meridians of those places. It is *after* noon, or **later**, at all places **east**, because the sun has already crossed the meridians of those places.

EXERCISES

261. **1.** What is the difference in longitude between a place 15° west of Greenwich and a place 30° west?

Which place has the earlier time and how much earlier?

2. How far westward must one go to pass from meridian 15° E. to Greenwich? from Greenwich to 15° W.? How many degrees west of 15° E. is 15° W.? Compare their times.

Compare the times of places on the following meridians:

3. 15° W. and 45° W.	**8.** 30° E. and 30° W.
4. 15° W. and 60° W.	**9.** 10° E. and 5° W.
5. 45° E. and 30° E.	**10.** 20° E. and 10° W.
6. 35° E. and 20° E.	**11.** 40° E. and 20° W.
7. 42° E. and 12° E.	**12.** 30° E. and 45° W.

TABLE OF LONGITUDES

West Longitudes	°	′	″	East Longitudes	°	′	″	East Longitudes	°	′	″
Boston	71	03	50	Batavia	106	48	37	Bombay	72	48	56
New York	74	00	24	Melbourne	144	58	35	Cape Town	18	28	40
Washington	77	03	06	Tokyo	139	44	30	Berlin	13	23	44
Chicago	87	36	45	Shanghai	121	28	55	Hamburg	9	58	25
Denver	104	58	00	Manila	120	58	06	Amsterdam	4	53	04
San Francisco	122	24	32	Canton	113	16	30	Paris	2	20	14

In the following exercises refer to this table for longitudes and find answers to the *nearest* second.

WRITTEN EXERCISES

262. **1.** When it is noon at San Francisco, what is the time at Washington?

$$\begin{array}{r} 122^\circ\ 24'\ 32'' \\ \underline{77^\circ\ \ 3'\ \ 6''} \\ 15\overline{)45^\circ\ 21'\ 26''} \\ 3^\circ\ \ 1'\ 26'' \end{array}$$

3 hr. 1 min. 26 sec. is the difference in time.

The difference in longitude is found to be 45° 21′ 26″.

Since 15° corresponds to 1 hr., 15′ to 1 min., and 15″ to 1 sec., the difference in time between San Francisco and Washington is as many hours, minutes, and seconds, respectively, as there are degrees, minutes, and seconds in $\frac{1}{15}$ of the difference in longitude.

Therefore the difference in time is 3 hr. 1 min. 26 sec., and since Washington is *east* of San Francisco, the time is *later* in Washington; that is, when it is noon at San Francisco it is 1 min. 26 sec. past 3 P.M. at Washington.

Find the true time in each of the following cities when the sun is on the meridian of Washington:

2. Boston
3. New York
4. Chicago
5. Berlin
6. Paris
7. London (Greenwich)
8. Hamburg
9. Amsterdam
10. Cape Town

11. What is the longitude of St. Petersburg, if its time is 6 hr. 57 min. 19 sec. later than that of New York?

hr.	min.	sec.					
6	57	19		104°	19′	45″	
		15		74°	00′	24″	W. (N.Y.)
104	19	45		30°	19′	21″	E. (St.P.)

104° 19′ 45″, diff. in longitude

Since 1 hr. corresponds to 15°, 1 min. to 15′, and 1 sec. to 15″, the difference in longitude between New York and St. Petersburg is as many degrees, minutes, and seconds, respectively, as there are hours, minutes, and seconds in 15 times the difference in time between the two places.

Therefore the difference in longitude is 104° 19′ 45″, and since the time of St. Petersburg is *later* than that of New York, St. Petersburg is *east* of New York; but New York is only 74° 00′ 24″ west of the prime meridian, and since the difference in longitude is greater than this, St. Petersburg must be east of the prime meridian. Subtracting, we find the longitude of St. Petersburg to be 30° 19′ 21″ E.

Find the longitude of places having the following times when the sun is on the meridian of Washington:

12. 2:21 P.M.	**15.** 10:18 A.M.	**18.** 7:32½ A.M.
13. 4:36 P.M.	**16.** 9:54 A.M.	**19.** 1:05¾ P.M.
14. 3:42 P.M.	**17.** 6:09 A.M.	**20.** 15 sec. before 8 A.M.

21. The longitude of Norwich, Eng., is 1° 18′ W.; of Norwich, Conn., 72° 4′ W. How much later does the sun cross the meridian of the American city than that of the English city?

22. An astronomer in Boston observed the moon entering the shadow of an eclipse at 1:13 A.M. At what times did astronomers in Chicago, Denver, and San Francisco observe the same?

23. When the people of Boston were celebrating the passing of the 19th century at midnight Dec. 31, 1900, how long had Parisians been living in the 20th century? How much of the 19th century was left for the people of Galveston, 94° 47′ 26″ W.?

24. At noon a ship's chronometer carrying Greenwich time indicated 1:05 P.M. In what longitude was the ship?

25. A ship's chronometer carrying Greenwich time was 35 minutes slow Saturday noon and 13 minutes fast the following Tuesday noon. In what longitude was the ship at each observation? How far east or west did she sail?

26. When it is 6 P.M. Jan. 10 at San Francisco, what is the time and date at Tokyo?

First Solution

Since San Francisco is 122° 24′ 32″ W. and Tokyo is 139° 44′ 30″ E., Tokyo is 262° 9′ 2″ *east* of San Francisco, and consequently has 17 hr. 28 min. 36 sec. *later* time. Counting this time on from 6 P.M. Jan. 10, the time at Tokyo is found to be 28 min. 36 sec. after 11 A.M. Jan. 11.

Second Solution

Reckoning in the other direction, Tokyo is 360° – 262° 9′ 2″, or 97° 50′ 58″ *west* of San Francisco, and its time is therefore 6 hr. 31 min. 24 sec. *earlier*.

This apparent contradiction of the first solution is explained thus: the time of Tokyo is 17 hr. 28 min. 36 sec. later than that of San Francisco, or a day later *lacking* 6 hr. 31 min. 24 sec. A day later than 6 P.M. Jan. 10 is 6 P.M. Jan. 11, and a day later *less* 6 hr. 31 min. 24 sec. gives the Tokyo time, 28 min. 36 sec. after 11 A.M. Jan. 11..

NOTE. — When a ship sails *westward* over the 180th meridian the calendar is set *forward* one day; sailing *eastward* its calendar is set back one day. An irregular line corresponding in general with this meridian marks the place where the calendar changes. It is called the **international date line.**

When the sun is on the meridian of New York, on the first day of May, find the true time and date in:

27. Manila **29**. Canton **31**. Bombay

28. Batavia **30**. Melbourne **32**. Cape Town

33. At noon, March 10, a ship weighed anchor at Hong-Kong, 114° 10′ 02″ E., and sailing eastward reached Honolulu, 157° 51′ 34″ W., at noon March 25. How long was the voyage?

Standard Time

263. In 1883, the railroads of the United States and Canada agreed upon a system of **standard time** that has come into general use because of its convenience. Under this system there are five **time belts**, each approximately 15° of longitude in width, and each having the time of its central meridian.

Each railway has selected the most convenient city on its own road at which to change from the standard time of one belt to that of the next. Since such towns on the various roads are seldom on the same meridian, the line connecting them forms an irregular boundary between the various belts; hence these time belts, shown on the following map, are neither equal in size nor uniform in shape.

264. The time belts are called **Atlantic**, with the time of the meridian of 60° W.; **Eastern**, with the time of the meridian of 75° W.; **Central**, with the time of the meridian of 90° W.; **Mountain**, with the time of the meridian of 105° W.; and **Pacific**, with the time of the meridian of 120° W.

It is evident that the time of the various belts differs by *hours*, the minute and second hands of all correct timepieces being the same at any instant. Central time is 1 hr. earlier than Eastern time; Mountain time is 1 hr. earlier than Central time; and Pacific time is 1 hr. earlier than Mountain time.

Correct time is telegraphed each day to all parts of the United States from the Naval Observatory at Washington.

265. Standard time has been adopted by most civilized governments of the world, the time meridian chosen being, with few exceptions, some multiple of 15° from the prime meridian through Greenwich.

In exercises refer to the map on page 155 for the time meridians of cities in the United States. The standard time meridian for each foreign city will be given in parentheses in the exercises where it is needed.

WRITTEN EXERCISES

266. **1.** A certain business transaction was reported by telegraph from Chicago at 10:30 A.M., to New York, New Orleans, San Antonio, and Portland, Ore. Allowing no time for transmission, when did the message reach each city?

2. If the news of the opening of the St. Louis Exposition at 12:15 P.M. was immediately telegraphed all over the world, at what time did Salt Lake City receive it? Portland, Me.? Manila (120° E.)? Bombay (75° E.)? Berlin (15° E.)? London (0°)? Tokyo (135° E.)?

3. The news of the *Maine* disaster was cabled to Madrid (0°) from Havana (75° W.) at 2 A.M. Neglecting the time of transmission, find the time when the news reached Madrid.

4. Dewey's flagship, the *Olympia*, opened fire on the defenses of Manila (120° E.) at 5:41 A.M., May 1, 1898, and ceased firing at 7:40 A.M. to allow time for breakfast. Between what hours and on what day, Washington time, did the assault occur?

5. The first news of the assault reached Madrid (0°) at 6:20 P.M. the same day. How long was this after the *Olympia* opened fire?

6. Cable communication with Manila was severed at 10 A.M., London time, May 2, 1898. What was the clock time in Manila (120° E.) when the cable was cut?

7. A London paper received a dispatch from Cairo (30° E.) reporting an explosion in the British barracks. If the dispatch was received in London at 3:15 P.M., at what time was it sent?

8. If $1\frac{1}{4}$ hours later the news was cabled to New York (75° W.), find the time that it reached New York.

9. The first shock of the earthquake at San Francisco April 18, 1906, was recorded at 5:12 A.M. at the University of California, and at 20 seconds after 8:19 A.M. at Washington. How long did it take the shock to cross the continent?

10. The dispatch regarding the final surrender of Port Arthur (120° E.) was sent from that place at 10 P.M., Jan. 2, 1905. At what time was the news received in Tokyo (135° E.)? in St. Petersburg (30° E.)? in London? in New York?

11. The *Atlantic*, the winner of a yacht race from Sandy Hook (75° W.) to the Lizard, Eng. (0°), started at 12:15 P.M., May 17, and finished at 9:18 P.M., May 29. Find the *Atlantic's* time.

12. A ship in distress reports by wireless telegraph that she is in longitude 21° W. and has been pumping water 5 hours. A station on the English coast catches the message immediately at 2:27 P.M., Greenwich time. When did the pumps begin work?

13. Paris uses the time of her meridian, 2° 20′ 14″ E. The Paris stock exchange, or *Bourse*, closes at 3 P.M. At what time will closing quotations reach New York, if cabled immediately?

14. At 12 P.M., Saturday, Dec. 31, 1904, the chief of the U. S. Signal Service sent a message from Washington around the world *via* Chicago, Denver, San Francisco, Manila (120° E.), Tokyo (135° E.), Melbourne (150° E.), Bombay (75° E.), Berlin (15° E.), and London. What was the date and the clock time in each city at the instant of sending the message?

METRIC SYSTEM

267. The **metric system** is a *decimal* system of weights and measures, having for its **principal unit** the **meter**, to which all other units are related.

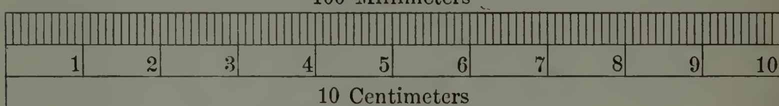
This system originated in France. Its use is required by law in many countries, and permitted in many others, including the United States. Congress has made it the official system in our Philippine possessions.

Its most general use is in the arts and sciences, where its convenience and accuracy have specially commended it.

MEASURES OF LENGTH

268. The primary unit of **length** is the **meter**, the unit of the system.

100 Millimeters



$\frac{1}{10}$ Meter, or 1 Decimeter

The length of the meter was intended to be a **ten-millionth** part of the distance from the equator to either pole, but subsequent calculations have shown it to differ slightly from that.

The length of the standard meter in the United States is 39.37 inches.

Other metric units of length are **decimal parts** of the meter and **multiples of 10** times the meter.

269. The primary units of surface, volume, capacity, weight, etc., are likewise subdivided and multiplied *decimally*, giving the other units of those measures. Consequently, in the metric

system names are simplified and made to show the size of each unit by giving to each *primary unit* the following *Latin prefixes* to indicate the *decimal parts* and *Greek prefixes* for the *multiples*.

LATIN		GREEK	
deci means	.1	**deka** means	10
centi means	.01	**hekto** means	100
milli means	.001	**kilo** means	1000

Thus, **decimeter** means $\frac{1}{10}$ of a meter; **dekameter** means 10 meters; etc. Another prefix sometimes used is **myria**, meaning 10,000.

270. Table of measures of length.

10 **millimeters** (mm.)	= 1 **centimeter** (cm.)
10 centimeters	= 1 decimeter (dm.)
10 decimeters	= 1 **meter** (m.)
10 meters	= 1 dekameter (Dm.)
10 dekameters	= 1 hektometer (Hm.)
10 hektometers	= 1 **kilometer** (Km.)

Abbreviations for *parts* of the primary unit begin with small letters, those for *multiples* of it with capitals.

The tables indicate important units by **heavy type**.

The **kilometer** (about .6 mi.) is used for long distances; the **meter** (about $3\frac{1}{4}$ ft., a little over 1 yd.), for shorter distances and to measure cloth, etc.; the **millimeter**, in the sciences and to express such small measurements as the thickness of wire.

271. The decimal relation of the units and their correspondence to the successive orders of units in the decimal notation is illustrated by the following:

Km.	Hm.	Dm.	m.		dm.	cm.	mm.
1	1	1	1	.	1	1	1

Since the meter occupies units' place, this number is read the same as 1111.111 m. Reduction to any other unit in the table is accomplished by simply moving the decimal point, thus:

$$1111.111 \text{ m.} = 1.111111 \text{ Km.} = 111111.1 \text{ cm.} = \text{etc.}$$

EXERCISES

272. Express in meters or decimals of a meter:

1. 2 Km.
2. 4 mm.
3. 7 Hm.
4. 9 cm.
5. 5 Dm.
6. 8 dm.
7. 75 cm.
8. 37.5 cm.
9. 12.5 cm.
10. 250 mm.
11. 981 mm.
12. 3.275 Km.
13. 5 Km. 300 m.
14. 40 Km. 75 m.
15. 26 Km. 33.5 m.
16. 706 m. 82 cm.
17. 530 m. 75 cm.
18. 48 cm. 9.65 mm.

19. About how many feet are there in 40 meters? meters in 13 feet? miles in 15 kilometers? kilometers in 60 miles?

20. During practice marches the German soldier walks 30 Km. per day. About how many miles does he walk per day?

21. A foot is about 30 cm. About how many steps must a man take to walk 100 ft., if each step is $\frac{5}{6}$ of a meter long?

MEASURES OF SURFACE

273. **1.** Draw a rectangle 10 cm. long and 1 cm. wide, and divide it into square centimeters. What is its area?

10 Square Centimeters

2. What is the area of a rectangle 10 cm. by 2 cm.? 10 cm. by 3 cm.? 10 cm. by 5 cm.? 10 cm. by 10 cm.?

3. Since 10 cm. = 1 dm., how many square centimeters are there in 1 square decimeter?

4. How many square decimeters equal 1 square meter? How many square meters equal 1 square dekameter?

274. *In metric square measure it requires* 100 *units of any denomination to make* 1 *unit of the next higher denomination.*

275. The primary unit of **surface** is the **square meter.**

The official equivalent of the square meter in the United States is 1.196 square yards.

276. Table of measures of surface.

100 square millimeters	=	1 square centimeter
100 square centimeters	=	1 square decimeter
100 square decimeters	=	1 **square meter**
100 square meters	=	1 square dekameter
100 square dekameters	=	1 square hektometer
100 square hektometers	=	1 **square kilometer**

The **square meter** (about 1.2 sq. yd.) is used for ordinary surfaces, such as floors, walls, etc.; the **square kilometer** (nearly .4 sq. mi.) for such large surfaces as the areas of countries.

277. From § 274 and the table, it is seen that the successive metric units of square measure occupy *two* orders of figures, thus:

sq. Km.		sq. Hm.		sq. Dm.		sq. m.			sq. dm.		sq. cm.		sq. mm.
1	0	1	0	1	0	1	.	0	1	0	1	0	1

Sometimes the abbreviation m^2 is used for sq. m.

278. The primary unit of **land measures** is the **are**, which is a *square dekameter*.

The official equivalent of the are is 119.6 square yards.

279. Table of land measures.

100 centares	=	1 **are**
100 ares	=	1 **hektare**

The **hektare** is nearly $2\frac{1}{2}$ acres.

280. The successive land units occupy two orders of figures:

Ha.		a.			ca.
1	0	1	.	0	1

EXERCISES

281. Reduce to square meters or decimals of a square meter:

1. 62.5 sq. dm.	**3.** 122.5 sq. Hm.	**5.** .125 sq. Dm.
2. 37.5 sq. dm.	**4.** 1.875 sq. Km.	**6.** 1000 ares.

7. Reduce to square millimeters: .5 sq. cm.; 1.25 sq. cm.

8. Reduce to square centimeters: 875 sq. mm.; 1500 sq. mm.

9. Find the area of a post card 14 cm. by 8 cm.

10. Find the area, in hektares, of a city block 1 Hm. square.

11. The large sulphur mines at Askhabad, Russia, cover an area of 6000 hektares. About how many acres do they cover?

MEASURES OF VOLUME

282. **1.** How many centimeters are there in 1 decimeter?

2. Into how many cubic centimeters may a rectangular solid be divided, if it is 1 dm. long, 1 dm. wide, and 1 cm. thick? 2 cm. thick? 5 cm. thick? 10 cm., or 1 dm., thick?

3. Then how many cubic centimeters are there in a rectangular solid 1 dm. by 1 dm. by 1 dm., that is, in 1 cu. dm.?

4. How many cubic decimeters equal 1 cubic meter? How many cubic meters equal 1 cubic dekameter?

283. *In metric cubic measure it requires* 1000 *units of any denomination to make* 1 *unit of the next higher denomination.*

284. The primary unit of **volume** is the **cubic meter.**

The official equivalent of the cubic meter is 1.308 cubic yards.

285. Table of measures of volume.

1000 cubic millimeters = 1 **cubic centimeter**
1000 cubic centimeters = 1 **cubic decimeter**
1000 cubic decimeters = 1 **cubic meter**

and so on. The higher units, however, are little used.

286. From § 283 and the table, it is evident that the successive metric units of cubic measure occupy *three* orders:

cu. Km.			cu. Hm.			cu. Dm.			cu. m.			cu. dm.			cu. cm.			cu. mm.
1	0	0	1	0	0	1	0	0	1.	0	0	1	0	0	1	0	0	1

Sometimes the abbreviation m^3 is used for cu. m.

287. The primary unit of **wood measures** is the **stere**, which is a *cubic meter*.

288. Table of wood measures.

10 decisteres = 1 stere
10 steres = 1 dekastere

WRITTEN EXERCISES

289. 1. A book is 19 cm. long, $14\frac{1}{2}$ cm. wide, and 2 cm. thick. Express its volume in cubic centimeters.

2. How many steres of wood can be piled in a shed 6 m. long, 5 m. wide, and 3.2 m. high?

3. How many cubic centimeters of water will fill a cubical box, each inside dimension of which is 1 dm.?

4. A wall along the side of a park 1 Km. long is 1 m. thick and 1.6 m. high. Find the solid contents in cubic meters.

5. How thin must 1 cu. cm. of gold be beaten to cover a rectangle 40 cm. long and 15 cm. wide?

6. A block of white marble quarried at Paros, Greece, was 4 m. long and 1.25 m. square. Find its value at $28 per cu. m.

7. In a recent year 320 cu. Hm. of gas were produced in New York City. How many cubic meters were produced?

8. Timbers of imported Spanish mahogany are usually 61 cm. square and 3.05 m. long. Find the volume of such a timber.

MEASURES OF CAPACITY

290. The primary unit of **capacity** for both liquid and dry measures is the **liter** (lētẽr), which contains 1 *cubic decimeter*, or 1000 *cubic centimeters.*

The official equivalent of the liter is 1.0567 qt. (liquid). The liter is .908 qt. (dry).

291. Table of measures of capacity.

10 milliliters	=	1 centiliter
10 centiliters	=	1 deciliter
10 deciliters	=	1 **liter**
10 liters	=	1 dekaliter
10 dekaliters	=	1 **hektoliter**

The **liter** (about 1 qt., liquid or dry) is used to measure moderate quantities; the **hektoliter** (nearly 2.84 bu.) is used to measure grain, fruit, vegetables, etc., in large quantities.

292. The successive units of capacity correspond to the orders of units in the decimal notation, thus:

Hl.	Dl.	l.		dl.	cl.	ml.
1	1	1	.	1	1	1

WRITTEN EXERCISES

293. 1. How many liters of rice will it take to fill a box the inside dimensions of which are 60 cm., 50 cm., and 40 cm.?

2. Find the cost, in Mexico, of 24 hektoliters of corn at $12 per 200 liters.

3. A Manila merchant bought 400 liters of olive oil at 25¢ per liter and retailed it at 30¢ per $\frac{3}{4}$ liter. Find his gain.

4. The French wheat harvest one year was $110\frac{1}{2}$ million hektoliters, grown on $6\frac{1}{2}$ million hektares. Find the yield per hektare.

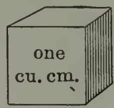
5. A Spanish olive orchard of 750 trees yielded 30 liters of olives per tree. How many hektoliters of olives did it yield?

6. An electric fire engine such as is used in France throws 350 liters of water per second. How many cubic meters of water do 4 such engines throw on a fire in an hour?

MEASURES OF WEIGHT

294. The primary unit of **weight** is the **gram**, which is the weight of 1 *cubic centimeter* of pure water at its greatest density.

The weight of 1000 cubic centimeters (1 cubic decimeter) of water, that is, of 1 *liter* of water, is called a **kilogram**, or a **kilo**.



The official equivalent of the kilogram is 2.2046 pounds avoirdupois.

295. Table of measures of weight.

10 milligrams = 1 centigram	10 grams = 1 dekagram
10 centigrams = 1 decigram	10 dekagrams = 1 hektogram
10 decigrams = 1 **gram**	10 hektograms = 1 **kilogram**

Higher units are:

10 kilograms = 1 myriagram
100 kilograms = 1 **metric quintal**
1000 kilograms = 1 **metric ton**

The **gram** (about $\frac{1}{5}$ the weight of a 5-cent piece) is used in weighing medicines, precious metals, etc.; the **kilogram** (about 2.2 lb.), in weighing groceries; and the **metric ton** (about 2205 lb.), in weighing bulky articles, such as hay, coal, etc.

A 5-cent piece weighs about 5 grams, and is about 2 cm. across and 2 mm. thick.

296. The successive units of weight and the orders of units in the decimal notation correspond, thus:

M.T.	Q.	Mg.	Kg.	Hg.	Dg.	g.		dg.	cg.	mg.
1	1	1	1	1	1	1	.	1	1	1

EXERCISES

297. Reduce to grams or decimals of a gram:

1. 3.5 Kg.	**3.** 2.5 dg.	**5.** 350 mg.	**7.** 49 Kg. 755 g.
2. 4.2 Hg.	**4.** 7.5 Dg.	**6.** 8400 mg.	**8.** 15 g. 200 mg.

9. Make a table of the units *milligram*, *gram*, *kilogram*, and *metric ton.*

10. If a 6-liter jar weighing $2\frac{1}{2}$ Kg. is filled with water, find the total weight of the jar and the water.

11. Find the weight of a cubic meter of water.

12. Ice is .92 as heavy as water. Find the weight of a cube of ice 30 cm. on each edge (about 1 cu. ft. of ice).

13. A boy who weighs 40 kilos weighs about —— pounds.

14. About how many 5-cent pieces weigh 1 kilo?

EQUIVALENTS

298. Hereafter, unless stated to the contrary, refer to the following equivalents when changing from one system to the other:

Common to Metric	Metric to Common
1 yd. = $\frac{3600}{3937}$ m. = .9144 m.	1 m. = 39.37 in.
1760 yd. = 1 mi. = 1.60935 Km.	1 Km. = .62137 mi.
1 sq. yd. = .836 sq. m.	1 sq. m. = 1.196 sq. yd.
1 A. = .4047 Ha.	1 Ha. = 2.471 A.
1 cu. yd. = .765 cu. m.	1 cu. m. = 1.308 cu. yd.
1 qt. (dry) = 1.1012 l.	1 l. = .908 qt. (dry)
1 qt. (liquid) = .94636 l.	1 l. = 1.0567 qt. (liquid)
1 bu. = .35239 Hl.	1 Hl. = 2.8377 bu.
1 oz. (troy) = 31.10348 g.	1 Kg. = 32.1507 oz. (troy)
1 lb. (av.) = .45359 Kg.	1 Kg. = 2.2046 lb. (av.)
1 T. = .90718 M.T.	1 M.T. = 1.1023 T.

WRITTEN EXERCISES

299. In changing from one system to the other, estimate results in advance; then use the equivalents on the preceding page, giving final inexact results to the nearest thousandth.

In exercises 1–9 reduce kilometers to miles, meters to feet, centimeters and millimeters to inches:

1. Height of spire of Strassburg Cathedral, 142 m.
2. Height of Pyramid of Cheops, 139.5 m.
3. Height of Leaning Tower of Pisa, 54.5 m.
4. Height of "203-meter Hill" near Port Arthur, 203 m.
5. Length of St. Gothard tunnel, Switzerland, 14.9 Km.
6. Length of Panama Railroad, Panama to Colon, 78 Km.
7. Length of French cable, Brest to New York, 5318.9 Km.
8. Length of hairspring of a watch, 23 cm.
9. Bore of Mauser rifle, 7 mm.

10. In Paris the height of a building to the eaves may not exceed the width of the street by more than 6 m., and may not exceed 20 m. in any case. How many feet high may the walls of a building be on a street 12 m. wide? on one 18 m. wide?

11. In a Parisian building 20 m. high to the eaves, the ceilings on the first floor must be at least 9 ft. 2 in. high. Express this height in meters.

12. There are 600,000 hektares of cork trees in Portugal, 300,000 in Spain, and 80,000 in Italy. How many square kilometers of cork tree forests are there in each country?

13. How many metric tons of lard are there in a shipment of 1000 tubs, each holding $12\frac{1}{2}$ Kg. of lard?

14. A South American sewer tunnel 1,278 m. long, 3.65 m. high, and 3 m. wide was cut through rock. How many cubic meters of rock were removed?

15. A large dredge removed 4500 cu. m. of mud per hour at a cost of 3 pfennigs per cubic meter. Find the cost in marks of running the dredge 12 hours.

16. Short cotton, grown in Algeria, averages 3 metric tons per hektare. Find the value of 5.5 Ha. of Algerian cotton at $9.25 per metric quintal.

17. A New York firm sent a letter to London weighing 135 g. How much postage was paid at 5¢ per 15 g.?

18. A coffee tree yielded 800 g. of berries. If $\frac{1}{5}$ of their weight was husk, how much marketable coffee was obtained?

19. In a recent year the output of gold in French Guiana amounted to 2541.35 Kg. How many troy ounces of gold were produced?

The **specific gravity** of a substance is its weight as compared with that of an equal volume of water.

20. If two liters of milk weigh 2.06 Kg., what is the specific gravity of milk, or how many times as heavy as water is it?

Solution

1 l. of water weighs 1 Kg.; 1 l. of milk weighs $\frac{1}{2}$ of 2.06 Kg., or 1.03 Kg. Therefore the specific gravity of milk is 1.03.

Find the specific gravity of the following substances:

21. Olive oil, if 40 cu. cm. weigh 36.6 g.

22. Mercury, if $\frac{1}{2}$ l. weighs 6.799 Kg.

23. Cork, if a piece 10 cm. by 8 cm. by 2 cm. weighs 384 dg.

24. Find the weight of 25 l. of naphtha, specific gravity .848.

25. A wholesale dealer carefully weighed a sample of cloth 40 mm. by 50 mm. and found its weight to be 62 cg. What was the weight of this cloth per meter, if it was 140 cm. wide?

26. Norway in a recent year exported 31,000,000 kilos of fresh codfish and 1,500,000 kilos more than half that amount of dried codfish. How many pounds of each did she export?

27. In a city in Greece, 800 private lights burning a certain kind of gas are in use. If each jet consumes 15 liters of gas per hour and burns on an average 4 hours per day, how much gas is consumed in the city per week?

28. The substance from which this gas is made gives out 300 liters of gas per kilo. How many boxes containing 50 kilos each are used per week?

29. A quart of water is less than a liter, but if frozen will make more than a liter of ice. Find how many cubic centimeters more, if water expands $\frac{2}{23}$ of its volume in freezing.

30. Find the cost of 17 metric quintals of chestnuts at 15 francs per sack of 100 kilos.

31. The length of a new French locomotive is 16.2 m. How much longer, to the nearest inch, is it than a large English locomotive whose length is 41 ft. 4 in.?

32. A shipment of coffee from Bahia, Brazil, to Bordeaux, France, consisted of 500 bags holding 60 Kg. each. Find the cost for freight in francs at 30 francs per 900 Kg.

33. In a quarry in Saxony an undercut resulted in the fall of 58,000 cu. m. of stone. Find the weight of the stone in metric tons, if its specific gravity was 2.7.

34. Greece sent to Great Britain one year 64,000 metric tons of currants, and to Holland and the United States $\frac{1}{2}$ and $\frac{5}{16}$ as much, respectively. Express these quantities in short tons.

35. The Bank of France pays 3437 francs for 1 kilogram of fine gold. Find, to the nearest cent, the equivalent price per troy ounce.

36. At a Berlin motor boat exhibit, an American exhibitor paid an entrance fee of 100 marks, and rented a floor space of $4\frac{1}{2}$ square dekameters at 10 marks per square meter. Find his expense in United States money.

PRACTICAL MEASUREMENTS

MEASURES AND EQUIVALENTS

300. In delicate weighing, a small unit of weight called a **grain**, originally the weight of a grain of wheat, is used.

The grain is the same for both avoirdupois and troy weight, and is therefore the connecting link between them.

Avoirdupois	Troy
1 pound = 7000 grains	**1 pound = 5760 grains**
1 ounce = $437\frac{1}{2}$ grains	**1 ounce = 480 grains**

Avoirdupois weight is used for weighing bulky articles; troy weight, for gold, silver, platinum, and some kinds of jewels; and apothecaries' weight, in filling prescriptions, though druggists buy and sell by avoirdupois weight.

301. Water in large quantities is measured by the barrel (bbl.) of $31\frac{1}{2}$ gallons, by the cubic foot, and by the thousand or million gallons.

A **gallon** is equivalent to **231 cu. in.**; hence, **1 cu. ft.** (1728 cu. in.) $= \frac{1728}{231}$ gal. $= 7.48^{+}$ gal., or nearly **$7\frac{1}{2}$ gal.**

302. In ordinary calculations, the weight of a **cubic foot** of water is taken as **$62\frac{1}{2}$ lb.**, or **1000 oz.**

Then since 1 gal. (231 cu. in.) $= \frac{231}{1728}$ cu. ft., a **gallon** of water weighs $\frac{231}{1728}$ of $62\frac{1}{2}$ lb. $= 8.355^{+}$ lb., or about **$8\frac{1}{3}$ lb.**

303. A **bushel** is equivalent to **2150.42 cu. in.**, but for all practical purposes it is considered sufficiently accurate to take **$1\frac{1}{4}$ cu. ft.** (= 2160 cu. in.) as the equivalent.

304. Hereafter, unless the contrary is expressly stated, the following measures and approximate equivalents should be used:

1 gal. = 231 cu. in.	1 bbl. = $31\frac{1}{2}$ gal.
1 cu. ft. = $7\frac{1}{2}$ gal.	1 bu. = $1\frac{1}{4}$ cu. ft.
1 cu. ft. of water weighs $62\frac{1}{2}$ lb	
1 gal. of water weighs $8\frac{1}{3}$ lb.	

WRITTEN EXERCISES

305. 1. A one-cent copper piece weighs 48 grains. If I get $35 in new cent pieces, how many pounds do I have to carry?

2. One hundred cartridges for the American military rifle weigh 6 lb. 7 oz. $52\frac{1}{2}$ gr. The same number of cartridges for the German rifle weigh 6 lb. 2 oz. 149 gr. Find the weight in grains of each kind of cartridge.

3. *A diamond carat is* 3.168 *grains.* The weight of the largest diamond in the world is $3025\frac{3}{4}$ carats. How much more than $1\frac{1}{4}$ pounds avoirdupois does the diamond weigh?

4. A silver dollar weighs 412.5 grains, $\frac{9}{10}$ of which is pure silver. How many silver dollars must a silversmith melt to obtain 2 lb. 6 oz. 18 pwt. 18 gr. of pure silver?

5. If 420 tons of drinking water are consumed on a vessel during an Atlantic voyage, how many gallons are used? how many cubic feet?

6. A skating rink 75 feet by 216 feet was flooded with 2 inches of water. How many gallons were used?

7. How many gallons of water will the tank of a locomotive tender hold, if its capacity is 890 cubic feet?

8. The cargo of grain from a vessel filled 12 elevator bins, each $8\frac{1}{3}$ feet square and 90 feet deep. How many bushels of wheat did the vessel carry?

9. Find the capacity in bushels of a Duluth grain elevator having 2,250,000 cubic feet of storage space.

10. The specific gravity of glass is about 2.76. Find the weight of a piece of plate glass 12 in. by 10 in. by $\frac{1}{2}$ in.

11 What is the specific gravity of paraffin, if a block 6 in. by 4 in. by 1 in. weighs $12\frac{1}{2}$ ounces?

12. The specific gravity of ice is .92. What must be the height of a room required to hold 184 tons of ice, making no allowance for packing, if the floor space is 20 feet square?

13. A cord of maple wood weighed 2814 pounds. If the specific gravity of the wood was .67, how many cubic feet of solid wood did the cord actually contain?

14. A body floating in water displaces its own weight of water. The steamship *Baltic* weighs 40,000 tons. How many cubic feet of fresh water would she displace? of salt water, specific gravity 1.03?

15. *A double eagle weighs* $21\frac{1}{2}$ *pennyweights.* Find to the nearest cent the value of 1 pennyweight of pure gold.

The value of a gold coin is the value of the gold it contains. U. S. gold coins are $\frac{9}{10}$ fine; British or sterling gold, $\frac{11}{12}$ fine.

16. How many grains does a new quarter eagle weigh?

17. Find the weight of $1,000,000 in new U. S. gold coin.

18. The English mint is compelled to coin any gold offered it, returning 77*s.* $10\frac{1}{2}$*d.* per ounce $\frac{11}{12}$ fine. Show that the standard weight of a sovereign is 123.27447 grains.

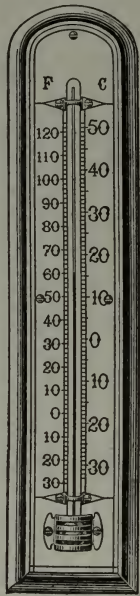
19. Find the weight of £1,000,000 in sterling money.

20. A stamping press at the mint stamps 80 double eagles per minute. How many pounds of coin does it stamp per hour?

21. The weight of gold lost every year by wear, sinking of vessels, etc., is estimated at 193,704 ounces. How many eagles could be made from this amount? How much gold would be left over?

MEASURES OF TEMPERATURE

306. On a Fahrenheit thermometer, the one in common use in America, the freezing point of water is marked 32°, or 32 *degrees*, and the boiling point 212°. For ordinary purposes, however, thermometers are not marked as far as the boiling point.



Since the difference between the freezing and boiling points of water, $212° - 32°$, is 180°, one **degree** Fahrenheit is $\frac{1}{180}$ of this difference.

A more convenient system, in which this difference in temperature is divided into 100 degrees, from 0° at the freezing point to 100° at the boiling point, has been devised; but it is not yet in general use in America. It is called the **centigrade** system of measuring temperature.

Temperatures below 0° are written with a minus sign, thus: $-1°$, $-3°$.

Since 100° C. = 180° F., 1° C. = $\frac{180}{100}°$ F., or $\frac{9}{5}°$ F.

Similarly, 1° F. = $\frac{100}{180}°$ C., or $\frac{5}{9}°$ C.

EXERCISES

307. 1. At 6 A.M. the temperature was $-10°$ F., and at noon it was 22° F. Find the rise in temperature.

2. At 2 P.M. the temperature was 27° F., and at 9 P.M. it was $-2°$ F. Find the fall in temperature.

3. How far below the freezing point of water is $-16°$ F.?

4. In a freezing mixture composed of ice and salt, the temperature fell from $+10°$ C. to $-18°$ C. Express the fall in degrees centigrade.

WRITTEN EXERCISES

308. 1. Express 15° C. in the Fahrenheit scale.

SOLUTION. — Since 1° C. = $\frac{9}{5}$° F. (§ 306), 15° C. = 15 × $\frac{9}{5}$° F. above the freezing point, that is, above 32° F.

$$15^\circ \text{ C.} = \overset{3}{\cancel{15}} \times \frac{9^\circ}{\cancel{5}} \text{ F.} + 32^\circ \text{ F.} = 59^\circ \text{ F.}$$

2. Express 59° F. in the centigrade scale.

SOLUTION. — Since 59° F. means 59° – 32°, or 27 Fahrenheit degrees above the freezing point of water, and since each Fahrenheit degree above the freezing point is equivalent to $\frac{5}{9}$ of a centigrade degree above 0° C.,

$$59^\circ \text{ F.} = \tfrac{5}{9}(59 - 32) \text{ degrees centigrade} = 15^\circ \text{ C.}$$

3. Blood heat is about 98° F. At how many degrees would a centigrade thermometer register blood heat?

4. Olive oil freezes at 30° F., and mercury at – 37.9° F. Express these freezing points in the centigrade scale.

5. When the temperature falls from 15° F. to – 10° F., how many Fahrenheit degrees does it fall? how many centigrade degrees?

6. Water is heaviest at 4° C. Express this temperature in degrees Fahrenheit.

7. Milk is pasteurized by heating it to 68$\frac{1}{3}$° C. and keeping it at that temperature for a certain time. Express the temperature in the Fahrenheit scale.

8. During the ascent of a mountain in Switzerland the temperature fell from 25° C. to 5° C. How many Fahrenheit degrees did it fall?

9. During a 5-year period the highest average monthly temperature in Manila was 28.7° C. in June; and the lowest was 25° C. in January. Express these temperatures in the Fahrenheit scale and find the difference between the highest and the lowest.

LUMBER MEASURE

309. The unit of lumber measure is the **board foot**. It is defined as a board 1 foot long, 1 foot wide, and 1 inch thick.

Then the number of board feet in a board 1 inch (or less) thick is the same as the number of square feet of surface in one side of the board.

A board 1 foot wide and 12 feet long contains 12 board feet, if it is 1 inch (or less) in thickness.

For brevity the word "foot" is used instead of board foot, and "thousand feet" or "M" instead of thousand board feet.

310. When lumber is more than 1 inch thick, the number of board feet is obtained by multiplying the *number* of *feet* in *length* by the *number* of *feet* in *width*, and that product by the *number* of *inches* in *thickness*.

A board 12 ft. long and 8 in. wide contains $(12 \times \frac{2}{3})$ ft., or 8 ft., if it is 1 in. (or less) thick; if it is 2 in. thick, it contains 2×8 ft., or 16 ft.; if $1\frac{1}{2}$ in. thick, $1\frac{1}{2} \times 8$ ft., or 12 ft.

Instead of 12 ft. by 8 in. by 2 in. we may write $12' \times 8'' \times 2''$.

WRITTEN EXERCISES

311. Find the number of feet in the following:

	Number of Pieces	Size		Number of Pieces	Size
1.	120	$2'' \times 6'' \times 14'$	8.	180	$3'' \times 8'' \times 16'$
2.	100	$3'' \times 4'' \times 13'$	9.	40	$4'' \times 7'' \times 12'$
3.	150	$2'' \times 4'' \times 10'$	10.	480	$2'' \times 12'' \times 12'$
4.	206	$2'' \times 8'' \times 14'$	11.	36	$3'' \times 10'' \times 22'$
5.	240	$4'' \times 6'' \times 18'$	12.	450	$6'' \times 14'' \times 14'$
6.	160	$4'' \times 4'' \times 12'$	13.	215	$4'' \times 10'' \times 18'$
7.	125	$4'' \times 8'' \times 15'$	14.	110	$8'' \times 16'' \times 20'$

15. The pasting table used by a paper hanger consisted of four $\frac{3}{4}$-inch white pine boards 6 feet long and 10 inches wide. How many feet of lumber did it contain?

16. For a combination bookcase, 45 feet of plain oak were used at \$30 per M, 10 feet of 3-ply oak at \$95 per M, and 8 feet of quartered oak at \$60 per M. This cost was $\frac{2}{11}$ of the cost of the materials used. Find the entire cost of materials.

17. A slab of ash 38 inches long, 11 inches wide, and $2\frac{3}{4}$ inches thick makes 4 baseball bats, which sell at 8¢ each at the mill. How many feet of ash are required to fill the order of a sporting goods company for baseball bats to the value of \$46.08?

18. Find the cost, at \$16 per M, of this lumber for a shed designed to cover an acre of growing pineapples:

463 posts $4'' \times 4'' \times 9'$	3900 boards $1'' \times 3'' \times 16'$ (roof)
266 stringers $2'' \times 6'' \times 16'$	450 boards $1'' \times 12'' \times 16'$ (sides)

19. A man who had 16 rows of celery, each row 160 feet long, blanched the celery by placing boards on both sides of each row and banking earth against them. The boards were 1 inch thick and 14 inches wide. Find the cost at \$21 per M.

20. To preserve the celery in winter it was placed in trenches 16 inches deep, the walls of which were lined with rough 1-inch boards. There were 40 trenches, each 60 feet long. What was the cost of the lumber at \$12 per M?

21. Find the cost of this lumber for a house, at \$20 per M: sills $4'' \times 8''$, total length $192'$; 10 posts $4'' \times 7'' \times 24'$; ties and plates $4'' \times 6''$, total length $522'$; 70 beams $2'' \times 8'' \times 15'$; 60 beams $2'' \times 8'' \times 22'$; 15 rafters $3'' \times 5'' \times 20'$.

22. In making checkers of a certain size, a piece of lumber $12' \times 12'' \times 1\frac{1}{2}''$ was sawed into 7 strips, each foot of strip making 20 checkers. How many feet of lumber were used to make enough checkers to fill 2800 boxes holding 30 each?

ROOFING AND FLOORING

312. The unit of roofing and flooring is an area of 100 square feet called a **square**.

The average shingle is about 18 inches long and 4 inches wide. When laid "4 inches to the weather," the exposed surface of 1 shingle is (4×4) sq. in., or $\frac{1}{9}$ sq. ft. Consequently it takes 900 shingles to cover a square, but to allow for waste 1000 shingles per square are often estimated.

There are 250 shingles in a **bunch**.

A fractional part of a bunch cannot be bought.

WRITTEN EXERCISES

313. 1. The roof of a house was 38′ long and it was 30′ from the ridge to the eaves on each side. How many pieces of slate 9″ wide were required to cover the roof, if they were laid 6″ to the weather?

2. Find the weight of slate on the roof, if a square foot weighed $4\frac{1}{2}$ pounds.

3. The roofer averaged 2 squares in 10 hours. If he received 35¢ per hour, how much did he receive for laying the roof?

4. Find the cost of the slate at $5.75 per square.

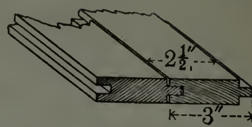
5. Adding $2.75 for hardware and breakage, what was the entire cost of the roof?

6. How many bunches of common shingles should be purchased to cover a roof 80′ × 32′, estimating 4 bunches per square?

7. Find the cost of the roof at $6 per square.

8. Good cypress shingles may be laid $4\frac{1}{4}$ inches to the weather. How many, to the nearest shingle, are required to cover a square? How many bunches must be purchased to cover a roof 38′ × 35′, estimating 875 shingles per square?

9. How many pieces of maple flooring, 14′ long and $2\frac{1}{2}''$ wide when laid, are required for the floor of a room 12′ × 14′?



10. The boards are $\frac{7}{8}''$ thick. Since they were 3″ wide before being tongued and grooved, they are sold as boards 3″ wide. Find the cost of the flooring at $35 per M.

11. How many days will be required to lay a mosaic tile floor in a room 48′ × 36′, if the tiler can lay in a day 20 sheets of tile, each sheet 2′ × 1′?

12. What will be the cost of the floor at $1.55 per square foot?

PLASTERING AND PAINTING

314. The unit of plastering, painting, and kalsomining is usually the **square yard.**

Laths are 4 feet long and there are 50 or 100 in a **bundle.** In this book a bundle means 100 laths. A fractional part of a bundle cannot be bought.

Custom is not uniform as to allowances for openings and baseboard. Consequently to avoid misunderstandings, a written contract should be made.

WRITTEN EXERCISES

315. The second story of a cottage consisted of 4 bedrooms. Each was 18′ × 12′ and 9′ high, with 2 windows, each 3′6″ × 6′ and 1 door 3′ × 7′. Find the following, deducting openings:

1. The number of laths required, if 1500 cover 100 sq. yd.

2. The cost of putting on the laths at 6¢ per sq. yd.

3. The number of days it will take a plasterer to lay on three coats, if he can plaster 144 sq. yd. of first coat per day, $\frac{2}{3}$ as much of second coat, and $\frac{1}{2}$ as much of third coat.

4. The cost of plastering the rooms at 33¢ per sq. yd.

5. In fireproof construction expanded metal or wire laths are used instead of wooden ones. In plastering a room $30' \times 21'$ and $11'$ high, deducting $\frac{1}{2}$ for 3 windows, each $7' \times 3'\ 9''$, and for 1 door $10' \times 10'$, how much more will fireproof construction cost at 70¢ per square yard than ordinary construction at 40¢?

6. If 1 gallon of stain will cover 400 square feet of soft wood, how much stain will be required to give a coat of stain to a white pine wainscoting 5 feet high around a room $20' \times 17'\ 6''$, deducting 75 square feet for openings?

7. In computing the bill for painting one side of a house 49 feet deep and 36 feet high, a painter, after deducting the area of 8 windows, each $3' \times 8'$, charged 27¢ per square yard, and 50¢ for each window. Find the cost to the owner.

PAPERING AND CARPETING

316. Wall paper is sold in **single rolls** 8 yards long, or in **double rolls** 16 yards long. It is usually 18 inches wide. Consequently for such a width 2 vertical strips are required for each yard of the distance around the room.

Fractional parts of rolls are not sold.

Borders or friezes are sold by the **linear yard**. They vary in width.

317. Carpeting is sold by the **linear yard**. It is usually 1 yard or $\frac{3}{4}$ of a yard in width.

Matting, oilcloth, linoleum, etc., are of various widths.

318. In practice it is not always worth while, or even possible, to compute the exact cost of papering or carpeting rooms.

What is sought is the approximate cost, making liberal allowances for doors and windows, for matching patterns, etc.

No allowance for matching the pattern is necessary for the first strip.

WRITTEN EXERCISES

319. 1. A room $21' \times 18'$ has 3 windows and 2 doors. How many strips of wall paper 18 inches wide are necessary to paper the walls of the room, deducting 2 for each window and 2 for each door?

2. The room is 9′ high. Making allowance for matching and for the width of baseboard and border, it is found that 3 strips can be cut from a roll. Find the cost of paper for the room at 50¢ per roll.

3. How much does the border cost at 7¢ per foot?

4. In papering the ceiling the long way, a roll makes but 1 strip. How much does this paper cost at 35¢ per roll?

5. Find the expense of carpeting this room with carpet $\frac{3}{4}$ yd. wide, costing $87\frac{1}{2}$¢ per yard, strips running the long way.

6. How much cheaper is it to paper the walls of a room $14'\ 8'' \times 12'\ 9''$ with double rolls that give 7 strips each, than with single rolls that give 3 strips each, if 9 strips are allowed for openings, the price per roll being 25¢, per double roll 50¢?

7. A room in which I wish to lay matting 36″ wide and costing 45¢ per yard is $14' \times 18'$. Is it cheaper to lay the matting the long or the short way, and how much cheaper?

8. Find the cost of enough linoleum 2 yards wide to cover the floor of an office $24' \times 16'\ 4''$, at $1.17 per square yard, strips running the short way.

9. Find the cost of carpeting the New York State senate chamber $60' \times 55'$ with Victoria Wilton carpet 27 inches wide at $2.35 per yard, making no allowance for matching the pattern.

10. A floor $16' \times 12'$ is to be covered with Axminster carpet 27 inches wide, costing $1.75 per yard. Allowing in all $1\frac{1}{4}$ yd. for matching the pattern, how much carpet will be required if the strips run the long way? Find the cost of the carpet.

MISCELLANEOUS MEASUREMENTS

WRITTEN EXERCISES

320. **1.** In getting sod for a park in an Illinois city, strips were cut 12″ wide and 9′ long, 80 strips making a wagon load. If the number of wagon loads was 250, how many *square yards* of sod were used?

2. The cost per square yard was: cutting 1.6¢, carting .9¢, laying 2.7¢, watering .6¢. Find the total cost of the sod.

3. Find the cost of excavating a cellar 28′ × 45′ and 9′ deep, at 30¢ per *cubic yard.*

4. A short ton of Lehigh egg coal occupies about 35 cubic feet. What must be the depth of a car 30′ × 8′ to hold 40 tons?

5. Find the cost of making a brick sidewalk 12′ × 318′, including the following items. Find also the cost per *square yard.*

Excavating and leveling ground, $4.75.
6 loads of sand at 40¢; carting, $6; spreading, $3.13.
Bricks, 38 per square yard, at $7.50 per *thousand.*
Labor, 1 man laying 53 square yards per day, at $3 per day.

6. Find the cost of asphalt paving for 405 feet of street 60 feet wide in a Southern city, if the concrete foundation cost 65¢ per square yard and the asphalt $1.15 per square yard.

7. How much did it cost to construct a macadam road $\frac{1}{2}$ mile long and 36 feet wide, at $.98$\frac{1}{2}$ per square yard?

8. Find the cost of paving the same road with asphalt at $2.36 per square yard.

9. One year 18,457 square yards of brick pavement were laid in the streets of Helena, Montana. If 58 bricks were required for a square yard, how much did the bricks cost at $20 per M?

10. How many bricks are required for a wall 25′ long, 12′ high, and 3′ thick, if $22\frac{1}{2}$ bricks with mortar occupy 1 cubic foot?

11. Find the cost of the bricks at $7.50 per M.

12. Find the cost of the wall at $12 per M bricks, laid.

13. How many *cubic yards* of mortar are required, if 1000 bricks are laid in .7 of a cubic yard of mortar?

14. Find the weight of the wall, if 1 cu. ft. weighs 125 lb.

15. How much will it cost to build a wall 80′ long, 4′ high, and 3′ thick of hollow concrete blocks $32'' \times 9'' \times 12''$, costing 25¢ each, laid?

16. The front wall of a house 22′ wide and 54′ high was of ashlar (cut stone) masonry 2′ thick. Deducting $\frac{1}{6}$ of the wall for openings, find the cost at $25.20 per cubic yard.

17. Find the cost, at 50¢ per *square foot*, of dressing a stone doorstep 5′ long, 3′ wide, and 7″ thick, on the top, ends, and one long side.

18. How many cubic yards were there in a concrete wall 5′ high, 2′ thick, and 54′ long?

19. If 1 cubic yard of this concrete consisted of $1\frac{4}{5}$ *barrels* of cement of $3\frac{3}{4}$ cubic feet each, twice as much sand, and four times as much broken stone, how many barrels of cement and how many *cubic yards* of sand and of stone were used in all?

20. The cement cost the contractor $2 per barrel; the sand 25¢ per cubic yard for loading, 20¢ for carting, and 15¢ for screening; and the broken stone 90¢ per cubic yard. If $1 was the additional cost per cubic yard for mixing and laying the concrete, find the total cost to the contractor.

21. The specific gravity of clay is about 1.2. Find the weight of a cubic yard of clay.

22. The specific gravity of lead is 11.35. What is the value of a cubic foot of lead at $4\frac{1}{2}$¢ per pound?

23. A body immersed in water is buoyed up by a force equal to the weight of water displaced. If a man's body displaces $2\frac{1}{2}$ cubic feet of water, how much will he be buoyed up when swimming in fresh water? in salt water, specific gravity 1.03?

24. How much less force is required to lift a stone $5' \times 2' \times 1'$ to the surface of a stream than to lift it out?

25. A floating timber 20′ long and 1′ square is three fourths submerged. The water displaced weighs as much as the timber. Find the weight of the timber and its specific gravity.

The speed of vessels at sea is measured in **nautical miles,** or **knots,** per hour. The U. S. Coast Survey knot is **6080.27 feet,** or about the length of 1′ of arc on the equator.

A sailor at the stern of a vessel throws overboard a log, or float, that remains stationary as the vessel proceeds. He then counts the number of knots of the line that are reeled out in a given time, usually 28 or 30 seconds. These knots are at such distances apart that the number reeled out in the given time is equal to the number of nautical miles an hour the ship is sailing. Thus, if the log falls behind 14 knots while the sand in the glass runs out, the ship is sailing 14 knots an hour. In the following use **1 knot = 1.15 statute miles.**

26. If the *Brooklyn* is making 22 knots an hour, how many statute miles an hour is she sailing? how many feet per second?

27. The new turbine Cunard steamships average $24\frac{1}{2}$ knots an hour. How many statute miles do they sail in an hour? in a day of 24 hours?

28. The cruiser *Washington* in a 4-hour trial made an average speed of 22.27 knots per hour. How many statute miles, to the nearest hundredth, did she travel per hour?

29. A ship sailed on the equator from 21° 50′ west longitude to 28° 14′ west longitude in 24 hours. Find her approximate speed in knots per hour.

REVIEW PROBLEMS IN INDUSTRIES

321. 1. A Kansas farmer had two fields sown to wheat, one $\frac{3}{4}$ of a mile long and $\frac{1}{2}$ of a mile wide, the other $\frac{3}{8}$ of a mile by $\frac{5}{8}$ of a mile. How many acres of wheat had he?

2. For plowing, 3 gang plows were used, each turning $12\frac{1}{2}$ acres per day of 10 hours. How many days and hours did it take to plow the 390 acres of both fields?

3. If $5\frac{1}{2}$ pecks of seed were sown per acre, find the cost, at 60 ¢ per bushel, of seed used on the 390 acres.

4. The wheat of the smaller field (150 acres) was reaped and bound into sheaves by a self-binding harvester, and afterward thrashed. If the yield was 3600 bushels, find the yield per acre. Find the cost of thrashing the wheat from this field at $4\frac{1}{2}$ ¢ per bushel.

5. If the larger field (240 acres) produced the same number of bushels per acre, find the whole yield of both fields.

6. On the larger field a combined harvester and thrasher was used, which reaped and thrashed the wheat and put it into 2-bushel sacks at the rate of 27 sacks per hour. How many days of 10 hours each did it take to reap the 5760 bushels?

7. The owner had his harvest of 9360 bushels carted to an elevator, where it was cleaned and loaded on freight cars at a cost of $1\frac{1}{2}$¢ per bushel. Find this item of expense.

8. Eighteen freight cars were used to carry this wheat. A bushel of wheat weighs 60 pounds. Find the load per car.

9. If the freight bill to Chicago amounted to $1010.88, what was the rate per 100 pounds?

10. At Chicago the charge for elevator storage was $\frac{3}{4}$¢ per bushel for the first 10 days, and $\frac{3}{8}$¢ for each succeeding 10 days. Find the cost of storing the 9360 bushels from August 8 to September 16 inclusive.

11. The farmer's entire expense was $2885.75. If the wheat was sold at 65¢ per bushel, what was his total gain?

12. It cost $4\frac{3}{4}$¢ per bushel to transport the 9360 bushels of wheat to New York by way of the Great Lakes, Erie canal, and Hudson River. The rate by rail was 16¢ per 100 pounds. How much more would the freight charges have been by rail?

13. If this crop of 9360 bushels made 4 pounds more than 2063 barrels of flour of 196 pounds each, what decimal part of the weight of the wheat was the weight of the flour?

14. How many bushels of wheat, to the nearest hundredth, did it take to make one barrel of flour?

15. If 100 pounds of flour make 135 1-lb. loaves of bread, how many loaves can a baker make from 5 barrels of flour?

16. Recently the average yield of wheat per acre in various countries was as follows: Great Britain, 31.8 bushels; France, 26; Germany, 19.4; Austria, 16.4; United States, 13.4; Russia, 9. Find the yield from 25 acres in each of these countries.

17. Our country's leading wheat states one year were Minnesota with 68,000,000 bu., Kansas with 65,000,000 bu., and North Dakota with 54,000,000 bu. If these states produced .34 of our entire crop, how much did we produce?

18. In the fall Harvey Smith accepted a position as timekeeper in a lumber camp. At $1.85 per day, how much did he earn in 156 working days?

19. With him in the camp were 34 men employed to chop down the trees and saw them into logs. If each received $2.25 per day, what were their combined wages for the 156 days?

20. The wages of 13 teamsters employed for this time amounted to $3853.20. How much did each receive per day?

21. The other occupants of the camp were: 1 cook at $45 per month, 1 chore boy at $30, and 1 scaler, who measured and marked the logs, at $45 per month. Find their combined wages for 6 months.

22. Another set of men was employed to make roads to the river. If each received $1.50 per day, how much did the 23 road makers receive in 12 weeks of 6 working days each?

23. The logs were drawn over these roads and rolled into the river. After the breaking up of the ice they were guided down to the mill by log drivers. Seventeen log drivers worked 35 days and received $3123.75 in wages. Find the daily wage of each.

24. What were the total wages paid to the men employed to procure the logs and get them to the mill?

25. At the mill the largest logs went to the band saw. The setter, who places the logs in position, received $2.55 per day;

the dogger, who fastens them, \$2; and the sawyer, \$5.50. Find the cost of this labor, if the mill was running 184 days.

26. The smaller logs went to the circular saw where they were squared. If the sawyer received \$3.75 per day and his assistant $\frac{8}{15}$ as much, find the earnings of each during 184 days.

27. These logs next went to the gang saw where they were sawed into boards of different thicknesses. How much was paid during the season for work at the gang saw, if the sawyer received \$2.75 per day and his 2 assistants \$2 each?

28. The final operation was edging and trimming, for which 2 edgers were employed at \$2 per day each, and 2 trimmers at \$1.85 each. Find their total wages for the season of 184 days.

29. The broken lumber and trimmings were then sorted by a boy and all suitable pieces cut into laths by a lathman. During the season, the boy and man together received \$690, of which $\frac{2}{5}$ was the boy's share. How much did each receive per day?

30. As the sawdust and waste were used for fuel, the only cost for power was the engineer's wages, \$3.50 per day, and the fireman's wages, \$2.25 per day. Find the cost for power.

31. The daily wages of the other employees were: saw filer, \$6.50; assistant, \$2; millwright, \$3.50; assistant, \$2.50; and 12 lumber handlers at \$1.85 each. Find their total earnings.

32. How much in all was paid for labor in the mill?

33. The average daily output of the mill was 95,000 feet of lumber. What was the output for the whole season?

34. Find the cost of transporting one day's output from the mill to the market at \$1.75 per M; the season's output.

35. If $\frac{3}{5}$ of the lumber was spruce worth \$22.25 per M, $\frac{1}{10}$ pine worth \$28.75 per M, and the rest hemlock worth \$19.75 per M, what was the value of one day's output of 95,000 feet?

36. If these were the average prices received, find the receipts from the lumber supplied by the mill during the season.

37. The entrance or shaft to an anthracite coal mine averaged 12 feet by 18 feet, and extended to a depth of 720 feet. How many cubic yards of material were removed for the shaft?

38. Find the cost of sinking and timbering the shaft at $3.75 per cubic yard.

39. If the average time required to sink and timber the shaft was 365 days per 292 feet, how many days did it take for this shaft?

40. For tunneling the underground passages, the cost per lineal yard was 90¢ for powder and $3.60 for labor. Find the cost of tunneling a passage 567 feet in length.

41. The cars in which the coal was carried from the miners to the shaft weighed 850 pounds and had a capacity of one long ton. What was the total weight of a train of 8 loaded cars?

42. The rails upon which these cars ran weighed 35 pounds per yard. At a cost of $13.20 per long ton, find the cost of rails for a track 180 yards long.

43. The cage in which the coal was raised from the mine ascended at the rate of 24 feet per second. How long did it take to reach the surface 720 feet above?

44. A miner and his laborer averaged $5\frac{1}{2}$ tons per day, for which they received 69¢ per ton. What were the earnings of each per day, if the miner received $\frac{2}{3}$ and the laborer $\frac{1}{3}$?

45. If they worked 24 days in November and mined $5\frac{1}{2}$ tons of coal per day, find the miner's share of the earnings.

46. It required, on the average, $10\frac{2}{3}$ ounces of powder to dislodge one ton of coal. Find the cost of the powder

used during the month, if a keg holding 25 pounds cost $1.50.

47. The miner's other expenses for November were: file, 18¢; sharpening and repairing tools, 96¢; fuse, 15¢; $1\frac{3}{8}$ gal. lard oil @ 48¢. What were his total expenses?

48. Deducting expenses from the $60.72 received for his labor, what were the miner's net earnings for the month?

49. The cost of running a mine locomotive included $4.25 per day for labor, 50¢ for coal and oil, and 60¢ for repairs. Find the cost for 24 days.

50. The mine locomotive did the same work formerly done by 15 mules and their drivers. Find the cost of mule power for 24 days, if feeding, shoeing, and tending the mules cost $9 per day and drivers' wages amounted to $7.25 per day.

51. How much was saved by using the locomotive instead of the mules?

52. After the coal was mined it was sent to the breaker where the slate was picked out by breaker boys, and the coal sorted into sizes. If the 43 breaker boys were paid $219.30 in 6 days, how much did each receive per day?

53. Of the coal mined, $\frac{3}{16}$ was lost by blasting, breaking, and adhering to the slate. Find the loss in one day when 1365 long tons of merchantable coal were obtained.

54. The mine operator's expenses on 850 long tons of chestnut coal that he sold at $3.40 per ton were $2.08 per ton for mining and 96¢ per ton for handling. Find his profits on this sale.

55. It cost the retailer who bought this coal 85¢ per ton for freight, and 75¢ per ton for handling. If he sold the coal at $6.25 per short ton, how much did he gain?

56. One year, when there was a miners' strike, Pennsylvania produced 36,940,710 tons of anthracite coal, and the next year increased the output by 23,301,750 tons. How many tons were produced the year after the strike?

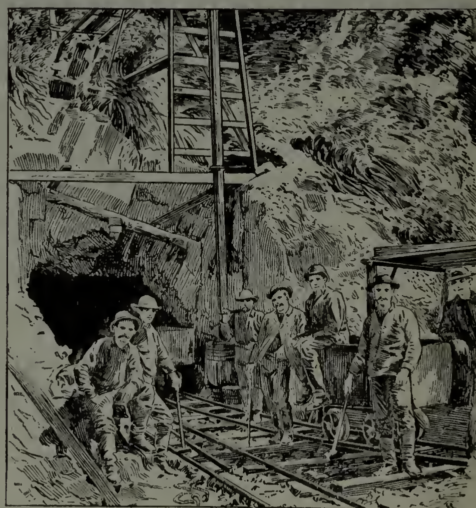
57. A new method of obtaining gold is by dredging on the surface. If one dredge picks up 85 cubic yards of earth weighing $1\frac{1}{2}$ tons each per hour, how many tons does it pick up in a day of 24 hours?

58. If each cubic yard of earth yields gold to the value of $17\frac{1}{2}$ ¢, what is the value of the gold dredged in one day?

59. If the amount of gold per cubic yard averages the same, find the value of gold dredged in 365 days.

60. Find the value of gold from $12\frac{1}{2}$ acres of gold-bearing earth, if it takes 11 days to dredge 1 acre.

61. Another method of mining gold is similar to that used in coal mining. How much did it cost to sink a gold mine shaft 102 feet, the expense being $16.75 per foot?



62. At certain levels horizontal passages called drifts were extended from the shaft. The various drifts had lengths of 184, 213, 239, 180, 93, 198, 41, and 156 feet, respectively. Find the total length of the drifts.

63. In making these drifts 1015 lb. of No. 1 powder at 16 ¢ per lb. and 12,760 lb. of No. 2 at 12 ¢ per lb. were used. Find the cost of powder.

64. To ignite the powder 37,500 feet of fuse at 52 ¢ per 1000 feet and 98 boxes of caps at 60 ¢ per box were required. How much did the fuse and caps cost?

65. If 1520 pounds of candles were used at a cost of \$114, how much did they cost per pound?

66. Find the cost of fuel, if $167\frac{1}{2}$ cords of wood were used at \$5 per cord and 5800 pounds of Cumberland coal at \$15 per ton.

67. The other expenses were: labor, \$9,750.75; tools, \$216.46; timbers and ties, \$85.86; steel rails, \$283.86; water, \$414.25; water pipe, \$521.23; incidentals, \$148.21. Find the entire expense of developing the drifts.

68. Find the cost per foot of the 1304 feet of drift.

69. The following men were employed in a gold mine, at the given daily wage for each man: 2 foremen, \$4; 2 brakemen, \$2.50; 36 miners, \$2.75; 2 smiths, \$4; 2 smith's helpers, \$2; 8 laborers, \$2.50. Find their combined wages during a month of 26 working days.

70. Adding \$249.15 for implements and \$260.25 for materials used, what was the total expense for labor, implements, and materials?

71. If 1020 tons of ore were mined during the month, find the expense of mining per ton.

72. If it cost \$2.48 in the mill to extract the gold from each ton of ore, find the cost of milling 1020 tons of ore.

73. What, then, was the expense of mining and milling each ton of ore? 1020 tons?

74. If $\frac{2}{3}$ of an ounce of gold was the average extraction per ton of ore, how many pounds of gold did the 1020 tons yield?

75. Find the value of the gold at \$20.67 per ounce.

76. How much did this exceed the cost of mining and milling?

77. If the cost of power for this mine was \$21 per horsepower by steam, or \$7 per horsepower by electricity, find the saving on 1370 horsepower by using an electric motor instead of a steam engine.

PERCENTAGE

322. The term **per cent** means *per hundred*, or **hundredths**, and the sign for it is %.

6 per cent, 6 %, $\frac{6}{100}$, and .06 represent the same thing, 6 hundredths.

323. The general name given to that part of arithmetic that treats of per cents is **percentage**.

It is therefore merely an application of decimal fractions.

WRITTEN EXERCISES

324. 1. Express decimally; then as a common fraction in its lowest terms: 5 %; 100 %; 125 %.

SOLUTIONS

$$5\% = .05;\ \text{also,}\ 5\% = \tfrac{5}{100} = \tfrac{1}{20}.$$
$$100\% = 1.00;\ \text{also,}\ 100\% = \tfrac{100}{100} = 1.$$
$$125\% = 1.25;\ \text{also,}\ 125\% = \tfrac{125}{100} = \tfrac{5}{4}.$$

2. Express decimally; then as a common fraction in its lowest terms: $33\frac{1}{3}$ %; $\frac{1}{2}$%.

SOLUTIONS

$$33\tfrac{1}{3}\% = .33\tfrac{1}{3};\ \text{also,}\ 33\tfrac{1}{3}\% = \frac{33\frac{1}{3}}{100} = \frac{1}{3}.$$
$$\tfrac{1}{2}\% = .00\tfrac{1}{2};\ \text{also,}\ \tfrac{1}{2}\% = \frac{\frac{1}{2}}{100} = \frac{1}{200}.$$

It is evident that $\frac{1}{2}$% means $\frac{1}{2}$ of 1 %. What does $\frac{1}{4}$% mean? $\frac{3}{8}$%?

Express decimally as hundredths:

3. 6 %	**8.** $8\frac{1}{2}$ %	**13.** 200 %	**18.** $\frac{1}{4}$ %
4. 8 %	**9.** $6\frac{1}{4}$ %	**14.** $12\frac{1}{2}$ %	**19.** $\frac{3}{5}$ %
5. 17 %	**10.** $3\frac{1}{2}$ %	**15.** 175 %	**20.** $\frac{5}{4}$ %
6. 25 %	**11.** $17\frac{3}{8}$ %	**16.** $87\frac{1}{2}$ %	**21.** $\frac{5}{8}$ %
7. 75 %	**12.** $15\frac{1}{4}$ %	**17.** $287\frac{1}{2}$ %	**22.** $\frac{16}{25}$ %

Express as a common fraction in its lowest terms:

23. 20 %	**27.** $12\frac{1}{2}$ %	**31.** $\frac{3}{4}$ %	**35.** 125 %
24. 45 %	**28.** $16\frac{2}{3}$ %	**32.** $\frac{5}{8}$ %	**36.** 165 %
25. 75 %	**29.** $37\frac{1}{2}$ %	**33.** $2\frac{1}{2}$ %	**37.** 295%
26. 56 %	**30.** $33\frac{1}{3}$ %	**34.** $6\frac{1}{4}$ %	**38.** $162\frac{1}{2}$ %

39. To what per cent is $\frac{3}{4}$ equivalent? $\frac{2}{3}$? $1\frac{1}{5}$?

Suggestion. — To how many *hundredths* is $\frac{3}{4}$ equal?

Express first decimally as hundredths; then as per cent:

40. $\frac{1}{5}$	**43.** $1\frac{3}{4}$	**46.** $\frac{3}{8}$	**49.** $\frac{3}{50}$	**52.** $\frac{2}{3}$	**55.** $\frac{6}{5}$
41. $\frac{1}{2}$	**44.** $1\frac{1}{3}$	**47.** $\frac{3}{20}$	**50.** $\frac{7}{20}$	**53.** $\frac{4}{25}$	**56.** $\frac{5}{4}$
42. $\frac{3}{5}$	**45.** $1\frac{2}{5}$	**48.** $\frac{1}{20}$	**51.** $\frac{9}{25}$	**54.** $\frac{7}{25}$	**57.** $\frac{7}{2}$

325. 1. What is $\frac{1}{4}$ of 24? .25 of 24? 25 % of 24?

2. What part of 24 is 6? How many hundredths of 24 is 6? What per cent of 24 is 6?

3. Six is $\frac{1}{4}$ of what number? 6 is .25 of what number? 6 is 25 % of what number?

In 1 you have found *a per cent of a given number;* in 2, *what per cent one number is of another;* in 3, *a number when a per cent of it is given.*

These are the three chief types of problems in percentage.

326. The number of which some per cent is found is called the **base.**

327. The *number* of hundredths found is called the **rate,** or the **rate per cent.**

328. The result obtained by finding a per cent of the base is called the **percentage.**

Thus, in 25 % of 24 = 6, 24 is the *base*, 25 % the *rate*, and 6 the *percentage.*

329. The sum of the base and the percentage is called the **amount,** and the base less the percentage is called the **difference.**

330. Memorize and apply whenever possible:

$10\% = \frac{1}{10}$	$50\ \% = \frac{1}{2}$	$62\frac{1}{2}\% = \frac{5}{8}$	$83\frac{1}{3}\% = \frac{5}{6}$
$20\% = \frac{1}{5}$	$25\ \% = \frac{1}{4}$	$87\frac{1}{2}\% = \frac{7}{8}$	$125\% = \frac{5}{4}$
$40\% = \frac{2}{5}$	$75\ \% = \frac{3}{4}$	$33\frac{1}{3}\% = \frac{1}{3}$	$112\frac{1}{2}\% = \frac{9}{8}$
$60\% = \frac{3}{5}$	$12\frac{1}{2}\% = \frac{1}{8}$	$66\frac{2}{3}\% = \frac{2}{3}$	$133\frac{1}{3}\% = \frac{4}{3}$
$80\% = \frac{4}{5}$	$37\frac{1}{2}\% = \frac{3}{8}$	$16\frac{2}{3}\% = \frac{1}{6}$	$166\frac{2}{3}\% = \frac{5}{3}$

331. **Finding a per cent of a number.**

EXERCISES

1. Find 2 %, or .02, of 400; 3 % of 500; 7 % of 200.

2. How many are 25 %, or $\frac{1}{4}$, of 36? $12\frac{1}{2}$ % of 40? $33\frac{1}{3}$ % of 60?

Find:

3. 6 % of 300	**6.** 20 % of 35 bu.	**9.** $37\frac{1}{2}$ % of \$48
4. 4 % of 600	**7.** 25 % of 60 yd.	**10.** $66\frac{2}{3}$ % of \$39
5. 3 % of 800	**8.** 40 % of 80 lb.	**11.** $87\frac{1}{2}$ % of \$56

12. Of the 20 Hawaiian Islands, 60 % are uninhabitable. How many uninhabitable islands are there in the group?

13. French capitalists have 300 million francs invested in Mexico, of which $33\frac{1}{3}$ % is in mines. How much of their capital is invested in Mexican mines?

14. Of the 200 toy factories in one district of Germany, 75 % make nothing but metal toys. How many make metal toys only?

WRITTEN EXERCISES

332. **1.** Find $45\frac{1}{3}$ % of 185.76; also $37\frac{1}{2}$ % of 110.08

$$\begin{array}{r} 185.76 \\ .45\frac{1}{3} \\ \hline 6192 \\ 9\ 2880 \\ 74\ 304 \\ \hline 84.2112 \end{array}$$

$37\frac{1}{2}$ % of 110.08 =

$$\frac{3}{\cancel{8}} \text{ of } \overset{13.76}{\cancel{110.08}} = 41.28$$

The percentage *equals the base multiplied by the rate.*

Find:

2. 15 % of \$4424
3. 90 % of \$7500
4. 40 % of 850 T.
5. $46\frac{1}{2}$ % of 5280 ft.
6. 125 % of 5424 gal.
7. $\frac{1}{4}$ % of 160 cu. in.
8. $16\frac{2}{3}$ % of 14,046
9. $87\frac{1}{2}$ % of 15,992
10. $183\frac{1}{3}$ % of 58,254
11. $162\frac{1}{2}$ % of 58,464

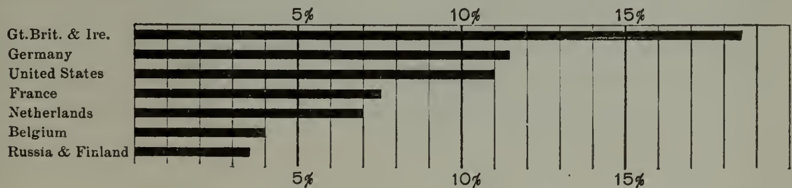
12. One year when 8400 books were published in the United States, 22 % of the books were fiction. How many works of fiction were published in the United States that year?

13. A man willed $37\frac{1}{2}$ % of his fortune of \$240,000 to a hospital. How much did the hospital receive?

14. Red hematite is an ore containing 70 % of iron. How many pounds of iron ore can be obtained from a long ton of red hematite?

15. In Oregon the average amount of standing timber per acre of forest land is 12,200 board feet, of which 66 % is red fir, 18 % pine, 5 % spruce, 5 % hemlock, 2 % cedar, and 4 % other kinds. Find the amount of each per acre.

16. When the commerce of the world for a year amounted to \$21,900,000,000, the shares of the chief nations were approximately as represented by the following graphs. Find the value of each nation's share.



17. If $57\frac{1}{2}$ % of the commerce of the United States was exports, what per cent was imports? What was the value of the imports? of the exports?

333. **Finding what per cent one number is of another.**

EXERCISES

1. What part of 30 is 6? How many hundredths of 30 is 6? What per cent of 30 is 6?

2. What part of .48 is .12? What per cent of .48 is .12?

3. What part of $\frac{1}{2}$ is $\frac{1}{8}$? What per cent of $\frac{1}{2}$ is $\frac{1}{8}$?

What per cent of

4. 20 is 15?	**6.** \$10 is \$50?	**8.** $\frac{3}{4}$ is $\frac{1}{2}$?
5. 44 is 11?	**7.** \$35 is \$14?	**9.** $\frac{2}{3}$ is $\frac{1}{4}$?

10. An elephant at the park eats 100 lb. of hay daily, a hippopotamus, 50 lb. What per cent of the larger amount is the smaller?

11. If Olive and John used 3 qt. of cream and 2 qt. of milk in making ice cream, what per cent of the mixture was cream?

12. Of the 300 coal mines in operation in India recently, 250 were in the province of Bengal. What per cent were in Bengal?

WRITTEN EXERCISES

334. **1.** What per cent of 1566 is 93.96?

$$\begin{array}{r} .06 \\ 1566\overline{)93.96} \\ \underline{93\ 96} \end{array}$$

Since 93.96 is $\frac{93.96}{1566}$ of 1566, and $\frac{93.96}{1566} = .06$, 93.96 is 6% of 1566.

Test. 6% of 1566 = 93.96.

Since the percentage equals the product of the base and the rate (§ 332), *the* rate *equals the percentage divided by the base.*

What per cent of

2. \$60 is \$14?	**6.** \$1600 is \$32?	**10.** 30 is $\frac{1}{5}$?
3. \$75 is \$60?	**7.** 150 yd. is 900 yd.?	**11.** $\frac{1}{2}$ is $\frac{1}{4}$?
4. \$14 is \$16?	**8.** 900 yd. is 150 yd.?	**12.** 1 is .625?
5. \$16 is \$14?	**9.** 8 hr. is 2 hr. 40 min.?	**13.** .75 is $.12\frac{1}{2}$?

14. Of the 6000 men's clothing establishments in this country, 2500 are in New York and 900 in Illinois. What per cent of the whole number is in each of these states?

15. What per cent of the cocoa from 1000 trees, whose average yield is $1\frac{3}{4}$ pounds apiece, is cocoa butter, if the cocoa butter amounts to 910 pounds?

16. Of the 20,000,000 bushels of wheat exported by the United States one year, 9,000,000 bushels went to Great Britain. What per cent of the wheat went to Great Britain?

17. If 150 lb. of "lead," used in a medium grade of lead pencils, consisted of $88\frac{1}{2}$ lb. of graphite and the rest clay, find the per cent of each substance in the mixture.

18. The Kohinoor diamond when first cut weighed 279 carats. It was cut a second time and then weighed 106 carats. Find, to the nearest per cent, its loss in weight due to the second cutting.

19. The New York State Library contains 575,000 volumes and the Library of Congress 1,345,000 volumes. What per cent, to the nearest tenth, of the number of volumes in the Congressional Library is the number in the New York State Library?

20. Find, to the nearest tenth of a per cent, what per cent the population of each continental division is of the population of the world; what per cent the area of each division is of the total land area.

Continental Divisions	Area in Sq. Miles	Population
North America	9,430,000	105,806,000
South America	6,856,000	38,482,000
Europe . . .	3,842,000	396,406,000
Asia	17,056,000	906,674,000
Africa . . .	11,512,000	140,274,000
Australia, etc.	3,456,000	6,458,000
S. Polar Lands	254,000	....
Totals	....	....

21. From the graphs on page 195 find, to the nearest tenth, what per cent the commerce of each nation was of the commerce of Great Britain and Ireland; also what per cent the commerce of each nation was of the commerce of the United States.

335. Finding a number when a per cent of it is given.

1. If $\frac{4}{5}$ of a number is 20, what is $\frac{1}{5}$ of it? what is $\frac{5}{5}$ of it, or the whole number?

2. If $\frac{9}{100}$ of a number is 81, what is $\frac{1}{100}$ of it? $\frac{100}{100}$ of it?

3. If 12% of a number is 48, what is 1% of it? What is the number? How does the result compare with $48 \div .12$?

4. If $37\frac{1}{2}$%, or $\frac{3}{8}$, of a number is 33, what is the number?

5. If $83\frac{1}{3}$%, or $\frac{5}{6}$, of a number is 35, what is the number?

Find the number of which

6. 60 is 2% **8.** 15 is 125% **10.** 160 is $16\frac{2}{3}$%

7. 80 is 4% **9.** 140 is 140% **11.** 333 is $37\frac{1}{2}$%

WRITTEN EXERCISES

336. 1. Find the number of which 939.75 is 35%.

SOLUTIONS

1. Since 35% of the number = 939.75,

$\frac{35}{100}$ of the number = 939.75,

$\frac{1}{100}$ of the number = 939.75 ÷ 35, or 26.85,

and the number = 2685.

2. Or, let x = the number.

Then, $.35x = 939.75$,

and $x = 939.75 \div .35 = 2685$.

Test. 35% of 2685 = 939.75.

2. Find the number of which 50.55 is $62\frac{1}{2}$%.

SOLUTION

$$50.55 \div .62\tfrac{1}{2} = 50.55 \div \tfrac{5}{8} = \overset{10.11}{\cancel{50.55}} \times \frac{8}{\cancel{5}} = 80.88.$$

Test. $62\frac{1}{2}$%, or $\frac{5}{8}$, of 80.88 = 50.55.

Since the percentage equals the product of the base and the rate, *the* **base** *equals the percentage divided by the rate.*

Find the number of which

3. \$2184 is 91%	**6.** \$63.90 is 15%	**9.** 1365 is $87\frac{1}{2}$%
4. \$864 is $2\frac{1}{4}$%	**7.** \$2.25 is 5%	**10.** 5200 is $6\frac{1}{4}$%
5. 5500 is $62\frac{1}{2}$%	**8.** \$5.73 is $33\frac{1}{3}$%	**11.** 4525 is $62\frac{1}{2}$%

12. What is the combined area of the parks in Denver, if the largest contains 319 acres, or 58% of the whole area?

13. How much petroleum did California produce one year, if Kern County alone furnished 72% of it, or 18,000,000 barrels?

14. San Bernardino County, Cal., furnished 37,800 tons of borax, or 84% of one year's production of the whole country. How much borax did the United States produce that year?

15. If 13,580, or $48\frac{1}{2}$%, of the passenger coaches on French railroads recently were third class, how many passenger coaches were there in all?

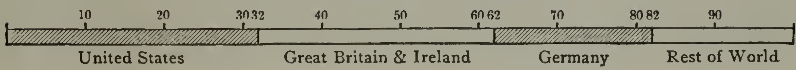
16. One year 151,590 skilled laborers arrived in the United States This was $18\frac{3}{5}$% of the total number of immigrants. How many immigrants arrived that year?

17. Lake Erie has an area of 9960 square miles. If it were larger by 24 square miles, its area would be 32% of that of Lake Superior. How large is Lake Superior?

18. One year the expenses of a Massachusetts electric light company were \$1,875,000. The cost of coal for the year was 25% of the expenses and 15% of the income. Find the income.

19. In a year when the United States produced 352,000,000 short tons of coal, find from the following graph the production of the world; of Great Britain and Ireland; of Germany.

Coal Production — Per cents



337. Sum or difference of a number and some per cent of it.

1. How much is 25 increased by $\frac{1}{5}$ of 25? 25 decreased by $\frac{1}{5}$ of 25? $25 + \frac{1}{5}$ of $25 = ?$ $\frac{6}{5}$ of $25 = ?$ $25 - \frac{1}{5}$ of $25 = ?$ $\frac{4}{5}$ of $25 = ?$

2. How much is 25 increased by 20% of itself? 25 decreased by 20% of itself? 100% + 20%, or 120%, of 25 = ? 100% − 20%, or 80%, of 25 = ?

3. If the number 20 is increased to 25, how much is it increased? By what per cent is it increased?

4. If a number is decreased from 30 to 15, what is the per cent of decrease?

WRITTEN EXERCISES

338. 1. How much is 4800 increased by $16\frac{2}{3}$% of itself?

2. How much is 1800 decreased by $37\frac{1}{2}$% of itself?

3. Under the sweat-shop system, a coat that formerly cost $2.50 for making, costs 40% less. Find the decreased cost.

4. How many farms in California were being irrigated at the beginning of this century, if there were then $87\frac{1}{2}$% more than there were ten years earlier, when the number was 13,600?

5. During five years the number of horses in this country increased from 13,500,000 to 17,010,000. What was the per cent of increase?

6. The tonnage of Russia's navy was 450,000 tons before the war with Japan and 225,000 tons after the war. Find the per cent of decrease.

7. One year the United States received 20,904 tons of currants from Greece. If $12\frac{1}{2}$% of the weight was *tare*, boxes and coverings, what was the actual weight of the currants?

8. The bark from a cork tree, 48 pounds in all, was reduced $33\frac{1}{3}$% in weight by boiling. It was then scraped, its weight being thus reduced 25%. Find its final weight.

9. On account of competition between steamship companies at one time, the steerage rate of \$40 from Germany to America was cut down to \$16.60. Find the per cent of decrease.

10. The average value of watch cases made one year in the United States was \$4.17. In Illinois the value was 55 % less and in New York 111 % more. Find, to the nearest cent, the average value in each of these states.

339. Finding a number when the number increased or decreased by some per cent of itself is given.

1. If $1\frac{1}{4}$ times a number is 15, what is the number? If a number increased by $\frac{1}{4}$ of itself is 15, what is the number?

2. If 125 % of a number is 15, what is the number? If a number increased by 25 % of itself is 15, what is the number?

3. If $\frac{3}{4}$ of a number is 24, what is the number? If a number decreased by $\frac{1}{4}$ of itself is 24, what is the number?

4. If 75 % of a number is 90, what is the number? A number decreased by 25 % of itself equals 90. Find the number.

What number increased by

5. 35 % of itself = 13,500?

6. 14 % of itself = 22,800?

What number decreased by

7. 12 % of itself = 8800?

8. $12\frac{1}{2}$ % of itself = 7000?

WRITTEN EXERCISES

340. 1. What number increased by 55% of itself equals 310?

Solutions

1. Since 310 is equal to a certain number increased by 55 % of itself, 310 is 155 % of the number, or 1.55 times the number.

Hence the number is 310 ÷ 1.55, or 200.

2. Let $x =$ the number.

Then, $x + 55\% \text{ of } x = x + .55x = 310;$

that is, $1.55x = 310.$

Hence, $x = 310 \div 1.55 = 200$, the number.

Test. 200 + 55 % of 200 = 200 + 110 = 310.

2. What number decreased by $37\frac{1}{2}\%$ of itself equals 660?

SOLUTIONS

$$\begin{array}{r} 5)\underline{660} \\ 132 \\ \underline{8} \\ 1056 \end{array}$$

1. Since 660 is equal to a certain number decreased by $37\frac{1}{2}\%$ of itself, or by $\frac{3}{8}$ of itself, 660 is $\frac{5}{8}$ of the number.

Since 660 is $\frac{5}{8}$ of the number, $\frac{1}{8}$ of the number is $\frac{1}{5}$ of 660, or 132, and the number is 8 times 132, or 1056, found as in the margin, or by cancellation.

2. Or, let $x =$ the number.

Then, $x - 37\frac{1}{2}\%$ of $x = x - \frac{3}{8}x = 660$;

that is, $\frac{5}{8}x = 660.$

Dividing both members by 5, $\frac{1}{8}x = 132.$

Multiplying both members by 8, $x = 1056$, the number.

Test. $1056 - 37\frac{1}{2}\%$ of $1056 = 1056 - 396 = 660.$

What number increased by

3. 15% of itself $= 138$?

4. 25% of itself $= 365$?

What number decreased by

5. $12\frac{1}{2}\%$ of itself $= 2345$?

6. $35\ \%$ of itself $= 1612$?

7. An orange tree yielded 50% more oranges the second time it bore than the first time. If the second crop amounted to 420 oranges, how many were there in the first crop?

8. A barge load of grain damaged by water was sold for \$1408, which was 78% less than its original value. What was its original value?

9. The rice crop of Japan one year was 204.6 million bushels, or 7% less than normal. What is the normal crop?

10. The Hudson River ice crop in a poor season was 1,440,000 tons, or 68% less than the crop of the previous season. How many tons of ice were harvested the previous season?

11. In April of one year 121,000 cubic yards of dirt were cleaned from the streets of Chicago. This was 120% more than the amount removed in June. How much dirt was removed in June?

MISCELLANEOUS EXERCISES

341. Find the value of x in each of the following exercises:

Base	Rate	Percentage	Base	Rate	Amount
1. \$375	16%	\$$x$	**9.** 7910	13%	x
2. \$$x$	$12\frac{1}{2}$%	\$2.25	**10.** \$$x$	113%	\$10,650
3. .75	x%	.125	**11.** x mi.	$133\frac{1}{3}$%	3640 mi.
4. x	250%	$\frac{3}{4}$	Base	Rate	Difference
5. 2364	$\frac{1}{4}$%	x	**12.** 1281 lb.	4%	x lb.
6. $\frac{5}{16}$	x%	$\frac{1}{16}$	**13.** \$$x$	24%	\$1247.92
7. x	47%	$23\frac{1}{2}$	**14.** 6036	$83\frac{1}{3}$%	x
8. 6 gal.	x%	4 qt.	**15.** \$$x$	$66\frac{2}{3}$%	\$4800.40

16. An automobile truck with its load weighed 18,700 pounds. If the load was 64% of the total weight, how many pounds did the load weigh?

17. How many miles of telegraph and telephone wires were in operation in a year when 35% of the amount, or 1,715,000 miles, was underground?

18. In the manufacture of 120,000 postal cards, 414 pounds of the material used was spruce pulp and 132 pounds was poplar pulp. If 1000 postal cards weigh 5 pounds, what per cent of the material was spruce? what per cent poplar?

19. The importation of cheese into Great Britain in 1880 was 125,000,000 lb. from the United States and 30,000,000 lb. from Canada. Twenty-five years later the amount from the United States was 80% less, and that from Canada 600% more. How much did Great Britain then import from each country?

20. The production of steel in Germany during a recent year was 920,000 tons, or 68% less than the amount produced in this country, and $2\frac{2}{9}$% more than the amount produced in England. What was the output of steel in each country?

PROFIT AND LOSS

342. 1. A house that cost \$3200 was sold at a gain of 25%. What was the gain? the selling price?

2. A farm that cost \$3200 was sold at a loss of 25%. What was the loss? the selling price?

3. If a farm costing \$3200 was sold at a loss of \$800, what was the per cent of loss?

4. If a man gains or loses \$30 on some goods and his gain or loss is $33\frac{1}{3}$% of the cost, what is the cost?

5. The selling price of some linen is 24¢ a yard. What was the cost, if there is 20% gain? if there is 20% loss?

343. The per cent of gain or of loss is reckoned on the *cost* or on the *sum invested.*

344. In marking goods to show the *cost* and the *selling price*, merchants often use as a private **key** some word or words with ten different letters to represent the ten Arabic numerals; they write the cost above a horizontal line and the selling price below it. To avoid giving a clew by using the same letter twice in succession an extra letter is used as a **repeater**.

Thus, if the key is

p	*l*	*a*	*n*	*e*	*f*	*r*	*o*	*g*	*s*
1	2	3	4	5	6	7	8	9	0

and the repeater *x*, an article that cost \$3.55 and is sold for \$4.20 may be marked $\frac{aex}{nls}$.

345. Some system of arbitrary symbols is often used in marking goods. For example, taking such a diagram as is shown here to aid the memory, the nine digits may be represented by the parts of the lines that inclose them, thus:

1	2	3
4	5	6
7	8	9

┘	⊔	└	⊐	□	⊏	┐	⊓	┌
1	2	3	4	5	6	7	8	9

In this system △ may be used for 0 and V for a repeater.

WRITTEN EXERCISES

346. Interpret the following marks according to the key given in § 344 and find the per cent of gain or of loss:

1. $\frac{les}{asx}$ **3.** $\frac{af}{nl}$ **5.** $\frac{pxs}{gx}$ **7.** $\frac{le}{as}$ **9.** $\frac{pfle}{pges}$

2. $\frac{anx}{aor}$ **4.** $\frac{re}{fs}$ **6.** $\frac{pls}{pxs}$ **8.** $\frac{no}{nl}$ **10.** $\frac{lnfs}{asre}$

For exercises 11–26 use the key given in § 345. Mark goods costing as follows to be sold at 25 % above cost:

11. \$4.00 **12.** \$6.48 **13.** \$24.56 **14.** \$15.20 **15.** \$.96

Mark goods costing as follows to be sold at $12\frac{1}{2}$ % below cost:

16. \$1.20 **17.** \$3.60 **18.** \$48.72 **19.** \$10.00 **20.** \$.88

Give marks according to indicated cost and gain per cent:

21. Sheeting, 5¢, 20 %

22. Jeans, 12¢, $16\frac{2}{3}$ %

23. Blueprints, 4¢, 25 %

24. Silk, \$1.28, $12\frac{1}{2}$ %

25. Velvet, \$2.50, 50 %

26. Broadcloth, \$2.75, 40 %

27. What per cent is gained by selling tea at 55 cents per pound, if it cost 40 cents?

28. An ice dealer who bought 300 tons of ice for \$750, sold it at the rate of 25¢ per 100 pounds. Find his per cent of gain.

29. The gain on a sale of machinery was \$157.53, which was 30 % of the cost. Find the cost.

30. A printer sold 3000 seed catalogues to a firm @ $3\frac{1}{4}$¢, thereby gaining 25 %. How much did all cost him?

31. A fruit dealer paid \$30 for 2 cases of washed figs, each case holding 100 one-pound baskets. At what price per basket did he sell the figs, if he gained $33\frac{1}{3}$ %?

32. What per cent is gained by buying coal by the long ton at \$4.20 per ton, freight paid, and selling it by the short ton at \$5.40, paying 40 cents per short ton for delivery?

33. A dealer bought goods at 20% less than their market value, and sold them at 20% above the market value. What per cent did he gain?

SUGGESTION. 80% of market value represents the cost, and 40% of market value the gain. What part of 80% is 40%? what per cent?

34. What per cent is gained by buying clothing at 10% below market value and selling it at $12\frac{1}{2}$% above?

35. At what price must goods that cost $1.80 per yard be marked so that the seller may deduct 10% from the marked price and still gain 20%?

SUGGESTION. $\frac{6}{5}$ of \$1.80 = 90% of marked price.

36. A dry goods merchant sold tablecloths at 20% less than the marked price, yet gained $16\frac{2}{3}$%. At what price were they marked, if they cost him $3.60 apiece?

37. A manufacturer sold 50 pieces of velveteen at auction at $48 per piece. On one half of them he gained 20% and on the remainder he lost 20%. Did he gain or lose on all and how much?

COMMISSION AND BROKERAGE

347. A person who buys or sells goods or transacts business for another is called an **agent**, a **commission merchant**, or a **broker.**

In general a commission merchant actually receives the goods, while a broker simply arranges for their sale or purchase, the goods being shipped directly from the seller to the purchaser.

348. The compensation of an agent is often reckoned as some per cent of the *value* involved, and is called **commission**, or **brokerage.**

Thus a seller's commission is some per cent of the amount of sales; a buyer's commission is some per cent of the cost; a collector's commission is some per cent of the money collected.

349. The sum left after the commission and other expenses have been paid is called the **net proceeds**.

WRITTEN EXERCISES

350. **1.** A commission merchant sold 1450 barrels of beef at \$15 per barrel, commission 2%. He paid \$130 for freight, \$15.75 for cartage, and \$15 for storage. Find his commission and the net proceeds.

SOLUTION

Amount of sales = 1450 × \$15 =		\$21,750.00
Commission = 2% of \$21,750 =	\$435.00	
Freight and cartage	145.75	
Storage	15.00	595.75
Net proceeds		\$21,154.25

Find the commission and the net proceeds:

	Article	Quantity	Price	Freight, etc.	Rate of Commission
2.	Apples	300 bbl.	\$4.00	\$90.50	5%
3.	Wheat	1000 bu.	$73\frac{1}{2}$¢	25.00	$\frac{1}{8}$¢ per bu.
4.	Beef	100 bbl.	18.50	56.00	3%
5.	Eggs	1500 doz.	20¢	5.00	5%
6.	Books	225	1.75	10.50	40%
7.	Cotton	4750 lb.	$10\frac{3}{4}$¢	45.75	$2\frac{1}{2}$%
8.	Silk	1000 yd.	1.75	25.00	7%

9. A collector succeeded in collecting 75% of a debt of \$4000 and charged 15% commission. How much did the creditor receive?

10. How much brokerage did a broker receive on 20,000 bushels of wheat, if he charged $\frac{1}{8}$¢ per bushel?

11. How much must I pay a note broker to sell my note for \$5000, if he charges me $\frac{1}{4}$% commission?

12. How many 5-dollar books must a book agent sell to earn \$240, if his commission is 40%?

13. If a real estate agent charged \$$2\frac{1}{2}$ a month for renting a house for \$$62\frac{1}{2}$ a month, what was his rate of commission?

14. Find the net proceeds from the sale of 1840 pounds of dressed poultry at 15 ¢ per pound, commission 5 %.

15. How much did a broker receive for purchasing 91,000 pounds of coffee for an importer @ 9 ¢, brokerage $\frac{1}{2}$ % ?

16. My agent sold a city house for me for \$5500, commission 2 %; also some country property for \$2800, commission 3 %. Find my net proceeds from the two sales.

17. One month an agent secured 346 subscriptions to a dollar magazine, earning \$86.50. Find his rate of commission.

18. If a salesman received \$3750 one year as commission on sales amounting to \$50,000, what was his rate of commission?

19. A commercial traveler receives a salary of \$25 a week and 2% commission on his sales. If his sales amount to \$75,000 in a year, what is his income for the year?

20. A salesman for an importing house is offered his choice of a salary of \$2500 a year, or \$1500 a year with a commission of 2% on his sales, or 6 % commission on all sales. He accepts the second offer and sells goods to the value of \$60,000. How much better off is he than by the first offer? how much worse off than by the third?

21. Maxwell & Thurston, commission merchants, buy for Stafford & Sons, grocers, 625 pounds of butter @ 19¢, 345 pounds of butter @ $17\frac{1}{2}$ ¢, 3000 eggs at \$1.50 per hundred, $126\frac{1}{2}$ pounds of cheese @ $11\frac{1}{2}$ ¢, and 20 barrels of sweet potatoes @ \$1.75; commission 2 %, cartage \$2.50. What is the indebtedness of the grocers to the commission merchants?

22. A grain dealer in Mobile bought 8000 bushels of shelled corn through a commission merchant in St. Louis @ 22 ¢, commission 2 %. The freight cost \$806.40. Unloading cost \$30.40, cartage \$80, and elevator charges were $\frac{1}{2}$ ¢ per bushel. Find the total cost and the cost per bushel in the elevator at Mobile.

COMMERCIAL DISCOUNT

351. A deduction from the price of anything is called a **commercial discount.**

Manufacturers and wholesale dealers issue catalogues and price lists with **list prices** quoted somewhat above the highest market prices of their goods, and sell to their customers at varying discounts according to the fluctuations of the market. Sometimes when the goods sold are to be paid for at some future time, a discount is allowed for payment before that time.

352. Several successive discounts are often allowed. The first is a discount from the list price, the second from the remainder, the third from the second remainder, and so on.

353. The price remaining after deducting all discounts is called the **net price.**

WRITTEN EXERCISES

354. 1. Find the net price of an automobile listed at \$2400, if the discounts are 25%, 20%, and 10%.

List price	\$2400
Less 25%	600
Remainder	\$1800
Less 20%	360
Remainder	\$1440
Less 10%	144
Remainder	\$1296

The first discount is 25% of \$2400, or \$600. Subtracting, we find the remainder, \$1800. The second discount is 20% of \$1800, or \$360, and the remainder \$1440. The third discount is 10% of \$1440, or \$144, and the remainder, \$1296.

Or, 90% of 80% of 75% of \$2400 = 54% of \$2400 = \$1296.

The first discount reduces the price to 100% – 25%, or 75%, of the list price, the second discount reduces it to 80% of this, or to 80% of 75% of the list price, and the third discount reduces it to 90% of this, or to 90% of 80% of 75% of the list price. This equals 54% of the list price, and 54% of \$2400 = \$1296.

2. In exercise 1, find the discounts in the order 10 %, 20 %, 25 %; then in the order 20 %, 25 %, 10 %; and compare results.

On the following bills find net amounts and total discounts:

3. $500, discounts 20 % and 10 %.

4. $750, discounts 30 % and 20 %.

5. $840, discounts 20 %, 10 %, and 5 %.

6. $932, discounts 25 %, 16 %, and 5 %.

7. $372.50, discounts 40 %, 10 %, and 5 %.

8. Find the net cost, to a dealer, of 280 pounds of Manila rope listed at $12\frac{1}{2}$¢ per pound, discounts 20% and 5%.

9. A plumber's bill for supplies amounted to $375, discount 40 %, a second discount of 4 % being offered for cash. He paid cash. What was the net amount of the bill?

10. How much less does a dealer pay for 360 yards of Brussels carpet listed at $1.25 per yard, if he makes his payment in 10 days with 4 % off instead of in 30 days with 3 % off?

11. What is the difference on a bill of $440 between a direct discount of 35 % and successive discounts of 20 % and 15 %?

12. Which gives the lower price for a piano listed at $600, a direct discount of 45 % or successive discounts of 25 %, 15 %, and 5 %? how much lower?

13. A manufacturer marked his goods down $12\frac{1}{2}$ % from the list price on account of a fall in their market value, and allowed an additional discount of 4 % for cash. What was the entire discount, expressed in per cent?

14. A retail druggist gets a discount of 40 % from the wholesale list price and sells at the list price. What per cent of profit does he make?

15. What single discount is equivalent to two successive discounts of 15 % and 10 %?

GOVERNMENT REVENUE

355. The expenses of government are paid, for the most part, by **taxes**, which are sums of money levied on persons, on property, or on business; and by **duties** or **customs**, which are charges on goods imported from other countries.

The expenses of our national government consist of outlays for the army and the navy, for pensions, for the salaries of officers, etc. The state expends money for salaries of state officers, for educational purposes, and for the care of criminals and defective persons; while local governments need money for maintaining their various departments, such as Fire Department, Police Department, Department of Education, etc.

Taxes

356. Fixed property, such as land and structures built upon it, is called **real estate.**

357. Movable property, such as money, bonds, mortgages, cattle, lumber, etc., is called **personal property**.

358. State and local taxes are usually a certain *per cent* of the *estimated value* of real estate and of personal property.

In addition to the property tax, some states collect a *poll tax* (head tax), varying from $1 to $3, from each male citizen over 21 years of age.

359. The officers who estimate, or *assess*, the value of the property subject to taxation are called **assessors.**

Property is not generally assessed at its full value.

360. The officer who collects the taxes, when there is a special officer for the purpose, is called a **collector**; he receives either a salary or a per cent of the tax collected.

The collector's fee is sometimes paid by the person who pays the tax. In such cases it is not included in the amount of tax to be raised.

361. The rate of taxation is usually expressed as a *number of mills* on each *dollar*, or a *number of cents* on each *hundred dollars* of assessed valuation. A tax rate of 1 mill means a tax of 1 mill on each dollar, and is therefore a tax of $\frac{1}{10}\%$.

EXERCISES

362. **1.** What is my tax on property assessed at $2000, if the tax rate is 10 mills on a dollar?

Find the tax on the following property:

2. $1500 @ 3 mills	**5.** $3600 @ $1\frac{1}{4}$ mills
3. $6000 @ 4 mills	**6.** $10,000 @ $5\frac{1}{2}$ mills
4. $7500 @ 2 mills	**7.** $20,000 @ $4\frac{3}{4}$ mills

Find the tax rate expressed as mills on a dollar:

	Valuation	Tax		Valuation	Tax		Valuation	Tax
8.	$1500	$4.50	**11.**	$10,000	$40	**14.**	$100,000	$100
9.	$2500	$5.00	**12.**	$15,000	$45	**15.**	$250,000	$625
10.	$4000	$6.00	**13.**	$75,000	$300	**16.**	$300,000	$900

Find the assessed valuation:

	Tax	Rate		Tax	Rate		Tax	Rate
17.	$10	1 mill	**19.**	$75	5 mills	**21.**	$5000	5 mills
18.	$40	2 mills	**20.**	$600	3 mills	**22.**	$8000	4 mills

WRITTEN EXERCISES

363. **1.** A union school district levied a tax of $3420 on an assessed valuation of $456,000. Find the tax rate; also the tax paid by a citizen whose real property was assessed at $9000 and personal property at $3000.

SOLUTION

$\$3420 \div \$456{,}000 = .0075$, or $7\frac{1}{2}$ mills on a dollar.
Tax on $\$9000 + \$3000$, or $\$12{,}000 = .0075$ of $\$12{,}000 = \$90$.

2. If the assessed valuation of a town is \$12,346,000, what will be the rate of tax to raise \$82,965.12?

3. How much must a citizen of this town pay, including the collector's fee of 1%, if his property is assessed at \$12,500?

4. The assessed valuation of a city is \$15,900,000, and the estimated cost of maintaining the schools is \$27,825. Find the rate of school tax in mills.

5. A town wishes to raise by taxation \$206,200. It is estimated to contain 9115 persons subject to a poll tax of \$1 each. Its assessed valuation is \$23,462,500. What will be the rate of property tax?

SUGGESTION. — First deduct the poll tax from the amount to be raised.

6. How much must a citizen of this town pay, if he is subject to a poll tax, and has real property assessed at \$40,000 and personal property at \$25,000, the collector's fee being $\frac{1}{2}$%?

7. How much county tax must be raised to pay for a courthouse that cost \$53,595, if the tax includes $\frac{3}{4}$% for collection?

SUGGESTION. \$53,595 = $99\frac{1}{4}$% of the tax to be raised.

8. The taxable property in a certain city consists of \$93,100,000 real estate and \$36,400,000 personal property. The tax rate is 18.6 mills per dollar on an assessed valuation of 80% of the actual value. Find the amount of tax raised.

9. Find, to the nearest cent, the tax for each borough of Greater New York in a year when the valuation and tax rates were:

BOROUGH	TAX RATE PER \$100	REALTY	PERSONALTY
Manhattan	\$1.413	\$3,483,793,382	\$549,843,253
Bronx	1.413	247,090,767	14,762,041
Brooklyn	1.489	853,742,357	100,052,348
Queens	1.475	123,781,723	10,176,900
Richmond	1.496	43,124,597	6,031,550

10. One year the tax rate of a city was \$2.14 per \$100; the next year an economical administration lowered it to \$1.90. Find the reduction in taxes on property assessed at \$64,500.

11. Find the total state tax raised in the State of New York in a year when the assessed valuation of the property in the state was \$7,738,165,640, the tax rate being $\frac{154}{1000}$ of a mill per dollar.

To facilitate calculation, **tax tables** are often used.

TAX TABLE. RATE, 1.74 MILLS ON \$1

PROP.	TAX	PROP.	TAX	PROP.	TAX	PROP.	TAX
\$1	\$0.002	\$10	\$0.017	\$100	\$0.174	\$1000	\$ 1.74
2	.003	20	.035	200	.348	2000	3.48
3	.005	30	.052	300	.522	3000	5.22
4	.007	40	.070	400	.696	4000	6.96
5	.009	50	.087	500	.870	5000	8.70
6	.010	60	.104	600	1.044	6000	10.44
7	.012	70	.122	700	1.218	7000	12.18
8	.014	80	.139	800	1.392	8000	13.92
9	.016	90	.157	900	1.566	9000	15.66

12. Find by the table Mr. Butler's tax on property assessed at \$9594.

SOLUTION

Tax on \$9000 = \$15.66
Tax on 500 = .87
Tax on 90 = .157
Tax on 4 = .007
Tax on \$9594 = \$16.694 = \$16.70

13. Mr. Hall's property is valued at \$2504, and his poll tax is \$1. Find his tax by the table.

Find by the table the tax on property valued at:

14. \$3556 **16.** \$70,345 **18.** \$643,200 **20.** \$750,000

15. \$4012 **17.** \$89,614 **19.** \$521,000 **21.** \$641,000

Duties or Customs

364. The income for the support of our national government is derived from **duties** or **customs** and from **internal revenue,** which consists chiefly of taxes on spirits, tobacco, etc.

365. The duties are collected at **customhouses,** which are maintained at *ports of entry.*

366. Imported goods may be divided into four classes: first, goods admitted free of duty; second, goods subject to an **ad valorem duty,** that is, a certain per cent of the cost of the goods; third, goods subject to a **specific duty,** a certain amount per yard, pound, etc., without regard to value; fourth, goods subject to both specific and ad valorem duties.

For example, coffee is admitted free; laces pay an ad valorem duty of 60%; the specific duty on wheat is 25¢ per bushel; and the duty on linen collars is 20% ad valorem and 40¢ per dozen.

Before computing specific duties allowance is often made for *tare,* or the weight of the box, bag, etc.

367. A schedule of duties on merchandise is called a **tariff.**

WRITTEN EXERCISES

368. Find the duty on:

	Imports	Value and Quantity	Tariff
1.	Blankets	$150, 450 lb.	30% + 22¢ per lb.
2.	Clocks	$4125	40%
3.	Eggs	3000 doz.	5¢ per doz.
4.	Quicksilver	100 flasks (76.5 lb. each)	7¢ per lb.
5.	Skins, tanned	$6430	20%
6.	Table knives	$600, 100 doz.	15% + 16¢ each
7.	Umbrellas	$7500	50%
8.	Wheat	10,000 bu.	25¢ per bu.

9. The specific duty on cheese is 6¢ per pound. Find the duty on 750 pounds of cheese.

10. The ad valorem duty on jewelry is 60%. Find the duty on a bracelet appraised at $550.

11. Find the duty on 400 boxes of raisins valued at $1.50 a box, each box containing 36 pounds, if the rate is 1¢ a pound and 35% ad valorem.

12. Find the duty on 25,000 cigars, invoiced at $62.50 per M, weighing on an average $10\frac{1}{2}$ ounces per box of 50 cigars. The tariff rate is $4.50 per pound and 25% ad valorem.

13. A merchant imported 80 dozen razors valued at $6 a dozen. He paid a duty of $1.75 a dozen and 20% ad valorem. Transportation cost $4.00. How much did each razor cost him?

14. Find the duty on 600 dozen linen handkerchiefs costing 7*s.* 6*d.* per dozen at 50% ad valorem.

15. Compute the duty; also the dutiable value:

MANIFEST NO. *751.* INVOICED AT *Bristol, Eng. June 21, 1906.*

INWARD FOREIGN ENTRY OF MERCHANDISE

IMPORTED BY *Bates & Holman* IN THE STEAMER *Holloway*

Henry Maxwell, Master. From *Liverpool, Eng.*

Arrived *July 5, '06.*

MARKS	NUMBERS	PACKAGES AND CONTENTS	QUANTITY	FREE LIST	VALUE	AD VALOREM DUTY		SPECIFIC DUTY		TOTAL DUTY
A.A.	*1624*	*8 cases woolen yarn*	*2400 lb.*		*£252.10.0*	*40%*	£ . . .	*38½¢ lb.*	$ . .	
⊕	*833*	*12 cases cotton hose*	*384 doz.*		*£149.12.0*	*15%*	£ . . .	*70¢ doz.*	$ . .	
									$ . .	
					£		£ . . .			
					$		$ . . .			$ . . .

16. A shipment of skein silk, costing 2754 francs in Lyons and weighing 324 pounds net, was taxed 40 cents per pound and 15 % ad valorem. How much duty was paid on it?

17. A man imported, from Paris, paintings valued at 125,400 francs upon which there was an ad valorem duty of 20 %. Freight and insurance charges were \$84.50. How much did his paintings cost him?

18. Find the entire cost in this country of 2000 pounds of macaroni bought in Italy for 500 lire, if the duty was $1\frac{1}{2}$¢ per pound and the freight charges were \$10.60.

19. A merchant imported cut and stained glass invoiced at 12,440 crowns (20 crowns = \$4.052). If the tariff rate was 60 % ad valorem, how much duty did he pay?

20. Find the duty on 3250 yards of English wool suiting costing 7*s.* $2\frac{1}{4}$*d.* per yard and weighing 1 pound per yard, at \$.44 per pound and 55 % ad valorem.

21. If it takes $3\frac{1}{4}$ yards of this material to make a suit of clothes, how much does the tariff add to the cost of a suit?

INSURANCE

369. Indemnity against loss or damage is called **insurance**.

Indemnity against loss of property by fire is called **fire insurance**; against loss of property at sea, **marine insurance**; against loss by personal injuries, **accident insurance**; against loss by sickness, **health insurance**; against loss by death, **life insurance**; against loss by injuries to employees, **liability insurance**. There are many other forms of insurance.

370. The contract or written agreement between the person protected and the insurance company is called the **policy**, and the amount of indemnity, or "protection," the **face of the policy**.

371. The price paid for insurance is called the **premium**.

Insurance rates are expressed in per cent, or in cents per \$100, or in dollars and cents per \$1000.

Property Insurance

372. The principal kinds of **property insurance** are fire insurance, marine insurance, and such special forms as plate glass insurance, tornado insurance, burglar insurance, credit insurance, etc.

WRITTEN EXERCISES

373. 1. A frame house with a shingle roof was insured for \$3500 at 70¢ per \$100 for 1 year. Find the premium.

2. If the roof had been tin, the rate would have been 60¢ per \$100. Find the difference in premiums for 10 years.

3. Find the annual premium on a brick dwelling insured for \$5000 at 25¢ per \$100.

4. If the rate for 3 years is only twice the annual rate, how much will be saved in 12 years by insuring a house for \$5000 in 3-year periods, the annual rate being 25¢ per \$100?

5. If the premium paid for a policy is \$9.60 and the rate is 60¢ per \$100, what is the face of the policy?

6. If the premium is \$19.50 and the rate is $\frac{1}{2}$%, what is the face of the policy?

7. If the premium on a policy for \$3800 is \$15.20, what is the rate in cents per \$100? in per cent?

8. A house worth \$6000, insured for \$4500, was damaged to the extent of \$5000. Find the owner's loss; also the company's loss.

9. A store insured for \$5000 in the Phœnix, \$3000 in the Firemen's, and \$2000 in the Protective, was damaged by fire to the extent of \$4000. Find the loss of each company.

10. A merchant insured his stock of silk for \$15,000 for 15 days, and paid 14% of the annual rate, which was 2%. Find the amount of premium that he paid.

11. A canal boat contains 8000 bushels of corn worth 42¢ a bushel. If the corn is insured for $\frac{7}{8}$ of its value at 85¢ per \$100, what will be the owner's net loss in case the cargo is destroyed?

12. A jeweler paid \$3.50 a year to insure each of his two plate glass windows, and he also insured his stock for \$25,000 against burglary, at \$15 per \$1000. Find his yearly premium for both.

Personal Insurance

374. The principal kinds of insurance of persons, or **personal insurance**, are life insurance, health insurance, accident insurance, and liability insurance. Life insurance is the most important kind.

375. The most important kinds of life insurance policies are:

1. **Ordinary Life Policies.** — The holder of an ordinary life policy pays a certain premium at the beginning of each year from the time he secures the policy until his death; at his death the company pays the face of the policy to his estate or to the persons named in the policy as his beneficiaries.

2. **Limited-payment Life Policies.** — These are paid for in a limited number of years, after which they are said to be *paid up*. Thus, a "20-payment life" policy is fully paid up in 20 years.

3. **Endowment Policies.** — These run for a specified number of years. The face of the policy is paid to the insured if he lives to the end of the term, or to his estate or beneficiaries if he dies before that time. Premiums are usually payable annually, for the whole term.

4. **Term Policies.** — These give only temporary insurance. No part of the face of the policy is paid unless the insured dies during the term.

376. The average number of years a healthy person of a given age has yet to live is called his **expectation of life**. As shown in the table on page 220, a person's expectation of life decreases as his age increases.

377. Life insurance premiums are given in dollars and cents per \$1000. The rate depends upon the kind of policy and upon the age of the insured or his expectation of life.

Brief Table of Annual Premiums for Insurance of $1000 with Participation in Profits Annually

Age	Ordinary Life	20-Payment Life	10-Payment Life	20-Year Endowment	25-Year Endowment	30-Year Endowment	20-Year Term	Expectation of Life
Years								Years
20	$18.50	$26.90	$42.70	$48.10	$37.60	$31.00	$13.00	43.07
25	20.70	29.40	46.40	48.70	38.30	31.70	14.00	39.49
30	23.50	32.30	50.90	49.60	39.30	32.90	15.40	35.85
35	27.30	36.00	56.30	50.80	40.80	34.70	17.70	32.17
40	32.20	40.60	62.80	52.80	43.20	37.60	21.50	28.48
45	38.80	46.60	70.80	56.00	47.10	42.30	27.70	24.82
50	47.90	54.70	80.80	61.30	53.40	49.70	37.70	21.24
55	60.40	65.70	93.20	69.80				17.80
60	77.70	81.20	108.90					14.56

The **age** of the insured is his age at his nearest birthday.

Premiums, if annual, are paid at the *beginning* of each year, and **dividends** (profits) are returned at the *end* of each year or credited as a part of the next premium, or applied to purchase additional insurance.

WRITTEN EXERCISES

378. Use the table in solving exercises 1–7 inclusive.

1. How much will it cost annually to carry an ordinary life policy for $2000 (age 25)?

2. How much will it cost annually to carry a life policy for $5000 (age 25), if it is agreed that the insurance is to be fully paid for by the first 20 annual premiums?

3. How much will it cost annually to carry a 20-year endowment policy for $2500 (age 20)?

4. Suppose that a man 30 years old takes out an ordinary life policy for $1000 and dies at the end of his expectation of life. How many premiums will he have paid? If the dividends (including a post mortem dividend for .85 of a year) amount to $207.36, what will be the net cost of his insurance?

5. A man 25 years old takes out a 20-payment life policy for \$3000, and lives the full term. Find the total premium.

6. Suppose that he dies at the end of his expectation of life. If the dividends average \$7 annually for 20 years, and then \$2 annually for 19.49 years, find the net cost of his insurance.

7. A man 35 years old took out a 20-year term policy for \$4000. He paid 10 premiums, receiving \$300 in dividends and then gave up his policy. What was the net cost of his insurance?

8. A man took out a 15-year endowment policy for \$3000 at \$68 per thousand, and paid ten annual premiums, receiving the following dividends: \$8.12, \$9.22, \$9.75, \$9.50, \$10.10, \$9.40, \$8.93, \$9.90, \$10.75, \$11.00. At the end of the tenth year he surrendered the policy to the company, and received a cash value of \$580. Did he gain or lose, and how much?

9. What was the net cost per thousand of his "protection" for these ten years?

10. A man secured a health policy that would pay him \$30 per week indemnity in case of illness. For each \$5 of weekly indemnity he paid \$8.75 per year. Find the cost of carrying the policy for 12 years.

11. During this time he was confined by illness 6 weeks at home and 8 weeks in a hospital. While he was at the hospital he received double indemnity. Find the total indemnity.

12. A man obtained insurance for \$4000 against accident for 9 months, and was charged 85% of the annual premium of \$11.25 for each \$1000. Find the cost of his "protection."

13. A manufacturer paid a liability company \$172.50 per month to assume all expenses that might arise from claims for injuries sustained by employees in the performance of their duties. This was $\frac{3}{4}$% of the average monthly pay roll. Find the amount of the monthly pay roll.

REVIEW PROBLEMS IN INDUSTRIES

379. **1.** A Louisiana sugar-cane plantation contained 120 acres. If it was 726 feet wide, how long was it?

2. The sugar cane was planted in rows that ran the long way of the field and were $5\frac{1}{2}$ feet apart. How many rows of cane did the width of the field allow?

3. How much seed cane was used for the plantation, if $5\frac{3}{4}$ tons were planted on each of the 120 acres?

4. At harvest time in November, 2340 tons of cane were obtained. Find the yield per acre.

5. The cost of producing the cane was $37.05 per acre. What was the cost per ton?

6. At the sugar mill the planter received 80 pounds of sugar for each of the 2340 tons of cane. What were his receipts for cane, if he sold the sugar at $3\frac{1}{2}$¢ per pound?

7. What was his whole gain? his gain per acre?

8. In the sugar mill the cane was crushed between rollers to extract the juice. The juice extracted was 75 % of the weight of the cane. How much was extracted from the 2340 tons?

9. A test showed that the juice was $13\frac{1}{3}$ % sugar. How many tons of sugar did the 1755 tons of juice contain?

10. Not all of the sugar in the juice could be extracted. Only 210.6 tons were obtained. What part of the 234 tons of sugar in the juice was extracted?

11. What per cent of the weight of the raw cane, then, was actually manufactured into sugar?

12. Since 9% of the cane was converted into sugar, find the yield of sugar from 1 ton of cane; from an acre, $19\frac{1}{2}$ tons.

13. After paying 80 lb. of sugar for each of the 2340 tons of cane, how much of the 210.6 tons of sugar did the mill owner have? How much was it worth at $3.50 per 100 lb.?

14. A certain amount of vegetable fiber, or cane trash, remains after the extraction of the juice. As 75% of the cane was juice, how many tons of trash were there?

15. The 585 tons of trash were used for fuel and were equivalent to $97\frac{1}{2}$ tons of coal. Find its value per ton, if coal was worth $3.90 per ton.

16. By the addition of water to dissolve the juice while the cane is being crushed, the yield of sugar may be increased 5 pounds per ton of cane. How many more pounds are thus obtained by a mill that crushes 500 tons of cane per day for 95 days?

17. How much more value is obtained from a ton of cane by this process when sugar is worth 3.5¢ per pound?

18. If the water added amounts to 10% of the weight of cane used, how many pounds of water does one ton of cane take?

19. How many pounds of coal are needed to evaporate the 200 lb. of water added to a ton of cane, if 1 lb. of coal evaporates 8 lb. of water? Find the cost of the coal at $4 per ton.

20. If the only expense of the 5 extra pounds of sugar is for coal, how much is gained per ton of cane by adding water?

21. What, then, is the added gain during the season, if the mill crushes 500 tons per day for 95 days?

22. The average yield of sugar cane per acre is 18 tons in Louisiana, 34 tons in Cuba, and 55 tons in Hawaii. If the sugar extracted averages 8% of the weight of the cane in Louisiana and 12% in Cuba and Hawaii, find the average yield of sugar per acre in each locality.

23. A farmer in Michigan sowed a 24-acre field with sugar beet seed, 20 pounds to the acre. Find the cost of the seed at 15 ¢ per pound.

24. It cost \$ 5.20 per acre for thinning, \$ 6.10 per acre for weeding, and \$ 5.75 per acre for pulling the beets and cutting off the tops. Find the sum of these expenses for 1 acre and for the whole field.

25. If these three expenses, \$409.20, were 55 % of the whole cost of the beets to the farmer, find the cost of the crop; the cost per acre.

26. The crop of sugar beets, amounting to 300 tons, was drawn to the factory in wagons holding $2\frac{1}{2}$ tons each. How many wagon loads were there?

27. At the factory the beets were bought by the ton but not until all dirt, called *tare*, had been removed. If the tare in this case was 8 % of the entire weight, how many tons of clean beets were there?

28. The factory paid \$ 4 per ton for beets that contained 12 % of their weight in sugar, and 25 ¢ more per ton for each additional per cent. These beets tested 14 %. How much did the farmer receive for his crop of 276 tons?

29. Find his profit on the crop.

30. Though testing 14%, each ton of beets yielded only 224 lb. of sugar. What per cent of the sugar was extracted?

31. The sugar extracted from the 276 tons of beets was sold at $4\frac{5}{8}$ ¢ per pound. How much was received for it?

32. If the total cost of the manufactured sugar to the operators of the factory was $4 per hundredweight, how much did they gain on each ton of clean beets?

33. This factory used 45,339 tons of beets during its working season of 105 days. How many tons did it use per day?

34. If 431.8 tons was only 85% of its daily capacity, find its capacity.

Find the cost of each of the following items, which were among this factory's principal expenses:

35. 9660 tons of coal @ $2.50.

36. 3220 tons of lime rock @ $1.90.

37. 21,000 pounds of sulphur @ $2\frac{1}{4}$ ¢.

38. 8350 yards of filter cloth @ 15 ¢.

39. 19,300 sugar bags @ 9 ¢.

40. 24,135 sugar barrels @ $26\frac{1}{3}$ ¢.

41. In a recent year Louisiana produced 283,500 tons of cane sugar, which was 7 % of the world's production. How much cane sugar was produced that year?

42. That year 24% of the whole yield of 4,050,000 tons was produced in Cuba. How many tons did Cuba produce?

43. The world's production of beet sugar in the same year was 5,800,000 tons. How many more tons of beet sugar were produced than of cane sugar?

44. How much of the beet sugar was furnished by the United States, if we produced $3\frac{2}{5}$ % of the world's crop?

45. What per cent of the combined production of cane and beet sugar did Europe furnish, if 5,614,500 tons were produced there?

Cotton from the field consists of seed and fiber together. At the cotton gin the seed is separated from the fiber, or lint, which is pressed into bales, covered with cloth called bagging, and bound with heavy iron wire called ties. Before long-distance shipment the bales are compressed to fill a still smaller space. The weight of a bale is spoken of as "gross," when it includes bagging and ties, "net," when the lint alone is considered.

46. A bale of cotton before compression occupies about 45 cubic feet of space. When compressed for shipment it is about 54 in. by 27 in. by 16 in. How much less space does a compressed bale occupy?

47. How many compressed bales of $13\frac{1}{2}$ cu. ft. can be put into the space that would be occupied by 3 uncompressed bales?

48. The average gross weight of a bale of American cotton is 500 pounds. If the weight of bagging and ties is 5 % of the gross weight, what is the net weight of a bale?

49. The average gross weights of bales in some other cotton countries are:

India, 400 pounds	Peru, 182 pounds
Brazil, 230 pounds	Egypt, 735 pounds

What per cent of the American 500-pound bale is each of these bales?

50. If it takes 1500 pounds of seed cotton from the field to produce a bale in which the lint weighs 480 pounds, what per cent of the seed cotton is lint? What per cent is seed?

51. If 20 acres produce 12,960 pounds of seed cotton, and $33\frac{1}{3}\%$ of it is lint, how many bales of 480 pounds net does it make?

52. What per cent of a bale does one acre produce?

53. How long would it take a laborer to pick enough seed cotton to make a bale of lint of this weight, if he picks 120 pounds of seed cotton per day?

54. If it takes the cotton gin 1 hr. 2 min. 30 sec. to separate 500 pounds of lint from the seed, how long does it take to gin one pound?

55. At the rate of 2 pounds of lint in 15 seconds how long does it take to gin a bale of 480 pounds net?

56. On 16 acres a cotton farmer raised 720 pounds of seed cotton per acre. After giving 25% of his crop to the owner for rent, how much did he have left?

57. It cost $51.84 to have the entire crop of 11,520 lb. of cotton picked. How much did each picker receive per 100 pounds?

58. If $66\frac{2}{3}\%$ of the farmer's 8640 pounds of cotton consisted of seed, how many bushels of seed of 30 pounds each did he have? how many pounds of lint?

59. When the cotton was ginned and pressed, each bale of lint weighed 500 pounds, including 20 pounds of bagging and ties. How many such bales did the 2880 pounds of lint make?

60. How much did the farmer receive, if he sold the 5760 lb. of seed at 12¢ per bushel (30 lb.) and the 6 bales of lint (500 lb. each) at $6\frac{1}{2}$¢ per pound, gross weight?

61. Find his gain, if the total expense of raising and marketing the crop was 75% of the $218.04 received.

62. If the farmer had paid his rent in money, $\$3\frac{1}{2}$ an acre for 16 acres, and had sold the 4000 lb. gross of cotton @ $6\frac{1}{2}$¢ and the 256 bu. of seed @ 12¢, how much would he then have gained, the total expense aside from rent amounting to $167.53?

63. A New Orleans cotton buyer paid $1062.50 to ship 425 bales, averaging 500 pounds gross, to New York. Find the freight rate per hundredweight.

64. At that time the freight rate to Liverpool was only 35¢ per 100 pounds. How much less would it have cost to ship the same cotton to Liverpool?

65. A car load of cotton in the ordinary square bales consisted of 55 bales, averaging 500 pounds each in weight. How many pounds of cotton did the car carry?

66. Recently a large car load of cotton in the more compact round bales consisted of 160 bales, averaging $428\frac{1}{8}$ pounds each in weight. Find the weight of cotton carried by the car.

67. In a recent year the world's output of cotton was equal to 18,000,000 bales of 500 pounds net. Find the number of such bales contributed by each of the following countries according to the per cents given:

Egypt, $6\frac{2}{3}\%$	East Indies, $16\frac{2}{3}\%$
Brazil, $1\frac{2}{3}\%$	United States, 75%

68. If Great Britain used 3,600,000 of these bales, the continent of Europe 5,100,000, and the United States 4,320,000, what per cent of the whole crop did each consume?

Cotton-seed oil mills obtain several products from the cotton seed as follows: *linters*, the short fibers not removed in ordinary ginning, used for spinning purposes or made into batting; *hulls*, sold for cattle feed; *cotton-seed oil*, used in cooking, for soap, as salad oil, etc.; and *oil cake*, or *meal*, the meat of the seed after the oil is pressed out, used for feeding cattle and for fertilizer.

69. An oil mill that uses 30 tons of cotton seed in a day of 24 hours uses how much per hour?

70. How many days must such a mill run to treat 6300 tons of seed?

The quantities and values of the products of one ton of cotton seed in a good year are:

Linters,	30 pounds	$1.05
Hulls,	950 pounds	1.33
Oil,	40 gallons	8.00
Meal,	675 pounds	6.75

Find:

71. Price of linters per pound.

72. Price of hulls per ton.

73. Price of oil per gallon.

74. Price of meal per hundredweight.

75. Value of products from one ton of seed.

76. Profit per ton, the expense being $13.75.

77. If the mill consumes 6300 tons of cotton seed during the year, find the total profit at $3.38 per ton.

78. How many pounds are there in a bale of hulls 1 ft. 2 in. by 1 ft. 6 in. by 1 ft. 8 in., if 50.4 cubic inches make one pound?

79. The oil is pressed out of the hulled seed under a plate whose area is 201 square inches. The total pressure is $351\frac{3}{4}$ tons. Express the pressure in pounds per square inch.

SIMPLE INTEREST

380. The sum paid for the use of money is called **interest**.

381. The money for the use of which interest is paid is called the **principal**.

382. The sum of the principal and the interest is called the **amount**.

383. Interest is reckoned as a certain *per cent* of the principal, and the **rate of interest** is the per cent paid for the use of the principal for *one year*.

384. In ordinary interest calculations, a month is regarded as 30 days, and a year as 12 months, or 360 days.

METHOD BY ALIQUOT PARTS

385. **1.** If a man pays \$6 for the use of \$100 for 1 year, how much must he pay for the use of \$100 for 2 yr.? for 3 yr.?

How much must he pay for the use of \$100 for 6 months, or $\frac{1}{2}$ of a year? for 2 mo., or $\frac{1}{3}$ of 6 mo.? for 30 days, or $\frac{1}{2}$ of 2 mo.? for 15 days, or $\frac{1}{2}$ of 30 days?

2. When 6 % is paid for the use of the principal for 1 year, what per cent of the principal is paid for its use for 2 yr.? for 6 mo.? for 2 mo.? for 6 da.?

3. Find the interest on \$400 at 5 % for 1 year; for 3 yr.; for 6 mo.; for 3 yr. 6 mo.

4. What is the interest at 4 % on \$600 for 1 year? for 2 months? for 15 days? for 1 yr. 2 mo. 15 da.?

WRITTEN EXERCISES

386. 1. Find the interest on \$325 for 3 years at 5%.

$$\begin{array}{r} \$325 \\ .05 \\ \hline \$16.25 \\ 3 \\ \hline \$48.75 \end{array}$$

We may find 3 times 5% of \$325; or 3 times 5%, that is 15%, of \$325.

$$\begin{array}{r} \$325 \\ .15 \\ \hline 16\ 25 \\ 32\ 5 \\ \hline \$48.75 \end{array}$$

The interest equals the principal multiplied by the rate, multiplied by the number expressing the time in years.

Find the interest on:

2. \$250 for 2 yr. at 5%
3. \$500 for 5 yr. at 3%
4. \$275 for 3 yr. at 7%
5. \$5000 for 2 yr. at $4\frac{1}{2}$%
6. \$6250 for 6 yr. at 4%
7. \$87.50 for 5 yr. at 3%
8. Find the interest on \$629 for 1 yr. 8 mo. at 7%.

SOLUTION

$$\begin{array}{rrl} & \$629, & \text{principal} \\ & .07, & \text{rate} \\ \hline & \$44.03, & \text{int. for 1 yr.} \\ \frac{1}{2} \text{ of int. for 1 yr.} = & 22.015, & \text{int. for 6 mo.} \\ \frac{1}{3} \text{ of int. for 6 mo.} = & 7.338, & \text{int. for 2 mo.} \\ \hline & \$73.38, & \text{int. for 1 yr. 8 mo.} \end{array}$$

NOTE.—It is sufficiently accurate to express final results to the nearest cent and intermediate results to the nearest mill.

Find, by aliquot parts, the interest on:

9. \$5000 for 1 yr. 6 mo. at 3%
10. \$5000 for 1 yr. 9 mo. at 4%
11. \$3225 from Aug. 1, 1906, to Nov. 1, 1907, at 4%
12. \$4175 from May 1, 1906, to Nov. 1, 1909, at $3\frac{1}{2}$%

13. Find the interest and the amount of $500 for 3 yr. 5 mo. 22 da. at 5 %.

SOLUTION

Principal				$500	
Int. for 1 yr., 5 % of $500	1 yr.	$25			
2 times int. for 1 yr.	2 yr.	50			
$\frac{1}{3}$ of int. for 1 yr.	4 mo.	8	333		
$\frac{1}{4}$ of int. for 4 mo.	1 mo.	2	083		
$\frac{1}{3}$ of int. for 1 mo.	10 da.		694		
$\frac{1}{3}$ " " " " "	10 da.		694		
$\frac{1}{5}$ of int. for 10 da.	2 da.		139		
Interest for 3 yr. 5 mo. 22 da.				86	94
Amount for 3 yr. 5 mo. 22 da.				$586	94

Find, by aliquot parts, the interest and the amount of:

14. $5000 for 1 yr. 3 mo. 3 da. at 6 %

15. $5000 for 1 yr. 6 mo. 6 da. at 5 %

16. $2500 for 1 yr. 7 mo. 15 da. at 7 %

17. $3650 for 2 yr. 3 mo. 21 da. at 5 %

18. $7500 for 3 yr. 2 mo. 18 da. at 4 %

19. $875.25 for 2 yr. 5 mo. 16 da. at 6 %

20. $169.75 for 3 yr. 11 mo. 10 da. at $3\frac{1}{2}$ %

21. $15,000 for 60 days at $4\frac{1}{2}$ %

22. $500,000 for 90 days at 2 %

23. $3264.18 for 63 days at 5 %

24. $4870.21 for 120 days at 6 %

25. $3000 from Oct. 11, 1906, to Jan. 26, 1907, at 5 %

26. $5000 from Aug. 16, 1907, to Dec. 28, 1909, at $3\frac{1}{2}$ %

27. $466.30 from Nov. 12, 1906, to June 30, 1912, at 6 %

28. $174.65 from Sept. 27, 1906, to Apr. 16, 1908, at 7 %

29. $50,000 from Dec. 6, 1907, to June 4, 1908, at $2\frac{1}{2}$ %

30. $125,000 from Oct. 22, 1906, to Sept. 11, 1907, at $4\frac{1}{2}$ %

31. $320,000 from May 26, 1907, to July 15, 1910, at 5 %

SIX PER CENT METHOD

387. The great bulk of money loaned in the money market is loaned for terms of 90 days, 60 days, or less. For such terms and a rate of 6 % the most convenient unit of time is 2 months, or **60 days**, $\frac{1}{6}$ of a year. Hence,

The interest for 60 *days at* 6 % *is* 1 % *of the principal, found by moving the decimal point two places toward the left.*

By adding or subtracting parts or multiples of the interest for 60 days, the interest for any term may be found.

The following processes illustrate the method of finding the interest at 6 % on \$5000, for the terms mentioned, from the interest for 60 days:

30 days	10 days	6 days	5 days
2)\$50.00	6)\$50.00	10)\$50.00	12)\$50.00
\$25.00	\$ 8.33	\$ 5.00	\$ 4.17

90 days	11 days	125 days	53 days
\$50.00	6)\$50.00	\$50.00	6)\$50.00, 60 da.
\$25.00	10) 8.333	12)50.00	– 8.33, 10 da.
\$75.00	.833	4.17	\$41.67, 50 da.
	\$9.17	\$104.17	+ 2.50, 3 da.
			\$44.17, 53 da.

This method is sometimes called the **banker's method.**

WRITTEN EXERCISES

388. Find the interest at 6 % on:

1. \$450 for 30 days
2. \$750 for 90 days
3. \$500 for 60 days
4. \$825 for 63 days
5. \$1200 for 45 days
6. \$5000 for 60 days
7. \$37,500 for 120 days
8. \$75,000 for 125 days

389. For convenience, bankers often use the following:

TABLE OF DAYS INTERVENING BETWEEN DATES

	JAN.	FEB.	MAR.	APR.	MAY	JUNE	JULY	AUG.	SEPT.	OCT.	NOV.	DEC.
Jan.	365	31	59	90	120	151	181	212	243	273	304	334
Feb.	334	365	28	59	89	120	150	181	212	242	273	303
Mar.	306	337	365	31	61	92	122	153	184	214	245	275
Apr.	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	333	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
Aug.	153	184	212	243	273	304	334	365	31	61	92	122
Sept.	122	153	181	212	242	273	303	334	365	30	61	91
Oct.	92	123	151	182	212	243	273	304	335	365	31	61
Nov.	61	92	120	151	181	212	242	273	304	334	365	30
Dec.	31	62	90	121	151	182	212	243	274	304	335	365

The number of days from any day of one month to the *same* day of another month is found by starting at the name of the first (in the left-hand column) and following across to the column headed with the name of the second. (Should Feb. 29th of a leap year intervene between dates, add 1 day.)

Suppose it is required to find the number of days from June 5 to Nov. 10. From the table we find that it is 153 days from June 5 to Nov. 5; adding 5 days, we find the required time to be 158 days.

WRITTEN EXERCISES

390. Find, by the six per cent method and the use of the table, the interest at 6 % on the following for the terms mentioned:

1. \$5000, Mar. 6 to May 5
2. \$2750, Apr. 10 to July 9
3. \$3000, June 2 to July 5
4. \$5000, July 5 to Nov. 2
5. \$4500, Aug. 8 to Oct. 7
6. \$7225, Sept. 1 to Dec. 3
7. \$8400, Nov. 30 to Feb. 1
8. \$1600, July 25 to Nov. 22
9. \$22.50, June 30 to Aug. 2
10. \$35.75, Sept. 26 to Nov. 15
11. \$15,000, Aug. 21 to Sept. 4
12. \$50,000, Dec. 22 to Jan. 7
13. \$125,000, May 4 to July 1
14. \$500,000, Nov. 6 to Feb. 2

391. Another form of the *six per cent method* often used makes the interest on \$1 for the given time the basis of computation.

Thus, the interest on \$1 for 1 year at 6% is \$.06; for 1 month, \$.005; for 6 days, \$.001; and for 1 day, \$.000$\frac{1}{6}$.

WRITTEN EXERCISES

392. **1.** Find the interest on \$240.50 for 2 yr. 4 mo. 14 da. at 6%.

SOLUTION

Int. on \$1 for 2 yr.	\$.12
Int. on \$1 for 4 mo.	.02
Int. on \$1 for 14 da.	.002$\frac{1}{3}$
Int. on \$1 for 2 yr. 4 mo. 14 da. =	.142$\frac{1}{3}$

The interest on \$240.50 = 240.50 × \$.142$\frac{1}{3}$, or \$34.23.

Find the interest on the following at 6%:

2. \$42.50 for 7 yr. 4 mo. 20 da.

3. \$39.75 for 5 yr. 3 mo. 15 da.

4. \$425.10 for 6 yr. 5 mo. 10 da.

5. \$365.42 for 8 yr. 10 mo. 24 da.

6. \$875.50 for 4 yr. 11 mo. 18 da.

7. \$920.65 for 4 yr. 10 mo. 17 da.

8. \$1500 from Mar. 14, 1906, to Apr. 3, 1907

9. \$4800 from June 24, 1907, to Dec. 15, 1908

10. \$3200 from Dec. 18, 1905, to Feb. 24, 1914

11. \$64.45 from Feb. 28, 1907, to Oct. 31, 1912

12. \$75,000 from Aug. 16, 1906, to Feb. 12, 1910

13. \$56,000 from Feb. 24, 1907, to Dec. 17, 1909

14. \$375.60 from May 22, 1903, to Mar. 16, 1908

15. \$840.07 from Dec. 10, 1902, to Nov. 20, 1910

16. \$225,000 from Oct. 4, 1906, to Feb. 11, 1908

17. \$500,000 from Aug. 1, 1906, to Jan. 31, 1907

393. From the interest on any principal at 6 % the interest at other rates may be found by adding or subtracting aliquot parts of the interest at 6 %, as follows:

Adding	$\frac{1}{6}$ of itself	for the interest at **7** %
Subtracting	$\frac{1}{6}$ of itself	for the interest at **5** %
Adding	$\frac{1}{3}$ of itself	for the interest at **8** %
Subtracting	$\frac{1}{3}$ of itself	for the interest at **4** %
Adding	$\frac{1}{4}$ of itself	for the interest at **7½** %
Subtracting	$\frac{1}{4}$ of itself	for the interest at **4½** %
Dividing by	2	for the interest at **3** %

and so on.

WRITTEN EXERCISES

394. Find, by the six per cent method, the interest and the amount of:

1. \$250 for 60 da. at 7 %

2. \$500 for 30 da. at 5 %

3. \$400 for 90 da. at 8 %

4. \$500 for 60 da. at 4 %

5. \$160 for 63 da. at 3 %

6. \$775 for 33 da. at 4 %

7. \$840 for 12 da. at 7½ %

8. \$450 for 15 da. at 4½ %

9. \$750 for 10 da. at 4 %

10. \$800 for 45 da. at 3 %

11. \$900 for 25 da. at 5 %

12. \$500 for 120 da. at 4½ %

13. \$5000 for 3 yr. 5 mo. 10 da. at 5 %

14. \$4000 for 2 yr. 7 mo. 24 da. at 8 %

15. \$5000 for 4 yr. 9 mo. 11 da. at 3 %

16. \$1400 for 2 yr. 1 mo. 22 da. at 4½ %

17. \$5000 for 4 mo. 2 da. at 3 %; at 3½ %

18. \$7500 for 2 mo. 5 da. at 5 %; at 5½ %

19. \$2440 from June 11, 1905, to Sept. 8, 1906, at 4 %

20. \$3685 from Feb. 17, 1906, to July 9, 1907, at 5 %

21. \$75.50 from Nov. 22, 1905, to May 10, 1906, at 7 %

22. \$825.25 from Sept. 8, 1904, to Apr. 3, 1910, at 7½ %

23. \$25,000 from May 16, 1907, to Aug. 14, 1908, at 3 %

CANCELLATION METHOD

395. This method is a modification of the six per cent method.

When the rate is 6 %, the interest for 60 days is 1 % of the principal, and for 6 days, $\frac{1}{10}$ % of the principal, or .001 of the principal. Hence,

The interest at 6 % *for any number of days is* .001 *of the principal multiplied by the number of days divided by* 6.

The interest at any other rate is found by taking such a part of the interest at 6 % as the given rate is of 6 %.

WRITTEN EXERCISES

396. **1.** Find the interest on \$6228 for 93 days at 6 % ; also at 4 %.

SOLUTIONS

$$\overset{\$1.038}{\$\cancel{6.228}} \times \frac{93}{\cancel{6}} =$$

$$\$1.038 \times 93 = \$96.53$$

$$\overset{\$1.038}{\$\cancel{6.228}} \times \frac{\overset{31}{\cancel{93}}}{\cancel{6}} \times \frac{\overset{2}{\cancel{4}}}{\underset{2}{\cancel{6}}} =$$

$$\$1.038 \times 31 \times 2 = \$64.36$$

Find, by cancellation, the interest on the following for the given time and rate :

2. \$3000, 12 days, 6 %

3. \$5000, 8 days, 4 %

4. \$7500, 15 days, 3 %

5. \$4365, 60 days, 5 %

6. \$8000, 10 days, 4 %

7. \$7200, 90 days, 4 %

8. \$3420, 42 days, 6 %

9. \$6250, 36 days, $4\frac{1}{2}$ %

10. \$7200, 93 days, $3\frac{1}{2}$ %

11. \$2400, July 6 to Oct. 4, 5 %

12. \$6300, June 4 to Aug. 3, 4 %

13. \$7800, Nov. 10 to Feb. 10, 3 %

14. \$8100, Dec. 12 to Feb. 1, 6 %

15. \$9600, May 16 to Aug. 14, 5%

16. \$2700, Mar. 1 to June 2, 7 %

17. \$75.60, Apr. 1 to June 30, $7\frac{1}{2}$ %

18. \$43.20, Nov. 4 to Jan. 3, $4\frac{1}{2}$ %

19. \$85.50, Aug. 7 to Dec. 11, 5 %

INTEREST TABLE METHOD

397. Bankers and others who have considerable interest to compute facilitate their work by the use of **interest tables.**

The following incomplete table illustrates the nature of such tables. This one is based on a year of 360 days and a rate of 6 %.

Years	$ 1000	$ 2000	$ 3000	$ 4000	$ 5000	$ 6000	$ 7000	$ 8000	$ 9000
1	60	120	180	240	300	360	420	480	540
2	120	240	360	480	600	720	840	960	1080
3	180	360	540	720	900	1080	1260	1440	1620
4	240	480	720	960	1200	1440	1680	1920	2160
5	300	600	900	1200	1500	1800	2100	2400	2700
6	360	720	1080	1440	1800	2160	2520	2880	3240
...	...	...	...	...	...	...	...	...	...

Months	$ 1000	$ 2000	$ 3000	$ 4000	$ 5000	$ 6000	$ 7000	$ 8000	$ 9000
1	5	10	15	20	25	30	35	40	45
2	10	20	30	40	50	60	70	80	90
3	15	30	45	60	75	90	105	120	135
4	20	40	60	80	100	120	140	160	180
5	25	50	75	100	125	150	175	200	225
6	30	60	90	120	150	180	210	240	270
7	35	70	105	140	175	210	245	280	315
8	40	80	120	160	200	240	280	320	360
...	...	...	...	...	...	...	...	...	...

Days	$ 1000	$ 2000	$ 3000	$ 4000	$ 5000	$ 6000	$ 7000	$ 8000	$ 9000
1	.167	.333	.50	.667	.833	1.00	1.167	1.333	1.50
2	.333	.667	1.00	1.333	1.667	2.00	2.333	2.667	3.00
3	.500	1.000	1.50	2.000	2.500	3.00	3.500	4.000	4.50
4	.667	1.333	2.00	2.667	3.333	4.00	4.667	5.333	6.00
5	.833	1.667	2.50	3.333	4.167	5.00	5.833	6.667	7.50
6	1.000	2.000	3.00	4.000	5.000	6.00	7.000	8.000	9.00
7	1.167	2.333	3.50	4.667	5.833	7.00	8.167	9.333	10.50
8	1.333	2.667	4.00	5.333	6.667	8.00	9.333	10.667	12.00
9	1.500	3.000	4.50	6.000	7.500	9.00	10.500	12.000	13.50
10	1.667	3.333	5.00	6.667	8.333	10.00	11.667	13.333	15.00
11	1.833	3.667	5.50	7.333	9.167	11.00	12.833	14.667	16.50
12	2.000	4.000	6.00	8.000	10.000	12.00	14.000	16.000	18.00
13	2.167	4.333	6.50	8.667	10.833	13.00	15.167	17.333	19.50
14	2.333	4.667	7.00	9.333	11.667	14.00	16.333	18.667	21.00
15	2.500	5.000	7.50	10.000	12.500	15.00	17.500	20.000	22.50
16	2.667	5.333	8.00	10.667	13.333	16.00	18.667	21.333	24.00
17	2.833	5.667	8.50	11.333	14.167	17.00	19.833	22.667	25.50
18	3.000	6.000	9.00	12.000	15.000	18.00	21.000	24.000	27.00
19	3.167	6.333	9.50	12.667	15.833	19.00	22.167	25.333	28.50
20	3.333	6.667	10.00	13.333	16.667	20.00	23.333	26.667	30.00
...	...	...	...	...	...	...	...	...	...

WRITTEN EXERCISES

398. **1.** Find, by the interest table, the interest on \$2348 for 3 yr. 2 mo. 7 da. at 6 %.

SOLUTION

Since the interest on any number of *hundred* dollars is .1 of that on the *same* number of *thousand* dollars; on any number of *tens* of dollars .01 of that on the *same* number of *thousand* dollars, etc., we have,

	3 YR.	2 MO.	7 DA.	
Int. on \$2000 =	\$360.00 +	\$20.00 +	\$2.333	
Int. on 300 =	54.00 +	3.00 +	.350	
Int. on 40 =	7.20 +	.40 +	.047	
Int. on 8 =	1.44 +	.08 +	.009	
Int. on \$2348 =	\$422.64 +	\$23.48 +	\$2.739	= \$448.86

NOTE. — Though the table is not complete, interest for any number of years, months, and days may be found by it. For example, the interest for 8 yr. is the same as for 6 yr. + 2 yr.; for 11 mo., the same as for 8 mo. + 3 mo.; for 25 da., the same as for 20 da. + 5 da.

The student is already familiar with methods of finding the interest at other rates, from the interest at 6 %.

Find, by the interest table, the interest on the following for the given time and rate:

2. \$2500, 12 days, 6 %

3. \$3200, 14 days, 6 %

4. \$4500, 11 days, 6 %

5. \$5000, 6 days, 6 %

6. \$4800, 29 days, 6 %

7. \$8700, 25 days, 6 %

8. \$7435, 5 mo., 3 %

9. \$2467, 3 mo., 5 %

10. \$5762, 11 mo., $4\frac{1}{2}$ %

11. \$3840, 10 days, 5 %

12. \$4070, 15 days, 4 %

13. \$3175, 21 days, 3 %

14. \$5000, Jan. 30, 1906, to Mar. 11, 1907, 4 %

15. \$8040, Feb. 16, 1906, to May 12, 1908, 6 %

16. \$5340.25, May 8, 1907, to Mar. 1, 1910, 5 %

17. \$8257.75, Aug. 4, 1905, to June 22, 1909, $4\frac{1}{2}$ %

18. \$3375.45, Jan. 16 to Mar. 10, in a leap year, 3 %

WRITTEN EXERCISES

399. Using any convenient method, find the interest at 6 %, 4 %, $4\frac{1}{2}$ %, 3 %, and 5 % on:

1. $6440 for 63 da.
2. $3200 for 75 da.
3. $5000 for 2 da.
4. $5000 for 5 da.
5. $5000 for 10 da.
6. $5000 for 30 da.
7. $5000 for 60 da.
8. $5000 for 90 da.
9. $5000 for 120 da.
10. $5000 for 125 da.
11. $133.20 for 21 da.
12. $786.32 for 27 da.
13. $432.60 for 11 da.
14. $635.80 for 57 da.
15. $752.19 for 93 da.
16. $563.22 for 3 mo. 18 da.
17. $287.10 for 5 mo. 12 da.
18. $1000 for 1 yr. 2 mo. 9 da.
19. $1200 for 2 yr. 7 mo. 3 da.
20. $16,000, Feb. 1 to Feb. 28
21. $45,000, Jan. 10 to Jan. 20
22. $500,000, June 7 to July 1
23. $2,500,000 for 60 da.
24. $13,200,000 for 30 da.

25. A man lived on the income from $50,000. If $\frac{1}{4}$ of it was invested at 7 % and the rest at 5 %, find his annual income.

26. A resident of Colorado Springs borrowed money in the East at 5 % and loaned it on good security at home at 8 %. Find his annual gain on an investment of $75,000.

27. A coal dealer purchased his coal at $3.20 per ton cash on the first of April, and sold it 6 months later. If money was worth 5 %, what was the real cost of the coal to him at the time of sale?

28. A man invested $10,000 in a gold mine and for 12 years received an annual return of 15 % on his investment. After 12 years, however, the ore was all mined and the man lost his principal. How much better or worse off was he than if he had invested his money in safe securities yielding 4 % interest, if he did not reinvest any interest in either case?

ACCURATE INTEREST

400. Interest computed by taking the exact number of days between dates and reckoning 365 days for a year is called **accurate, or exact, interest.**

Accurate interest, then, does not differ from ordinary interest for whole years, but for terms less than a year it is $\frac{360}{365}$ of ordinary interest, or it is ordinary interest less $\frac{5}{365}$, or $\frac{1}{73}$, of itself when the latter is reckoned for the actual number of days.

It may also be computed by accurate interest tables.

Accurate interest is used by the United States government, by some banks, and to some extent in other business.

WRITTEN EXERCISES

401. Find the accurate interest on:

1. \$200 for 90 da. at 6%
2. \$500 for 33 da. at 6%
3. \$750 for 60 da. at 5%
4. \$480 for 45 da. at 4%
5. \$5000 for 120 da. at 3%
6. \$6400 for 111 da. at 4%
7. \$7200 for 214 da. at 5%
8. \$8100 for 179 da. at $4\frac{1}{2}$%

Using the table for days on page 234, find the accurate interest at 6% on:

9. \$150, Mar. 2 to July 15
10. \$444, Aug. 1 to Dec. 1
11. \$765, May 1 to Nov. 1
12. \$480, June 1 to Dec. 1
13. \$5000, Apr. 1 to Oct. 1
14. \$5000, May 1 to Nov. 1
15. \$4640, June 1 to Nov. 4
16. \$3875, July 6 to Dec. 9

17. How much less is the accurate interest on \$10,000 from June 15 to Sept. 15 at 3% than the interest computed in the ordinary way?

18. The United States pays accurate interest. How much interest is saved thereby on \$2,500,000, borrowed from Mar. 1 to July 1 at $4\frac{1}{2}$%?

PROBLEMS IN INTEREST

402. To find the rate.

1. What is the interest on $200 for 2 years at 1%?

2. Since $200 yields $4 interest in 2 years at 1%, at what rate will it yield $8 interest in the same time? $12 interest? $24 interest?

3. At what rate will $500 yield $100 interest in 5 years?

4. At what rate will $1000 yield $120 interest in 2 years?

WRITTEN EXERCISES

403. 1. At what rate must $800 be invested to yield an interest of $176 in 5 yr. 6 mo.?

Solutions

1. The interest on $800 at 1% for $5\frac{1}{2}$ years is $5\frac{1}{2}$% of $800, or $44.
Since a rate of 1% yields $44, it will require a rate of as many per cent to yield $176 as $44 is contained times in $176, or 4%.

2. Let $x =$ the rate. Then the interest on $800 for $5\frac{1}{2}$ years is $5\frac{1}{2} \times \frac{x}{100}$ of $800, or $44x$ dollars. Since this interest is 176 dollars,

$$44x = 176.$$
$$\therefore x = 4, \text{ and the rate is } 4\%.$$

Test. — Interest on $800 for $5\frac{1}{2}$ years at 4% = $176.

The rate equals the given interest divided by the interest for the given time at 1%.

At what rate of interest will

2. $300 yield an interest of $60 in 4 yr.?

3. $500 yield an interest of $90 in 3 yr.?

4. $700 yield an interest of $70 in 2 yr. 6. mo.?

Find the rate of interest when the interest on

5. $500 for 5 yr. 3 mo. is $105

6. $900 for 3 yr. 4 mo. is $180

7. \$150 for 4 yr. 6 mo. is \$33.75

8. \$225 for 5 yr. 8 mo. is \$76.50

9. \$3200 for 3 yr. 1 mo. 15 da. is \$400

10. \$4500 for 6 yr. 4 mo. 24 da. is \$1296

11. I deposited \$3200 in a bank and 6 months later was credited with \$56 interest. What rate of interest was allowed?

12. A man borrowed \$3000 and gave a written promise to pay the lender \$3050 in four months. How much of the \$3050 was interest? What rate of interest was paid?

13. At what rate will \$600 amount to \$720 in 4 years?

14. A house that cost \$5400 rented for \$576 per year. Taxes and repairs cost the owner \$157.50 a year. Find the net annual income and the rate of interest realized on the investment.

15. The profits earned by a power plant in a year, after deducting all expenses and allowing for depreciation of the property, were \$8187.50. The plant cost \$25,000. What rate of interest was earned on the investment?

404. To find the time.

1. What interest will \$200 earn in 1 year at 6%?

2. In what time will \$200 at 6% earn \$12 interest? \$18? \$30?

WRITTEN EXERCISES

405. 1. In what time will \$750 at 6% yield \$250 interest?

SOLUTIONS

1. The interest for 1 yr. at 6% is 6% of \$750, or \$45.
Hence the time required is $\frac{250}{45}$ yr., or $5\frac{5}{9}$ yr., or 5 yr. 6 mo. 20 da.

2. Let x = the number of years. Then $\frac{6x}{100} \times 750 = 250$.

Hence, $45x = 250$

$\therefore x = 5\frac{5}{9}$, and the time is $5\frac{5}{9}$ yr.

Test. — Interest on \$750 for $5\frac{5}{9}$ years at 6% = \$250.

The number of years equals the given interest divided by the interest at the given rate for 1 year.

Find how long each of the following sums at the given rate must draw interest to amount to $1000:

SUGGESTION. — First find the total interest by subtracting the principal from the amount.

2. $600 at 8%	**4.** $900 at 4%	**6.** $625 at $4\frac{1}{2}$%
3. $800 at 5%	**5.** $840 at 6%	**7.** $875 at $3\frac{1}{2}$%

8. In how many years will any principal double itself at 4%? at 5%? at 2%? at 6%? at $4\frac{1}{2}$%?

SUGGESTION. — The interest to be earned is equal to the principal. How many times does the principal contain 4% of itself?

9. In how many years will any principal treble itself at 5%?

406. To find the principal.

WRITTEN EXERCISES

1. What principal invested at 5% per annum will yield a yearly income of $1000?

SOLUTIONS

1. Since $1 at 5% yields $.05 interest per year, as many dollars must be invested to yield $1000 per year as $.05 is contained times in $1000. $1000 ÷ $.05 = 20,000. Hence $20,000 must be invested.

2. Let x = number of dollars that must be invested.

Then,
$$.05x = 1000.$$
$$x = 1000 \div .05 = 20,000.$$

Hence the principal required is $20,000.

Test. 5% of $20,000 = $1000.

The principal equals the given interest divided by the interest on $1 for the given time at the given rate.

What principal will yield an annual income of

2. $125 at 5%?	**4.** $750 at 6%?	**6.** $1000 at 2%?
3. $500 at 4%?	**5.** $900 at $4\frac{1}{2}$%?	**7.** $3000 at $7\frac{1}{2}$%?

What principal will yield an interest of

8. \$375 in 4 yr. at 5 %? **11.** \$500 in 2 yr. 6 mo. at 4 %?

9. \$900 in 2 yr. at 6 %? **12.** \$480 in 3 yr. 4 mo. at 6 %?

10. \$840 in 7 yr. at 3 %? **13.** \$1000 in 60 da. at 5 %?

14. A man wishes to insure his life for an amount that invested at 4 % will provide an annual income of \$3000 after his death. For what amount shall he insure his life?

15. A man bequeathed to a college enough money so that the interest on it at $3\frac{1}{2}$ % would provide for 10 annual scholarships of \$350 each. Find the amount of the gift or endowment.

Present Worth and True Discount

407. **1.** What will be the amount of \$100 in 1 year, if loaned at 6 % interest? in 2 years? in 3 years?

2. If money is loaned at 6 %, what is the value *now* of \$106 due 1 year hence? What is the present value, or *present worth*, of \$112 due 2 years hence? of \$118 due in 3 years?

3. When money is worth 6 %, what sum should be deducted from a debt of \$106 paid 1 year before it is due? What *discount* should be allowed on \$1060 paid 1 year before it is due?

408. The **present worth** of a sum due at a future time is its *cash* value; or it is the sum that loaned at the current market rate would amount to the given sum in the given time.

409. The difference between the face value of a sum due at a future time and its present worth, or the sum that should be deducted for immediate payment, is sometimes called the **true discount.**

The present worth may be regarded as the *principal*, the true discount as the *interest*, and the sum due at a future time as the *amount*.

WRITTEN EXERCISES

410. **1.** A man owing me \$500 due in 1 yr. 3 mo. wishes to pay me now. How much should I accept in payment, money being worth 5 %? Find the true discount.

Solutions

1. Since every dollar put at interest at 5 % would amount to \$1.0625 in 1 yr. 3 mo., it will require as many dollars now to amount to \$500 in 1 yr. 3 mo. as \$1.0625 is contained times in \$500. \$500 ÷ \$1.0625 = 470.59 (to the nearest .01). Hence I should accept a cash payment of \$470.59.

The true discount is \$500 − \$470.59, or \$29.41.

2. In 1 yr. 3 mo., x dollars at 5 % will amount to $1.0625\,x$ dollars, or to the sum due at that time, \$500.

Hence, $$1.0625\,x = 500.$$
$$\therefore\ x = 500 \div 1.0625 = 470.59.$$

Hence present worth = \$470.59; true discount = \$500 − \$470.59, or \$29.41.

The present worth equals the given amount divided by the amount of \$1 *for the given time at the given rate.*

2. A merchant bought a bill of goods amounting to \$800 on 60 days' credit. Find the equivalent cash value of the goods, if money was worth 6 %.

3. What is the present worth of \$1000 due 2 yr. 6 mo. from date, if money can be borrowed at 6 %?

4. Find the true discount on \$5000 paid 6 months before it is due, if money is worth 5 % per annum.

5. What sum of money put at interest for 9 months at $3\frac{1}{2}$ % will amount to \$1642?

6. A person just 20 years old is named in a will as heir to \$5000 due when he is 21 years old. What is the present worth of his inheritance, money being worth 5 %?

7. I have an endowment policy for \$5000 due in 10 months. Find its present value, money being worth 6 %.

8. I am offered a credit of 90 days on a bill for \$2500, or $1\frac{1}{2}$ % off for cash. Which is the better offer, credit or cash, and how much, if money can be borrowed at 4 %?

9. Mr. Brown offers me \$8800 in cash for a house and lot, and Mr. White offers \$6000 in cash and \$3000 payable in 1 year without interest. Which is the better offer, and how much better, if money is worth 6 %? if money is worth 8 %?

Interest "Payable Annually"

411. In some states when a written agreement contains the expression "with interest payable *annually*," simple interest may be collected upon the principal and upon each year's interest that has not been paid when due. This is called **annual interest.**

WRITTEN EXERCISES

412. 1. No interest having been paid, find the amount due in 3 years 6 months 6 days on \$1000, with interest payable annually at 6 %.

SOLUTION

Principal	\$1000	
Simple interest on \$1000, at 6 %, for 3 yr. 6 mo. 6 da. . .	211	
The interest for each year is \$60		
The 1st annual int., \$60, remains unpaid for 2 yr. 6 mo. 6 da.		
The 2d annual int., \$60, remains unpaid for 1 yr. 6 mo. 6 da.		
The 3d annual int., \$60, remains unpaid for 6 mo. 6 da.		
Interest on \$60, at 6 %, for . . . . 4 yr. 6 mo. 18 da. =	16	38
Amount of \$1000	\$1227	38

The amount equals the principal, plus the simple interest for the entire time, plus the interest on each year's interest for the time it remains unpaid.

2. Find the amount of \$5850 in 4 years 3 months, with interest payable annually at 4 %.

3. Find the amount of \$24,000 in 3 years 3 months 12 days, with interest payable annually at 5 %.

COMPOUND INTEREST

413. Interest on the principal and its unpaid interest, combined at regular intervals, is called **compound interest.**

Interest may be compounded with the principal annually, or semi-annually, or quarterly, etc.

Compound interest cannot usually be enforced by law, even though it is specified in the contract. Nevertheless the subject is of importance since it is at the basis of computations concerning investments, for there is nothing to hinder a man from drawing his interest and investing it.

WRITTEN EXERCISES

414. 1. Find the amount of \$400 for 2 yr. 4 mo. at 6 %, interest compounded annually; also find the compound interest.

SOLUTION

Principal	\$400
Interest for 1st yr. at 6 %	24
Principal beginning 2d yr.	\$424
Interest for 2d yr. at 6 %	25.44
Principal beginning 3d yr.	\$449.44
Interest for 4 mo. at 6 %	8.99
Amount for 2 yr. 4 mo. at 6 %	\$458.43

Since the amount of \$400 at compound interest is \$458.43, the compound interest is \$458.43 – \$400, or \$58.43.

NOTE. — Unless otherwise specified, interest is understood to be compounded *annually*. If compounded semiannually, the rate must be considered one half the annual rate mentioned; if quarterly, one fourth, etc.

When the time consists of years, months, and days, the amount is to be found for the greatest number of entire periods, as years, half years, quarter years, etc., and the simple interest upon this for the rest of the time.

Find the amount and the compound interest of:

2. \$400 for 2 yr. at 5 %

3. \$1000 for 3 yr. at 4 %

4. \$1500 for 4 yr. 6 mo. at 6 %

5. \$5000 for 2 yr. at 6 %, payable semiannually

6. \$5000 for 2 yr. at 6 %, payable quarterly

7. \$8000 for 1 yr. 8 mo. at 4 %, payable quarterly

8. Find the amount of \$1000 for 10 years at 4 % compound interest; at 4 %, compounded semiannually.

Solutions

1. By the table at the end of the book, facing the Index, the amount of \$1 at 4 % compound interest for 10 years is \$1.480244. The amount of \$1000 for the same time at the same rate is 1000 × \$1.480244, or \$1480.24.

2. If the rate is 4 % compounded semiannually, there are 20 interest periods and the rate for each is 2 %. Therefore the amount of \$1000 for 10 years at 4 %, compounded semiannually, is the same as the amount of \$1000 for 20 years at 2 %, compounded annually. By use of the table this amount is found to be \$1485.95.

Find, by the table, the amount at compound interest of:

9. \$1000 for 10 yr. at 5 %

10. \$5000 for 20 yr. at $3\frac{1}{2}$ %

11. \$3250 for 12 yr. at $4\frac{1}{2}$ %

12. \$725.32 for 16 yr. at 6 %

13. \$4000 for 8 yr. 3 mo. at 4 %

14. \$6000 for 6 yr. 8 mo. at $2\frac{1}{2}$ %

15. \$7000 for 14 yr. 9 mo. at 3 %

16. \$86.48 for 18 yr. 10 mo. 15 da. at 6 %

17. \$75.50 for 15 yr. 6 mo. 3 da. at 4 %

18. \$5000 for 8 yr. at 6 %, payable semiannually

19. \$6500 for 4 yr. 4 mo. 10 da. at 4 %, payable quarterly

415. A written promise made by one person to pay to another a definite sum of money at a definite time is called a **promissory note**, or simply a **note**.

> *$4000.* *Chicago, Ill., May 15, 1906.*
>
> *-----Three months-----after date---I---promise to pay to the*
>
> *order of Walter Winters-----------------------------------*
>
> *Four thousand and* $\frac{00}{100}$ *~~~~~~~~~~~~~~~~~~~~~~~~~~~~ Dollars.*
>
> *Value received, with interest at 5%.*
>
> *John Simpson.*

416. The *essentials* of a note are as follows:

1. It must be signed by the person who promises to pay. This person is called the **maker**, or **drawer**.

2. It must designate by name, or otherwise, the person to whom the money is to be paid. This person is called the **payee**.

3. The *sum* to be paid must be definite. It is called the **face** of the note.

The face of a note is usually written both in words and in figures.

4. The *time* of payment must be definite, in the sense that it is a time that can be determined and is not ambiguous.

It may be payable *on demand;* at a *specified time* after date, which must appear on the note; on or before a *specified date;* etc. When no time of payment is mentioned, the note is payable on demand.

A promise to pay "when trade will permit" is not binding.

417. A note should be definite in regard to the following particulars, according to the agreement between the maker and the payee:

1. Unless the *place* of payment is expressly stated, the residence or place of business of the maker is understood, and this must appear on the note.

2. When a note reads "with interest," but does not give the rate of interest, the legal rate in the state where the note was made may be collected.

3. If a note does not contain the words "with interest," no interest may be collected for the time it has to run; but if the note is not paid when due, it then begins to draw interest at the legal rate.

NOTE. — The words "value received" are usually included in a note, though they are not essential.

418. The following illustrates a **demand** note:

$84.^{65}	*Baltimore, Md., Aug. 10, 1907.*
On demand---*I*---*promise to pay to*-------------------------	
Hiram Fairbanks----------------------------------*or order,*	
Eighty-four and $\frac{65}{100}$ ----------------------------------*Dollars.*	
	Silas Vincent.

The words "or order" mean that Silas Vincent will pay $84.65 to any one to whom Hiram Fairbanks may order it paid.

A note payable "to bearer" is payable to any one who presents it.

EXERCISES

419. Examine the note in § 418 and answer these questions:

1. Who is the maker? the payee?
2. When is the note payable?
3. Where is the note payable?
4. What is the face of the note?
5. Answer the same questions in regard to the note shown on the previous page.

420. When a note becomes payable it is said to **mature**.

1. At common law, arising from the custom of merchants in the past, a note matures three days after the time specified therein. These three days are called **days of grace**. The majority of the states and territories, however, have abolished days of grace.

2. If a note falls due on Sunday, a Saturday half-holiday, or a legal holiday, it is usually payable on the next *succeeding* business day; but in some states it must be paid on the preceding business day.

NOTE. — In regard to the legal time of maturity of notes the teacher should be governed by the law of the state in which the school is situated.

421. If a note is drawn payable "to bearer," the payee may sell it, or **negotiate** it, by delivering it to the purchaser ; and any purchaser may negotiate it in the same way.

If the note is drawn payable "to order," a simple delivery is not sufficient to negotiate it, for it is payable to the payee only, until he orders it paid to another person. To do this he writes his name on the back of the note together with any instructions or conditions he deems best. This is called **indorsing** the note.

If a note is drawn payable to the payee only, no words "to order" or "to bearer" being inserted, it is **non-negotiable**.

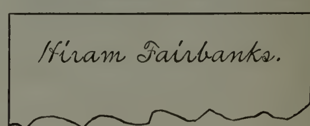
422. The person who indorses a note is called an **indorser** ; the person who holds a note is called the **holder**.

The payee is the first holder; if he sells the note, the purchaser becomes the holder, etc.

423. Indorsements are of various forms:

1. Suppose that Hiram Fairbanks, the payee of the note in § 418, sells the note to Seth Bell and indorses it **in blank**, as shown here.

INDORSEMENT IN BLANK

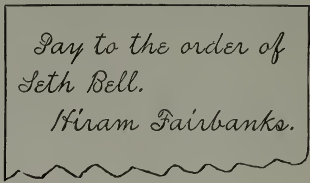


This indorsement is really an agreement between Mr. Fairbanks and all subsequent holders of the note to this effect: "For value received, I, Hiram Fairbanks, transfer to the bearer all my

right and title in this note. I guarantee that it is genuine and that it will be paid when due, if not by the maker, then by myself."

2. If Mr. Fairbanks indorses **in full** when he sells the note to Mr. Bell, no one except Mr. Bell can negotiate the note, for it is payable to his order only.

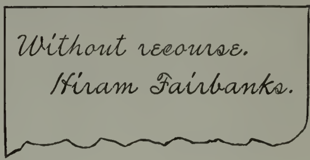
INDORSEMENT IN FULL



This form of indorsement is useful in sending notes through the mails, for if lost or stolen, they are valueless to persons obtaining them.

3. Suppose that Mr. Fairbanks wishes to avoid responsibility for the payment of the note. He will write "without recourse" over his name.

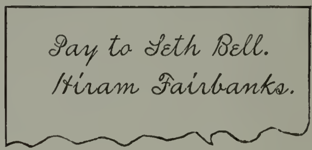
QUALIFIED INDORSEMENT



This is a **qualified** indorsement *in blank*.

4. If Mr. Fairbanks, in selling Mr. Bell the note, wishes to *restrict* its transfer so that Mr. Bell cannot sell it, he writes over his signature "Pay to Seth Bell only," or "Pay to Seth Bell."

RESTRICTIVE INDORSEMENT



This is a **restrictive** indorsement.

A restrictive indorsement makes the note non-negotiable.

424. If, when a note is unpaid at its maturity, the holder fails to **protest** it, that is, to notify the indorsers in a manner prescribed by law that it is unpaid, they are released from responsibility regarding its payment.

425. A higher rate of interest than that authorized by law is called **usury**.

The penalty for making usurious contracts varies in the different states, from the loss of the whole debt and interest to nothing.

WRITTEN EXERCISES

426. **1.** Write a negotiable note for $122.50, yourself the maker and Reuben Hamilton the payee. Make it payable ninety days after date in the place where you live, with interest at the legal rate in your state.

2. Write a non-negotiable demand note for $200 payable by yourself to John H. Fassett, without interest.

3. Write a note with three indorsements, the first and second *in full*, the last *in blank*.

4. Write a note with two indorsements in full, the second *without recourse*.

5. Write two forms of negotiable notes for $795.36 payable in three months to John C. Fenton, with interest.

6. Indorse them properly to transfer one to the bearer and the other to Richard Gray or order.

7. Write a demand note with interest. Write indorsements to show that it has been transferred four times, each time by a different kind of an indorsement, the last being *restrictive*.

8. Suppose that you have sold to Sanford & Williams merchandise for $1000, terms half cash and half payable in sixty days. Draw the note, with their signature. Make the note negotiable.

9. Indorse the note for selling it to J. K. Avery in such a way that he can sell it. Do this in two ways.

10. Suppose that Mr. Avery sells the note to Henry Baldwin, and indorses it in such a way as to avoid responsibility for its payment. Write the indorsement.

11. Suppose that Mr. Baldwin is the final holder of the note. If Sanford & Williams do not pay the note at maturity, who must pay it? Explain each indorser's liability.

12. Who are responsible to Mr. Baldwin for the payment of the note? Who is responsible to you?

PARTIAL PAYMENTS

427. It frequently happens that the maker of a note cannot pay the whole amount at one time, but instead he makes **partial payments.**

A record of the partial payments is written, or **indorsed**, on the back of the note with the dates when they were made.

United States Rule

WRITTEN EXERCISES

428. 1. A note for $1000, dated Jan. 1, 1904, interest 6%, was indorsed as follows: Mar. 22, 1904, $125; June 1, 1905, $50; Aug. 4, 1905, $75; Nov. 1, 1905, $150. Find the amount due Jan. 1, 1906.

SOLUTION

Principal, Jan. 1, 1904		$1000.00
Int. to Mar. 22, 1904, — 2 mo. 21 da.		13.50
Amount		1013.50
First payment		125.00
New principal, Mar. 22, 1904		$888.50
Int. to June 1, 1905, — 1 yr. 2 mo. 9 da.		63.53
Second payment (which is less than int. due)	$50.00	
Int. from June 1 to Aug. 4 — 2 mo. 3 da. on $888.50		9.33
Amount		961.36
Third payment (to be added to second)	$75.00	125.00
New principal, Aug. 4, 1905		$836.36
Int. to Nov. 1, 1905, — 2 mo. 27 da.		12.13
Amount		848.49
Fourth payment		150.00
New principal, Nov. 1, 1905		$698.49
Int. to Jan. 1, 1906, — 2 mo.		6.98
Amount due Jan. 1, 1906		$705.47

United States Rule. — *Find the amount of the principal to a time when a payment, or the sum of two or more payments, equals or exceeds the interest due, and from the amount subtract such payment or payments.*

With the remainder as a new principal proceed as before.

In other words, partial payments are applied first to pay the interest due, and then, if anything is left, to reduce the principal.

Most of the states have adopted the United States Rule for computing the indebtedness when partial payments have been made. In states where other methods are prescribed the rules of those states should be followed.

2. A note for \$1000, bearing 5% interest, and dated Jan. 2, 1906, had these payments indorsed on it: Apr. 2, 1906, \$200; July 2, 1906, \$300. How much was due Oct. 12, 1906?

3. A note for \$400, dated Apr. 1, 1905, had indorsed on it the following payments: July 1, 1905, \$25; Sept. 1, 1905, \$30; Dec. 11, 1905, \$100. How much was due Apr. 5, 1906, with interest at 4%?

4. A note for \$1600, bearing 6% interest, dated Nov. 1, 1904, had indorsed on it the following partial payments: Feb. 1, 1905, \$60; Oct. 20, 1905, \$20; Nov. 1, 1905, \$400. How much was due Apr. 10, 1906?

5. A note for \$4000, bearing 5% interest, dated Aug. 15, 1904, had the following partial payments indorsed on it: Jan. 2, 1905, \$40; May 11, 1905, \$500. How much was due June 1, 1906?

6. A note for \$1250, given Feb. 28, 1903, had indorsed on it the following payments: July 1, 1903, \$100; Dec. 16, 1904, \$200; Oct. 11, 1905, \$600. Find the amount due Apr. 4, 1906, with interest at 6%.

7. A note for \$80, dated Oct. 22, 1904, had written across the back of it, "Received on the within note, Feb. 1, 1905, \$47." How much was due July 1, 1905, with interest at 4%?

Mercantile Rule

429. Business men often settle notes and accounts on which partial payments have been made, and which do not run longer than one year, by the following rule:

Mercantile Rule. — *Find the amount of the principal at the time of settlement.*

Find the amount of each payment from the time it was made until the time of settlement.

From the amount of the principal subtract the sum of the amounts of the payments.

The following exercises are to be solved by this rule.

WRITTEN EXERCISES

430. 1. A note for \$1200 given May 18, 1904, had the following partial payments indorsed on it: Oct. 15, 1904, \$300; Mar. 10, 1905, \$50. How much was due May 18, 1905, with interest at 5%?

2. A note for \$500, dated May 15, 1906, had these payments indorsed on it: July 10, 1906, \$145; Oct. 16, 1906, \$175. How much was due Jan. 1, 1907, interest at 6%?

3. A note for \$850, dated June 1, 1905, had the following payments indorsed on it: Oct. 1, 1905, \$100; Feb. 1, 1906, \$150; May 1, 1906, \$75. How much was due May 16, 1906, with interest at 6%?

4. A note for \$175, dated June 20, 1906, had these indorsements: July 20, 1906, \$75; Aug. 15, 1906, \$25. How much was due Dec. 20, 1906, at 4% interest?

5. A note for \$1600, dated Jan. 1, 1905, had the following payments indorsed on it: Mar. 2, 1905, \$300; July 1, 1905, \$25; Oct. 17, 1905, \$80. With interest at 6%, how much was due Jan. 2, 1906?

Find also the amount due by the United States Rule.

BANKING

431. People do not usually carry much money about in their pockets or leave it in their houses or in their stores; they deposit it in banks for safe keeping.

If they intend to use the money soon, they place it in a **bank of deposit**, but if they do not expect to withdraw it for some time, it will be wiser to leave it in a **savings bank**, because the savings bank will pay interest upon small sums of money that are left for the *interest term*, which is commonly six months.

Banks of deposit do not usually pay interest on deposits unless the sums left with them are large.

432. The chief business of banks of deposit and discount, variously styled **commercial banks, national banks, state banks, private banks**, etc., is the receiving of deposits for safe keeping and the lending of money in various ways, as by buying notes at a discount. They also perform various financial services for business men and corporations.

433. National banks do business under the authority of national law. Besides the ordinary banking business, they issue demand notes, payable to bearer, called **bank notes**, or **bank bills**, which circulate as money.

434. A **trust company** is authorized under state law and is usually similar to a bank so far as loans and deposits are concerned, but it does other business peculiar to itself.

Trust companies, being restricted less than banks in regard to the kind of investments they may make, are usually able to pay interest on deposits. They do not issue bank notes.

435. Money that is deposited in a bank or trust company is usually payable to the depositor or his order on demand. The demand is made by a written order called a **check.**

Stub	Check
No. 460	No. 460 Omaha, Neb., Jan. 16, 1906.
Date Jan. 16, '06.	Second National Bank OF OMAHA
Payable to	
John Green	Pay to the order of John Green
For Mdse.	Forty-eight and $\frac{25}{100}$ Dollars.
Am't. \$48.^{25}	\$48.^{25} Samuel J. Slawson.

The terms **maker, payee, face, negotiable,** and the different kinds of **indorsement,** etc., apply to checks the same as to promissory notes.

To get this check cashed at the bank John Green, the payee, must indorse it, and then it will not be cashed unless he is known at the bank, or is identified by some one who is known there.

436. The **stub** that remains in the *check book* after the check has been torn out gives a complete record of the check.

After the check has been cashed at the bank, it will be canceled and later returned to Mr. Slawson. It then serves as a **receipt** from John Green since it has his name on the back of it.

437. When a depositor wishes to draw money for himself from his bank, he may write a check payable to the order of "Self," in which case he must indorse it before he can get it cashed; or, he may make it payable to the order of "Cash," in which case no indorsement is necessary.

438. A check payable to the payee "or bearer" is usually paid to any one presenting it at the bank on which it is drawn, if the bank is sure that the maker's signature is genuine.

439. Opening a Bank Account. — A person wishing to open an account at a bank takes his money there and leaves it with the "cashier" or "receiving teller" together with a **deposit slip** properly filled out.

He is then given a small book with the amount credited in it. He is usually furnished with a book of blank checks.

In order that the bank may have his true signature to refer to at any time, he is required to sign his name in a book of signatures.

When a stranger to the bank officials wishes to open an account, he usually has to be introduced by some reliable person or else he must furnish references.

DEPOSIT SLIP

DEPOSITED BY

Samuel J. Slawson

IN THE

Second National Bank

Omaha, Neb., Jan. 1, 1906.

Bills	$40	
Coin	14	75
Check on 1st Nat. Bank	76	17
" " Corn Exch. "	22	38
Total	153	30

A deposit slip is handed in each time a deposit is made, and the amount is credited in the depositor's bank book, which he brings with him.

WRITTEN EXERCISES

440. 1. A man's balance at the First National Bank was \$846.20. Find his balance after checking out \$45, \$3.75, \$75.50, \$13.62, \$175.25, and \$126.10.

2. On the first of April a man's balance at the Franklin National Bank was \$142.34. Apr. 3 he deposited \$112.50; Apr. 10 he deposited \$60.40 and a check for \$18.75; Apr. 17 he deposited a check for \$65; Apr. 24 he deposited \$92; Apr. 30 he drew out \$125. Find his balance in the bank on the first of May.

3. Henry Brown's balance at the bank May 1 was \$3472.38. During the month he deposited \$84.60, \$250, \$36.22, \$80.01, \$72.35, \$39.17, \$24.64, \$7.77, \$4.65, and \$96, and drew out \$125, \$36, \$72.50, \$18.10, \$23.90, \$500, and \$61.37. What was his balance June 1?

4. A man's balance in the bank at the beginning of a week was \$196.50. During the week he deposited \$234.60, \$490.75, \$325.50, \$416, \$325, and \$410.75, and paid by check a bill of goods for \$1250 less 3%, another for \$98.50 less 2%, and \$375 for store rent. Find his balance at the end of the week.

5. Suppose that you sell Charles Raymond of your city an automobile for \$1850 less 5% for cash, receiving his check on the Second National Bank for the amount. Write the check, supplying the necessary details.

6. Indorse the check in blank and deposit it to your credit in the Bank of Commerce, together with \$75 in bills and \$16.63 in coin, filling out a deposit slip in due form.

7. Draw against your deposit in the Bank of Commerce, check No. 1 for \$125, payable to "cash"; draw check No. 2 for \$80, payable to the order of "self" and indorse it; draw check No. 3 for \$83.65, payable to the order of Frank A. Culver.

8. Write the stub record of each check, the first two being for personal expenses and the third for automobile repairs.

9. Suppose that Frank A. Culver applies your check No. 3 in part payment of his bill at Carroll Brothers' grocery, and that Carroll Brothers deposit it in the Bank of Commerce. Write the indorsements that the check will show.

10. John Dixon pays George White \$37.50 per month for rent by check on the Mercantile Bank. Supplying the date and place, write one of these checks in such a form that Mrs. White, who is not known at the bank, may cash it there.

BANK DISCOUNT

441. Lending Money. — Besides lending that part of its own capital that is not needed for other purposes, a bank of deposit and discount is able to lend a large part of the money that is deposited with it, for it is not likely that the depositors will all want the whole of their money at the same time.

442. Borrowing Money. — A great part of the capital employed in business is borrowed. This money is largely obtained from banks. Banks lend for the most part on notes or on some other form of negotiable paper. As a rule the names of at least two persons are required on the paper; one person will be the borrower, the other, a person who becomes responsible with the borrower for the payment of the loan, by *indorsing* the note.

443. If Mr. Ford wishes to borrow money at a bank and Albert Ross agrees to indorse his note, Mr. Ford may make out a promissory note as follows:

$1000.00		*Philadelphia, Pa., Aug. 8, 1906.*
Ninety days after date, for value received, I promise to pay to the order of Albert Ross		
One thousand $\frac{00}{100}$ Dollars		
at The Merchants' National Bank.		
No. 73.	*Due*	*John Ford.*

When Albert Ross has endorsed the note, John Ford may get money on it at the bank, if the officials consider both men reliable. He will not, however, get the whole sum of $1000. The bank will take out *interest in advance* for the time the note has yet to run and will give Mr. Ford the balance.

444. Simple interest, collected in advance, upon the sum due on a note at maturity is called **bank discount.**

When the note bears interest, the bank discount is reckoned on the *amount* of the note at maturity.

445. The sum due on a note at its maturity less the bank discount is called the **proceeds** of the note.

446. The *number of days* from the time when a note is discounted to the time when it legally matures is called the **term of discount.**

In states that allow days of grace care should be taken to include them.

WRITTEN EXERCISES

447. 1. Find the date of maturity, the term of discount, the bank discount, and the proceeds of the note in § 443, if it was discounted Sept. 16, 1906, at 6 %.

Solution

The **date of maturity** is 90 days after Aug. 8, 1906, which is found to be Nov. 6, 1906.

The **term of discount** is from Sept. 16, 1906, to Nov. 6, 1906, or 51 days (see table, page 234).

The **bank discount** is the interest on $1000 for 51 days at 6 %, or $8.50.

The **proceeds** = $1000 – $8.50 = $991.50.

NOTES. — 1. If the time given in the note were "three months" instead of "ninety days," the date of maturity would be Nov. 8, 1906, instead of Nov. 6, 1906, and the other answers would be changed accordingly.

2. In some states the term of discount would include both Sept. 16 and Nov. 6, making the term 52 days instead of 51 days.

3. In states that allow days of grace, the date of maturity would be 3 days later and the term of discount 3 days longer.

Local customs should govern in all these matters. The answers in this book, however, are based on the method shown in the above solution.

2. Find the date of maturity, the term of discount, the bank discount, and the proceeds of a 60-day note for $5000, without interest, dated Mar. 4, discounted Mar. 7 at 6 %.

Find the date of maturity, the term of discount, the bank discount, and the proceeds of the following:

3. A 30-day note for $5000, without interest, dated Aug. 27, discounted Aug. 27 at 6%.

4. A 15-day note for $10,000, without interest, dated May 1, discounted immediately at 5%.

5. A 90-day note for $7500, without interest, dated Nov. 18, discounted Nov. 19 at 6%.

6. A 10-day note for $18,000, without interest, dated July 28, discounted the same day at 5%.

7. A 120-day note for $3200, without interest, dated Sept. 30, discounted the same day at 7%.

8. A 60-day note for $840, with interest at 6%, dated Oct. 12, discounted Nov. 2 at 6%.

9. A 30-day note for $3000, with interest at 5%, dated July 3, discounted July 5 at 6%.

10. A 45-day note for $60,000, with interest at 4%, dated Dec. 28, discounted Jan. 2 at 5%.

11. A 90-day note for $8200, with interest at 6%, dated Feb. 1, 1907, discounted the same day at 7%.

12. A 60-day note for $25,000, with interest at 6%, dated Feb. 1, 1908, discounted Mar. 1 at 4%.

13. A note for $4000 on interest for 4 months at 6%, dated Jan. 1, 1907, discounted Jan. 10, 1907, at 6%.

14. A 60-day note for $75,000, without interest, dated Apr. 7, discounted Apr. 7 at 3%.

15. A note for $7200 on interest for 3 months at 6%, dated July 17, discounted Sept. 1 at 4%.

16. A 10-day note for $1000, discounted at $3\frac{1}{2}$% on the day it was made.

17. The following represents a bank's purchases of notes on the first of November:

Note for	Dated	Due	Interest Rate	Discount Rate	Cost of Note
$3000	Nov. 1	Dec. 1	4%	$4\frac{1}{2}$%	$ . . . .
4200	Oct. 31	Nov. 30	5%	$4\frac{1}{2}$%	. . . .
5000	Nov. 1	Dec. 1	none	$4\frac{1}{2}$%	. . . .
6400	Oct. 28	Dec. 27	none	5%	. . . .
5000	Oct. 30	Nov. 9	none	4%	. . . .
7500	Oct. 31	Nov. 15	5%	4%	. . . .
12,000	Nov. 1	Jan. 30	none	6%	. . . .

Find the amount invested in notes that day.

18. Find the face of a 60-day note that, discounted at 6% on the day it is made, will realize $9900 in cash.

Suggestion.—The proceeds of $1 discounted for 60 days at 6% = $.99.

19. For what sum must I draw my note for 60 days to obtain $2975 in cash, if the rate of discount is 5%?

20. For what sum must a 90-day note be drawn to realize $19,775 in cash when the rate of discount is $4\frac{1}{2}$%?

SAVINGS BANK ACCOUNTS

448. Savings banks pay the depositors compound interest, compounding it either monthly, quarterly, or semiannually.

449. The interval between the dates at which interest is paid is called the **interest term.**

Quarterly interest terms begin Jan. 1, Apr. 1, July 1, and Oct. 1; semiannual terms begin Jan. 1 and July 1, or Apr. 1 and Oct. 1.

450. The bank books of depositors in savings banks must be presented both when deposits are made and when money is drawn out, the amounts being credited or charged as the case may be. Books should also be presented at the bank at the end of each interest term to have interest credited.

451. The most general custom with savings banks is to credit interest, at the end of every interest term, on the *smallest balance* on deposit during the entire term.

Usually no interest is computed on the cents of the balance.

Custom varies greatly with different banks and in different localities. Frequently interest is allowed for the rest of the term on deposits made on the first (or within a few days of the first) of any month of the term. On the other hand, some banks subtract withdrawals from the balance at the beginning of the term and credit interest only on the difference obtained, which may be less than the smallest balance during the term.

452. The following is an illustrative statement of deposits and withdrawals with interest compounded quarterly at 3% per annum.

STATEMENT

DATE	DEPOSITED		DRAWN OUT		INTEREST		BALANCE	
1905								
Dec. 15	320						320	
1906								
Jan. 1							320	
Feb .6			75				245	
Apr. 1					1	83	246	83
June 10	40						286	83
July 1	25				1	84	313	67
Sept. 12			50				263	67
Sept. 20	125						388	67
Oct. 1					1	97	390	64

The statement shows that the deposit of Dec. 15, 1905, did not begin to draw interest until Jan. 1, 1906, and the interest for the quarter from Jan. 1 to Apr. 1 was computed on the *smallest* balance for the quarter, namely, $245. Similarly, $1.84 is the interest on $246 for 3 months, and $1.97 is the interest on $263 for three months.

NOTE. — The fractional part of a cent in the interest is usually dropped. Local customs, in regard to all points concerning which the usage of banks differs, should be followed. The answers in this book, however, are based on the method of solution just given.

WRITTEN EXERCISES

453. Arrange the following as in a savings bank book, and find the balance due Jan. 1, 1907:

1. Interest quarterly (Jan. 1, Apr. 1, etc.) at 4 %.

Deposits: Aug. 8, 1905, \$425; Oct. 2, 1905, \$100; June 30, 1906, \$150; Sept. 3, 1906, \$80.

Withdrawals: Sept. 18, 1905, \$35; Mar. 19, 1906, \$250.

2. Same as exercise 1, except interest semiannually (Jan. 1 and July 1).

3. Interest quarterly (Jan. 1, Apr. 1, etc.) at 4 %.

Deposits: Jan. 30, 1906, \$85; Feb. 18, 1906, \$45; Mar. 8, 1906, \$100; Mar. 29, 1906, \$50; June 2, 1906, \$125.

Withdrawals: Apr. 13, 1906, \$60; Sept. 1, 1906, \$80.

4. Same as exercise 3, except interest semiannually (Jan. 1 and July 1) at 3 %.

5. Same as exercise 3, except interest semiannually at $3\frac{1}{2}$ %.

6. Interest semiannually (Jan. 1 and July 1) at 3 %.

Deposits: Jan. 2, 1906, \$250; Jan. 15, 1906, \$75; Feb. 3, 1906, \$125; Mar. 12, 1906, \$175; June 4, 1906, \$60.

Withdrawals: Feb. 28, 1906, \$10; Apr. 2, 1906, \$50.

7. Interest quarterly (Jan. 1, Apr. 1, etc.) at $3\frac{1}{2}$ %. Balance, July 2, 1906, \$147.12.

Deposits: July 7, 1906, \$15.50; Sept. 12, 1906, \$25; Oct. 12, 1906, \$60; Nov. 10, 1906, \$22.50.

Withdrawals: Oct. 2, 1906, \$16; Dec. 12, 1906, \$35.

8. Interest semiannually (Jan. 1 and July 1) at $3\frac{1}{2}$ %.

Deposits: Jan. 2, 1902, \$400; Mar. 6, 1902, \$200; Aug. 5, 1902, \$150; Jan. 2, 1903, \$125; May 6, 1903, \$85; Oct. 1, 1903, \$110; Jan. 15, 1904, \$75; Sept. 23, 1904, \$120; Mar. 4, 1905, \$60; July 1, 1905, \$50; Dec. 2, 1905, \$12; Feb. 1, 1906, \$44; July 2, 1906, \$36.

Withdrawals: None.

EXCHANGE

454. Paying debts or collecting credits in distant places without actually transferring money is called **exchange.**

455. The various methods of exchange explained in the following pages are by **postal money order**, by **express money order**, by **bankers' association money order**, by **telegraphic money order**, by **check**, by **bank draft** (similar to a check), and by **foreign bill of exchange** (similar to a draft).

456. Exchange between two places in the same country is called **domestic**, or **inland, exchange**; exchange between two places in different countries is called **foreign exchange.**

DOMESTIC EXCHANGE

457. A **postal money order** is an order made by the postmaster in one place on the postmaster in another to pay to the person named therein a specified sum of money. It is *negotiable.*

In addition to the *face*, money orders cost as follows:

For orders for sums not exceeding	\$ 2.50	...... 3 cents.
Over \$ 2.50 and not exceeding	\$ 5.00	...... 5 cents.
Over \$ 5.00 and not exceeding	\$ 10.00	...... 8 cents.
Over \$10.00 and not exceeding	\$ 20.00	......10 cents.
Over \$20.00 and not exceeding	\$ 30.00	......12 cents.
Over \$30.00 and not exceeding	\$ 40.00	......15 cents.
Over \$40.00 and not exceeding	\$ 50.00	......18 cents.
Over \$50.00 and not exceeding	\$ 60.00	......20 cents.
Over \$60.00 and not exceeding	\$ 75.00	......25 cents.
Over \$75.00 and not exceeding	\$100.00	......30 cents.

NOTE. — The maximum amount for which a single order may be issued is \$100. When a larger sum is to be sent, additional orders must be obtained.

458. An **express money order** is similar to a postal order, the rates being the same. It is *negotiable.*

459. Telegraphic money order. — Telegraph companies make transfers in small amounts between their offices, subject to the following charges: 1% on all sums of \$25 or over, and 25¢ for any smaller sum, a further charge being added for telegraphic service not to exceed double the usual rate for a 10-word message between the two places.

Banks and express companies transfer for their customers by telegraph when haste is necessary.

460. Exchange by check. — A personal check drawn on a bank in which the drawer has money deposited may be sent by mail. The payee gets it cashed at some bank and this bank collects it from the bank on which it was drawn, sometimes charging the payee a small fee, called **exchange**, for collecting.

461. Exchange by bank draft. — The following illustrates the ordinary **bank draft**, or **bill of exchange**, which is simply a check drawn by one bank on another:

No. 522

First National Bank

Fort Wayne, Ind., Dec. 7, 1906.

Pay to the order of Henry Adams .... *\$ 500.00*

Five hundred $\frac{00}{100}$ ~~~~~~~~~~~~~~~~ *Dollars.*

To **Commercial Bank**
NEW YORK CITY

Ernest Rayburn,
Cashier.

The person who signs a draft is the **drawer**; the one who is directed to pay it, the **drawee**; and the one to whom it is to be paid, the **payee.**

462. Banks usually keep money on deposit in some bank, called a **correspondence bank**, in a large money center.

The draft shown on the previous page is an order on the Commercial Bank of New York to pay Henry Adams or his order $500, and to charge the amount to the account of the First National Bank of Ft. Wayne. By indorsing it Mr. Adams may get it cashed at any bank, which in turn may collect the cash from the New York bank or, what is more likely, deposit the draft as a credit to draw on in the New York bank. The latter may charge it to the Ft. Wayne bank's account, if it has one, or it may collect of the Ft. Wayne bank, or deposit the draft there as a credit. In the end, however, the Ft. Wayne bank must pay $500 in credit or money, though in the meantime it has had the use of the $500 paid in by the original purchaser, and has earned a fee charged for exchange.

463. **Bankers' association money orders** are bank drafts drawn by certain banks on a circuit of banks located in important money centers.

464. **Collecting by draft.** — Suppose that W. C. Atkins of Chicago owes John Best of Boston $500 on account, and that Mr. Best wishes to collect the debt at once. He may draw on Mr. Atkins by means of a **commercial draft**:

$\$500\frac{00}{100}$ *Boston, Mass., Jan. 6, 1906.*

-------*At sight pay to the order of* Merchants' Bank of Boston

Five hundred $\frac{00}{100}$ ~~~~~~~~~~~~~~~~~~~~ *Dollars.*

Value received, and charge to account of

To W. C. Atkins, *John Best.*

25 Federal St., Chicago, Ill.

The Merchants' Bank of Boston cashes this draft and sends it to some Chicago bank for collection. The Chicago bank either collects of Mr. Atkins and remits to the Boston bank, or if payment is refused, returns the draft to the Boston bank, which in turn notifies Mr. Best to "take up

the draft" and refund the money advanced on it. Some banks would not have cashed the draft until collected. In either case charges for collection would be deducted from the face of the draft.

Mr. Best might make the paper payable to "Self." It would then be merely a demand for payment, but would become a **sight draft** by his indorsement.

465. Suppose that Mr. Atkins' debt were not due for sixty days. Instead of "at sight" the draft would read "sixty days after sight." Such a commercial draft is called a **time draft.**

When presented to Mr. Atkins, if he intended to pay it, he would write across the face in red ink "Accepted" with the date, and sign his name underneath. The draft is then called an **acceptance** and is equivalent to a promissory note, due sixty days after the date when it was accepted.

An acceptance may be sold by indorsing it and discounting it for the time it has to run. In discounting, banks sometimes add several days to the discount term to allow for time to collect; they may also charge exchange.

Time drafts are sometimes made payable at a given time *after date*. Then the acceptor does not write the date of acceptance.

466. Exchange Market. — Bills of exchange are credits, and when indorsed may be bought and sold. The business of buying and selling them is carried on largely by banks and in the open market. Hence, as with any other commodity, when the supply offered on the market exceeds the demand, the price goes down and exchange is at a **discount,** or **below par**; but when the demand exceeds the supply, the price goes up and exchange is at a **premium,** or **above par.**

Thus, when the Chicago banks have large sums on deposit in New York banks and wish to use part of their money at home, they will sell sight drafts on New York at favorable rates, probably at par for small amounts, and slightly below par for large amounts.

When the deposits of Chicago banks in New York banks are not sufficient to meet the demands made upon them for New York drafts, they are obliged to send money to New York by express at considerable expense and they must charge something above par for their New York drafts.

467. How Exchange is Quoted. — The price of domestic exchange (for sight drafts) is quoted either at par, at a certain per cent premium or discount, or at a certain amount of money, premium or discount, for $1000.

The following, taken from a New York newspaper, are market quotations for exchange on New York at:

Boston	10 ¢ discount.
Chicago	10 ¢ premium.
New Orleans . .	commercial, par; bank, $1 premium.
Savannah . . .	buying, 50 ¢ discount; selling, 75 ¢ premium.
San Francisco .	sight, 3½ ¢ premium; telegraph, 7½ ¢ premium.

EXERCISES

468. 1. At the rates quoted above, how much will New York exchange for $5000 cost in Boston? in Chicago? In which city are the banks more eager to sell New York exchange?

2. Find the cost of a draft on a New York bank for $10,000, in New Orleans; in Savannah.

3. A cotton dealer in Savannah sold some cotton to a New York firm and drew on them at sight for $12,000. A Savannah bank bought the bill of him at a *discount* of 50¢ per $1000. How much did the bank pay for the bill?

4. At this time the Savannah bank had very little money in New York to draw against. Selling bank drafts on New York meant expressing money to meet them. Consequently, the bank charged a merchant who applied for a draft for $12,000, a *premium* of 75¢ per $1000. Find the selling price of this draft.

Find the cost of a postal or an express money order for:

5. $3.10	7. $ 5.75	9. $21.80	11. $50	13. $62.75
6. $8.50	8. $11.20	10. $17.65	12. $66	14. $99.80

15. How much will it cost Mr. Drew to telegraph $200 to his son in New Haven, the rate for telegrams being 25–2?

WRITTEN EXERCISES

469. 1. Find the cost of a draft for \$750 at $\frac{1}{10}\%$ premium.

2. What will be the cost of a draft for \$125,340 when exchange is at $\frac{1}{15}\%$ discount?

3. A. T. Stewart of Kalamazoo draws on W. Wetmore of New York for \$3500. The bank in Kalamazoo charges $\frac{1}{8}\%$ for collection. Find the proceeds of the draft.
Write the draft, supplying necessary details.

4. What will be the proceeds of a draft for \$2000, payable 30 days after date, discounted at 6%, if 6 days are allowed for the collection and the return of the money?

5. The Cleveland Electric Co. sent to Chas. Avery, their salesman in Nashville, a New York draft on the First National Bank for \$200, dated June 27. The draft was cashed for Mr. Avery by the proprietor of a hotel, James Hanlon, who deposited it in the Cotton Exchange Bank. This bank forwarded it to the First National Bank of New York as credit. Write the draft and its indorsements.

6. Find the proceeds of a 60-day draft for \$1885.50, discounted at 7% for the full term, if the bank charges 80¢ in addition for collection.

7. C. M. Clay of Charleston sold 200 barrels of tar @ \$1.80, 50 casks of crude turpentine @ \$3.20, and 250 barrels of D resin, @ \$2.05, to Hines & Co., New York. Hines & Co. remitted with the order a sight draft for \$275 on Colfax & Son, Charleston, and authorized C. M. Clay to draw at 60 days' sight for the balance. Write both drafts, supplying suitable dates.

8. C. M. Clay discounted the second draft for 63 days at 6% and was charged $\frac{1}{32}\%$ of the face of the draft for exchange. Find the net proceeds of the shipment.

9. A speculator in San Francisco sent his New York broker \$4000 by telegraph at $7\frac{1}{2}$¢ premium. The message cost \$1.35. Find the total cost.

10. A commission firm in Louisville received a car load of oranges, 396 boxes, from a dealer in New Orleans, paying \$77 freight charges. The firm sold 200 boxes @ \$2.50 and the rest @ \$2.40. They remitted the proceeds, after deducting freight charges, 2% commission, $2\frac{1}{2}$¢ per box for storage, and $\frac{1}{8}$% on \$1000 insurance, by New York exchange at par. What was the face of the draft?

11. A Cincinnati pork packer received an order from a New York customer for 75 barrels of mess pork @ \$13.25, freight charges paid to New York. He shipped the pork, drew at sight on his customer for the whole amount less \$54.64 for freight charges and insurance, and sold the draft at the bank at 30¢ discount. Find the net proceeds.

12. A merchant in Minneapolis received an order from a New York firm for 800 barrels of flour @ \$3.25 on the cars at Minneapolis, terms 60 days' sight draft. He shipped the flour. On receiving the New York firm's acceptance, he discounted it at the bank for 66 days, at 6%, and was charged 15¢ per \$1000 of the face of the acceptance for exchange. Find the net proceeds.

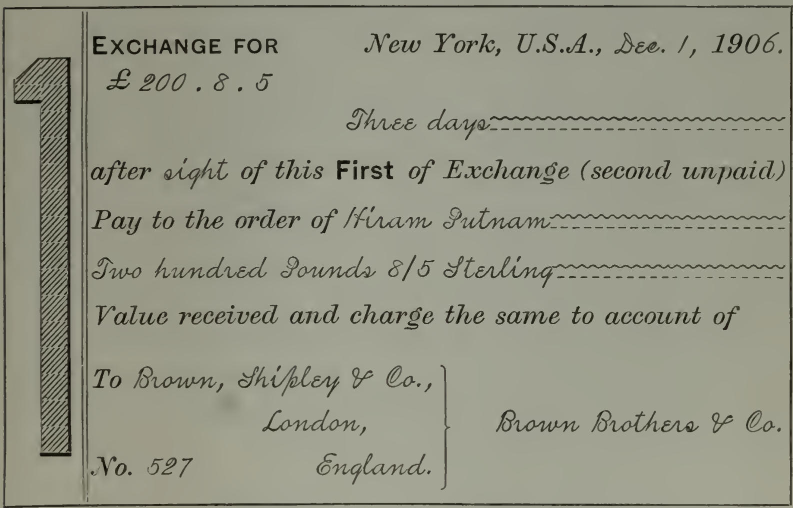
13. Express companies and the U. S. subtreasury charge 75¢ per \$1000 to transfer money from New York to New Orleans, 50¢ to St. Louis, Chicago, and the West, and 25¢ to near-by Eastern cities. One week in October when money was in demand for moving the crops, New York banks transferred through these agencies \$3,142,000 to New Orleans and the South, \$7,495,000 to St. Louis, \$3,280,000 to Chicago and the West, and \$3,746,000 to near-by Eastern cities. What was the total cost of the week's exchange?

FOREIGN EXCHANGE

470. Foreign exchange does not differ in principle from domestic exchange.

A **foreign bill of exchange** is similar to a bank draft and is payable in the money of the country on which it is drawn. **Commercial drafts** are also drawn and accepted as in domestic exchange.

471. Foreign bills of exchange are usually written in duplicate, called a **set of exchange**, illustrated as follows :


1

EXCHANGE FOR *New York, U.S.A., Dec. 1, 1906.*
£ 200 . 8 . 5

Three days

after sight of this **First** *of Exchange (second unpaid)*

Pay to the order of Hiram Putnam

Two hundred Pounds 8/5 Sterling

Value received and charge the same to account of

To Brown, Shipley & Co.,
London,
No. 527 England.

Brown Brothers & Co.

In the duplicate, "**Second** of exchange (first unpaid)" is substituted for "**First** of exchange (second unpaid)," and "2" for "1" in the left margin. When either one of the set is paid, the other becomes void.

472. Par of exchange between two countries is the value of the monetary unit of one expressed in that of the other.

The table in § 238 gives the par of exchange in the United States, on England, France, and Germany.

473. How foreign exchange is quoted. — Exchange on :

England, by giving the cost of a bill of exchange for £1; thus, 4.87 means that a bill for £1 costs $4.87.

France (and other countries using the same monetary system), by giving the number of francs of exchange that can be bought for $1: thus, 5.18 means that 5.18 fr. can be bought for $1.

Germany, by giving the number of cents that 4 marks of exchange cost; thus, $94\frac{5}{8}$ means that 4 marks cost $94\frac{5}{8}$¢.

Newspapers usually give exchange rates for cable transfers, demand bills, and sixty-day bills, thus:

	CABLES	DEMAND	60 DAYS
Sterling	$4.86\frac{1}{4}$	$4.85\frac{1}{4}$	4.82
Francs	$5.16\frac{7}{8}$	$5.18\frac{1}{8}$	$5.20\frac{5}{8}$
Marks	$95\frac{1}{4}$	$95\frac{1}{8}$	$94\frac{1}{2}$

474. Foreign exchange for small amounts is usually effected by means of **international** express and postal **money orders, at** fixed rates.

475. Letters of credit. — A person intending to travel abroad may deposit funds with an international banking house to draw against, and receive a **letter of credit** guaranteed by the bank.

The purchaser signs several signature blanks, one of which is sent to each correspondent bank. When he presents the letter at any one of these banks he is asked to sign a draft or check for the amount he wishes to draw. The cashier compares the signature with that on the signature blank, and if they correspond, the money is paid and charged on the letter, which is returned to the owner.

Letters of credit are usually written in sterling money, the holder paying London exchange when he purchases it, and a further exchange when he draws in any other country than England.

476. Travelers' checks. — These are guaranteed checks issued in denominations of $10, $20, $50, $100, and $200, by banks and express companies, at a fixed rate of $\frac{1}{2}\%$ of the face value.

The purchaser signs them on the face when purchased, and on the face or back when cashed. Identification is by comparison of signatures. These checks are readily received at European hotels, railroads, and business places.

WRITTEN EXERCISES

477. **1.** Find the cost of a cable transfer of 265 fr. at $5.16\frac{7}{8}$.

SUGGESTION. — One franc costs $\$1 \div 5.16\frac{7}{8}$.

2. How much will a 60-day bill for 250 M. cost at $94\frac{1}{2}$?

SUGGESTION. — One mark costs $\frac{1}{4}$ of $\$.94\frac{1}{2}$.

3. Find the cost of a demand bill for £ 75 6*s.* 4*d.* at $4.85\frac{1}{4}$.

SUGGESTION. — See exercise 11, page 141.

Find the cost, to the nearest cent, of exchange for :

4. £ 1200 @ 4.8365

5. £ 1525 @ 4.8420

6. £ 95 12*s.* @ $4.87\frac{1}{4}$

7. £ 225 10*s.* 2*d.* @ $4.83\frac{5}{8}$

8. 10,000 fr. @ 5.18

9. 25,000 M. @ $95\frac{1}{16}$

10. 1224.75 fr. @ $5.19\frac{7}{8}$

11. 4520.65 M. @ $94\frac{3}{4}$

12. A cotton exporter in Mobile drew a 60-day bill on London for £ 1075 against a shipment of 100 bales of cotton, and sold the bill at the bank at $4.83\frac{7}{8}$. Find the proceeds.

13. An American sent to his family in Milan a bill for 2000 lire, exchange at 5.19. What was the cost of the bill?

14. Find the cost of £ 5 at 4.87, sent to Dublin at Christmas time by international money order, the fee being 30 ¢.

15. Find the cost of travelers' checks for \$4000, at $\frac{1}{2}$ %.

16. The value of these checks in foreign money was according to the following values, printed on a 20-dollar check :

£ s. d.	FRANCS	MARKS	LIRE	CROWNS	FLORINS
4 1 2	102.50	82.50	102.50	73.39	49.02

The purchaser cashed \$700 in checks in England, \$900 in France, \$400 in Germany, \$200 in Denmark (crowns), \$500 in Belgium (francs), \$200 in Holland (florins), and \$600 in Italy. Find the amount of money of each kind that he received and the amount refunded to him at the end of the trip.

17. An importer purchased a sight draft on London for £15,000 at $4.86\frac{1}{2}$. How much did it cost him?

18. Suppose that he had purchased the draft through a broker and had obtained a slightly lower rate, 4.8640. The brokerage being $5 per £10,000, how much less would the draft have cost him?

19. Find the proceeds of a bill for 25,600 marks sold through a broker at $94\frac{1}{2}$, brokerage $\frac{1}{64}\%$.

20. A firm in Newark drew on a firm in Rio Janeiro at 60 days' sight for £25,000, representing a shipment of machinery, and sold the bill through a New York broker at $4.83\frac{7}{8}$, brokerage $5 per £10,000. What were the proceeds?

21. A New York bank contracted with cotton shippers in the South for the purchase of £100,000 of bills of exchange for cotton at $4.81\frac{1}{2}$, and deposited them with its London correspondent for collection. The London bank charged $\frac{1}{2}\%$ for collection and credited the proceeds to the New York bank. The New York bank drew demand bills against this credit and sold them in the open market at $4.85\frac{7}{8}$. Find the profit.

22. On Nov. 24, when money was worth 5%, a banker invested £10,000 in 60-day bills on London at 4.80. How much interest could he have obtained for this money, if he had not bought the drafts?

23. He sent the bills to London for acceptance. They were accepted and returned to him. He held them until they became due, when he sold them as demand bills at 4.87. How much did he gain?

24. A grain shipper sold 22,500 centals of wheat (1 cental = 100 pounds) at $1.60 per cental to a merchant in Havre, France, and drew on him for the amount at $5.20\frac{1}{4}$. Find the face of the bill.

STOCKS AND BONDS

478. Corporations. — When a number of persons wish to engage in a business requiring a large capital, they usually raise as much capital as they can among themselves, perhaps soliciting subscriptions from others, and organize a **stock company**, or **corporation**.

A corporation is authorized under the law; has certain powers and privileges; is subject to certain limitations; and is regarded by the law as a single person engaged in a stated business with a declared capital, or **stock**.

479. Shares. — The stock of a corporation is divided into equal parts, called **shares**.

A share is usually $100, but it may be more or less than that sum; unless stated to the contrary the face value of a share, in this book, means $100.

If the capital of a corporation is $400,000, it is divided, perhaps, into 4000 shares of $100 each.

480. Stockholders. — A person who becomes the owner of one or more of these shares is called a **shareholder**, or a **stockholder**.

Corporations are managed by officers elected by the stockholders, a stockholder having one vote for each share he owns.

A stockholder's liability for the debts of the corporation is usually limited to the original value of the stock he owns.

Stockholders of national banks, however, are responsible for *double* the original value of their stock.

481. Certificates of Stock. — Every stockholder receives a *certificate* showing the number of shares to which he is entitled and the *original*, or **par value**, of each.

482. Dividends. — When the business has been successful, the profits, after all expenses are paid and a working surplus is laid aside, are divided among the stockholders according to the number of shares each holds. This is called a **dividend.** It is usually a certain per cent of the par value of the stock.

Thus, if the capital stock is $100,000 and the net earnings are $5000, a 5 % dividend may be declared, and a man who owns stock to the par value of $1000 will receive $50 as his part of the profits.

483. Assessments. — If the business has been unsuccessful, the dividend may be omitted, or *passed;* and sometimes the stockholders may be required to make up the deficiency by an **assessment.**

484. Corporations often issue two kinds of stock, called **preferred** and **common.**

The holders of preferred stock are generally entitled to a fixed rate of dividend that must be paid before the holders of common stock are entitled to participate in the profits.

485. Market value. — Stock is said to be **at par**, when it is worth its face value in the market; **above par**, or **at a premium**, when it is worth more than its face value; and **below par**, or **at a discount**, when it is worth less than its face value.

When the dividends are large enough to pay stockholders good interest on their investment, the stock will usually be at or above par; otherwise below par.

486. A **corporate bond** is a corporation's formal written promise under seal to pay a certain sum of money to the purchaser, on or before a specified time, with interest at regular intervals at a fixed rate.

Corporate bonds must be secured by a **mortgage**, which is an agreement that the holder of the bond may sell the property of the corporation, if the bond is not paid.

487. Government bonds.—Bonds are issued by the United States, or other countries, by states, cities, counties, and other political divisions of a country, to raise money for various purposes of government.

Government bonds are not secured by mortgage.

488. When bonds are recorded by number and in the name of the person owning them, they are called **registered bonds**.

Registered bonds cannot be transferred without indicating the transfer in the records of the bonds.

489. Bonds to which interest certificates, called **coupons**, are attached, are called **coupon bonds.**

Coupon bonds are payable to the bearer. When interest is due, a coupon is cut off and presented for payment at a bank or elsewhere.

490. Stocks and bonds compared.—Stockholders of a corporation are owners of the property; bondholders are lenders to the corporation. Bonds bear interest at a fixed rate; the income from stocks depends upon the prosperity of the corporation and the rate of dividend declared.

491. How stocks and bonds are quoted.—The following quotations were found in a newspaper:

Stocks		Bonds	
Am. Locom.	$61\frac{3}{4}$	U. S. 3s reg.	103
Am. Locom. pf.	113	U. S. 4s coup., 1907. . . .	$103\frac{3}{8}$
N. Y., N. H., & H.	$194\frac{3}{4}$	Cen. Pac. 1st 4s	99

These quotations in order mean: American Locomotive common stock, at $61\frac{3}{4}$ % of its par value; American Locomotive preferred stock, at 113 % of par; New York, New Haven, and Hartford railroad common stock, at $194\frac{3}{4}$ %; United States registered bonds, paying 3 % interest, at 103 %; United States coupon bonds, paying 4 % interest and due in 1907, at $103\frac{3}{8}$ %; and Central Pacific railroad first mortgage bonds, paying 4 % interest, at 99 %.

492. A person whose business it is to buy and sell stocks and bonds for others is called a **stock broker**; the compensation he receives is called **brokerage**.

The customary brokerage, for 100-dollar shares, is $\frac{1}{8}$ % of the *par value* for buying and the same for selling.

Dealings in stocks are usually by "blocks" of 100 shares, but a smaller number of shares may be bought.

EXERCISES

493. 1. Which of the stocks and bonds quoted in § 491 are above par? below par?

2. Bonds are usually issued for $1000 each. From the quotations find the cost of one of the U. S. 3s reg.

3. The capital stock of a bank is $100,000. How many shares are there? How much of the capital do I own, if I have 12 shares? 30 shares? 50 shares?

4. If on a capital of $100,000 there is a net gain of $5000, what per cent of dividend may be declared?

5. If the semiannual dividend is $3\frac{1}{2}$ %, what amount of income do I receive yearly from 1 share? from 50 shares? from 100 shares?

6. What is the market value of 10 shares of stock when quoted at par? at 125? at 150? at 50?

7. If the par value of an industrial stock is $50, how much above par is it when a share sells for $75? for $100? How much below par, when it sells for $25? for $40?

8. A bank fails and the stockholders are assessed 25 %. How much must I pay, if I own 10 shares? 50 shares?

9. If a bank stock is quoted at 300, how much must be paid for 1 share? for 20 shares?

10. If I received $5 dividend on every share of a stock that I hold, what was the entire dividend declared on a capital of $100,000?

11. At $\frac{1}{8}$ % commission, what is the brokerage on the sale of 1 share of stock? 100 shares?

WRITTEN EXERCISES

494. **1.** Find the cost of 500 shares of C. M. and St. P. R.R. common stock at $163\frac{7}{8}$, brokerage at the usual rate.

SOLUTION

Brokerage per share $= \frac{1}{8}\%$ of \$100, or $\$\frac{1}{8}$
Total cost per share $= \$163\frac{7}{8} + \$\frac{1}{8} = \$164$
Then 500 shares cost 500 times \$164, or \$82,000.

2. How much must be paid, including brokerage, for 25 shares of Canadian Pacific railroad stock at $174\frac{5}{8}$?

3. How much will it cost me to buy 25 1000-dollar U. S. 3s reg., at $2\frac{3}{4}\%$ premium, brokerage $\frac{1}{8}\%$?

4. If U. S. Steel preferred is worth $106\frac{3}{8}$, how much will 100 shares of it cost me, including brokerage?

5. Find the cost, including brokerage, of 200 shares of Commercial Cable Co. stock at $192\frac{3}{8}$.

6. How much must be paid, including brokerage, for 500 shares of N. Y. Central R.R. stock at $137\frac{1}{2}$?

7. Find the cost of 500 shares of an industrial stock at $127\frac{3}{4}$, brokerage $\frac{1}{4}\%$, if the par value of a share is \$50.

8. A man purchased 100 shares of Republic Iron & Steel Co. pf. at $93\frac{1}{2}$ and sold them the same day at $95\frac{7}{8}$. Find his net gain, if he paid brokerage for both buying and selling.

9. A speculator bought 500 shares of railroad stock at $67\frac{1}{2}$ and sold them the same day at $68\frac{1}{4}$, in each case through a certain broker. How much did the broker receive? How much did the speculator gain?

10. A broker received an order to sell 200 shares of General Electric stock and to buy 400 shares of Illinois Central R.R. stock. With the order was a draft for \$7000. The broker sold the first at $166\frac{1}{2}$, bought the second at 170, and drew on his customer for the balance. Find the face of this draft.

11. When Baltimore & Ohio pf. is selling at 96, how many shares can be bought for $5767.50, brokerage at the usual rate?

SOLUTION

Total cost per share = $96 + $$\frac{1}{8}$ = $96.125
Number of shares = $5767.50 ÷ $96.125 = 60.

12. How many shares of Northern Pacific R.R. stock at $97\frac{1}{8}$ can be bought through a broker for $4862.50?

13. If Southern Ry. pf. stock can be bought at 100, how many shares of it can I buy through a broker for $8010?

14. How many shares of Pressed Steel Car stock pf. at $92\frac{3}{4}$ can be bought through a broker for $40,865?

15. I purchased 24 shares of Consolidated Gas stock at 138 and, after receiving one quarterly dividend of $1\frac{1}{2}$%, sold the stock at $141\frac{1}{2}$. How much did I gain, allowing brokerage for both buying and selling?

SOLUTION

Paid for 1 share, $138 + $$\frac{1}{8}$, or $$138\frac{1}{8}$
Paid for 24 shares, 24 times $$138\frac{1}{8}$, or $3315
Received for 1 share, $$141\frac{1}{2}$ – $$\frac{1}{8}$, or $$141\frac{3}{8}$
Received for 24 shares, 24 times $$141\frac{3}{8}$, or $3393
Dividend on 24 shares = $1\frac{1}{2}$% of $2400 = $36
Total amount received = $3393 + $36 = $3429
Gain = $3429 – $3315 = $114.

16. Mr. Brown bought 125 shares of N. Y. C. R.R. stock at $149\frac{1}{4}$, and after receiving one quarterly dividend of $1\frac{1}{4}$%, sold them at $152\frac{1}{2}$. Allowing brokerage in each case, how much did he gain?

17. My money in the savings bank was paying me $3\frac{1}{2}$% interest. I drew out $2002 and invested it in railroad stock at 125, paying the usual brokerage. This stock yielded a semiannual dividend of $2\frac{1}{2}$%. Was my semiannual income increased or diminished, and how much?

18. A man bought 300 shares of a certain stock at $87\frac{1}{2}$. Later he received 15 shares of the stock as a dividend on his 300 shares. Becoming alarmed because the dividend was not in cash, he sold his shares at $81\frac{7}{8}$. Find his loss, including brokerage for both buying and selling.

19. How much must be invested in Galveston Street Railway 5% bonds (denomination \$100) at 112, usual brokerage, to secure to the purchaser an annual income of \$1000?

SOLUTION

Since the income from each bond is \$5, it will take as many bonds to give an income of \$1000 as \$5 is contained times in \$1000, or 200 bonds.

At 112, each bond, including brokerage, will cost $\$112\frac{1}{8}$, and 200 bonds will cost 200 times $\$112\frac{1}{8}$, or \$22,425. Therefore \$22,425 must be invested to secure an annual income of \$1000.

20. What sum must be invested in Imperial Japanese 6s, at $99\frac{5}{8}$, to give an annual income of \$2400, allowing the usual brokerage?

21. What amount invested in Republic of Cuba 5s at $105\frac{3}{8}$, brokerage as usual, will secure an annual income of \$500?

22. What sum must I invest in American Hide & Leather 6s at $99\frac{7}{8}$, allowing brokerage, to secure an income of \$1200?

23. What per cent shall I make on my money, if I buy 4% bonds at 80?

SUGGESTION. \$4 is what per cent of \$80?

24. Find the per cent realized on an investment in 6% preferred stock at 75.

25. How much must I pay for 4% stock in order to realize 5% on my investment?

SUGGESTION. \$4 is 5% of what must be paid for a share.

26. How much must be paid for 6% stock to realize 5% on the money invested?

27. Find the cost of 3% stock when the money invested in it yields 8%.

28. A man receives a quarterly dividend of $50 on 25 shares of stock. Find the rate of dividend each quarter.

29. A company with a capital of $300,000 pays its stockholders $6000 quarterly. Find the quarterly dividend of a man who holds 40 shares of the stock.

30. During one year the net earnings of a company whose capital is $360,000 were $65,835. If it retained a surplus of $5835 and distributed the rest in dividends, how much was received by a man owning 60 shares?

31. What annual income will a man receive who invests $4835.25 in a 6% stock purchased at 115, brokerage at the usual rate?

32. A man bought at private sale 50 shares of a bank stock at 193, and afterward sold them through a broker at $191\frac{1}{2}$. How much did he lose?

33. How much must be paid, including brokerage, for a sufficient number of U. S. 4s at $133\frac{1}{2}$ to obtain an annual income of $1000?

34. At one time during the war between Japan and Russia, Japan borrowed a large sum of money by means of an issue of 6% bonds. They sold at $93\frac{1}{2}$. What rate of interest, to the nearest .01%, did Japan actually pay on a par basis?

35. Several years before the war Japan had sold an issue of 5% bonds at $103\frac{5}{8}$. Find the true rate of interest paid.

36. A stock company declared a dividend of $375,000 for one quarter. The stock consisted of $12,500,000 preferred and $12,500,000 common, the preferred stock being allowed 7% annually before the common stock could participate in the profits. Find the dividend of a stockholder who owned 50 shares of preferred and 10 shares of common stock.

RATIO AND PROPORTION

RATIO

495. **1.** Compare 15, 20, 25, and 30 with 5 in such a way as to tell how many times each contains 5.

2. What number expresses the relation, or *ratio*, of 15 to 5? of 20 to 5? of 12 to 3? of \$50 to \$10? of 32 qt. to 8 qt.?

3. What number expresses the ratio of 3 to 6? of 2 to 6? of 3 to 12? of \$10 to \$40? of 7 days to 28 days?

496. The relation of one number to another of the same kind, expressed by the quotient of the first divided by the second, is called the **ratio** of the first number to the second.

All ratios are abstract numbers.

Thus, the ratio of 15 to 3 is $15 \div 3$, or $\frac{15}{3}$, or 5.

497. The **ratio sign** is a colon (:).

Thus, the ratio of 15 to 3 may be written 15 : 3.

498. The first number in a ratio is called the **antecedent**, and the second number, the **consequent**.

$$3 \div 4 = \frac{3}{4} = 3:4 = \frac{\text{dividend}}{\text{divisor}} = \frac{\text{numerator}}{\text{denominator}} = \frac{\text{antecedent}}{\text{consequent}}.$$

The antecedent and consequent of a ratio are called its **terms**.

499. Since a ratio may be written in the form of a fraction, and since multiplying or dividing both terms of a fraction by the same number does not change the value of the fraction,

Multiplying or dividing both terms of a ratio by the same number does not change its value.

Thus, 4 : 8 reduced to lowest terms is 1 : 2; also 4 : 8 = 12 : 24.

EXERCISES

500. What is the ratio of

1. 14 to 7? 7 to 14? 18 to 6? 6 to 18?

2. 15 to 30? 12 to 60? 96 to 16? 100 to 20?

3. 2 to 1? 3 to 5? 7 to 2? 5 to 4? 17 to 3? 25 to 8?

4. 25 to 100? 40 to 100? 90 to 100? 120 to 100?

5. $12\frac{1}{2}$ to 100? $37\frac{1}{2}$ to 100? $62\frac{1}{2}$ to 100? $33\frac{1}{3}$ to 100? $112\frac{1}{2}$ to 100? 125 to 100?

6. 20 % to 1? 50 % to 1? 40 % to 1? $87\frac{1}{2}$ % to 1? 110 % to 1?

Reduce to lowest terms the ratios expressed by:

7. 10 : 2	**10.** 3 : 27	**13.** $\frac{12}{24}$	**16.** $75 \div 100$
8. 12 : 6	**11.** 4 : 40	**14.** $\frac{16}{48}$	**17.** $60 \div 120$
9. 16 : 4	**12.** 9 : 72	**15.** $\frac{60}{80}$	**18.** $80 \div 240$

19. What is the ratio of \$70 to \$100? of \$600 to \$1000?

20. What is the ratio of 15 days to 30 days? of 21 days to 1 week?

21. What is the ratio of 1 rod to 1 mile? of 1 ounce to 1 pound?

22. What is the ratio of a long ton to a short ton?

WRITTEN EXERCISES

501. Find the value of the following ratios:

1. $\frac{4}{5} : \frac{2}{5}$	**4.** .7 : .8	**7.** 10 : 4.5	**10.** 4 ft. : 16 in.
2. $\frac{3}{4} : \frac{1}{2}$	**5.** .4 : 10	**8.** 72 : 8.5	**11.** 2 yd. : 18 in.
3. $\frac{5}{8} : \frac{1}{4}$	**6.** 7.5 : 12	**9.** $2\frac{1}{2} : 7\frac{1}{2}$	**12.** 1 Km. : 62.5 m.

13. The capital stock of a street railway company was \$7,500,000, the gross earnings for a year \$1,500,000, and the net earnings \$600,000. Find the ratio of gross earnings to capital stock; of net earnings to gross earnings; of net earnings to capital stock. Then express each ratio in per cent.

14. In 1880, 17 % of the persons in the United States were illiterate; in 1890, 13.3 %; in 1900, 10.7 %. Express these per cents as ratios having 10,000 as a consequent.

SIMPLE PROPORTION

502. An equality of two ratios is called a **proportion.**

Thus, $4 : 6 = 8 : 12$ is a proportion; also $\frac{4}{6} = \frac{8}{12}$.

The double colon (::) is often used instead of the sign of equality.

The proportion $4 : 6 = 8 : 12$, or $\frac{4}{6} = \frac{8}{12}$, or $4 : 6 :: 8 : 12$ is read "the ratio of 4 to 6 is equal to the ratio of 8 to 12"; or more briefly, "4 is to 6 as 8 is to 12."

503. The first and last terms of a proportion are called the **extremes**, and the second and third terms the **means**.

In $4 : 6 = 8 : 12$, the extremes are 4 and 12; the means, 6 and 8.

504. 1. In the proportion $4 : 6 = 8 : 12$, what is the product of the extremes? of the means? How does the product of the extremes compare with the product of the means?

2. Compare the product of the extremes with the product of the means in $1 : 2 = 4 : 8$; in $1 : 2 = 5 : 10$; in $2 : 4 = 6 : 12$.

3. Form other proportions and compare the product of the extremes in each with the product of the means.

In the following compare the first ratio with the second to find whether or not they are equal; then compare the product of the extremes in each case with the product of the means.

4. Does $\frac{3}{6} = \frac{4}{8}$? $\frac{3}{6} = \frac{5}{10}$? $\frac{3}{6} = \frac{7}{14}$? $\frac{3}{6} = \frac{6}{12}$?

5. Does $2 : 4 = 5 : 10$? $3 : 6 = 9 : 18$? $4 : 8 = 5 : 10$?

6. Does $2 : 8 = 5 : 20$? $2 : 8 = 6 : 24$? $3 : 5 = 9 : 15$?

In any proportion the product of the extremes is equal to the product of the means.

WRITTEN EXERCISES

505. **1.** If $24 : 7 = 48 : x$, what is the value of x?

SOLUTION

$$\text{Product of extremes} = \text{product of means}$$
$$24x = 7 \times 48$$
$$x = 7 \times 2 = 14.$$

2. Solve the proportion $12 : 16 = x : 60$.

SOLUTION

Reducing $12:16$ to lowest terms, $3:4 = x:60$

$$\text{Product of means} = \text{product of extremes}$$
$$4x = 3 \times 60$$
$$x = 3 \times 15 = 45.$$

SUGGESTION. — Reduce ratios to lowest terms before solving.

Solve the following proportions:

3. $30 : 5 = 12 : x$

4. $10 : 5 = x : 7$

5. $15 : x = 3 : 6$

6. $15 : x = 6 : 2$

7. $15 : x = 10 : 2$

8. $5 : 20 = x : 32$

9. $\frac{1}{2} : \frac{3}{4} = x : 1$

10. $x : \frac{2}{3} = \frac{1}{2} : \frac{1}{4}$

11. $2 : 3 = \frac{1}{2} : x$

12. $\frac{7}{8} : \frac{3}{16} = x : \frac{5}{8}$

13. $50 : 75 = x : 3$

14. $24 : 72 = 3 : x$

15. Solve 3 tons : x tons = \$15 : \$40.

SUGGESTION. — Since $\frac{3 \text{ tons}}{x \text{ tons}} = \frac{3}{x}$ and $\frac{\$15}{\$40} = \frac{15}{40}$, the number of tons may be found from the proportion $\frac{3}{x} = \frac{15}{40}$, or $3 : x = 15 : 40$.

Solve the following proportions:

16. $\frac{18 \text{ yd.}}{24 \text{ yd.}} = \frac{\$x}{\$3.60}$

17. $\frac{15 \text{ bu.}}{x \text{ bu.}} = \frac{\$37.50}{\$100}$

18. $\frac{4 \text{ men}}{12 \text{ men}} = \frac{5 \text{ days}}{x \text{ days}}$

19. $\frac{x \text{ ft.}}{5600 \text{ ft.}} = \frac{4}{1120}$

WRITTEN EXERCISES

506. **1.** If 4 tons of coal cost $17, how much will 14 tons cost at the same price per ton?

SOLUTION

The cost of the coal is proportional to the quantity; that is,

$$\left\{\begin{matrix}\text{Cost of} \\ \text{14 tons}\end{matrix}\right\} : \left\{\begin{matrix}\text{cost of} \\ \text{4 tons}\end{matrix}\right\} :: 14 \text{ tons} : 4 \text{ tons}$$

or,
$$\$x : \$17 = 14 \text{ tons} : 4 \text{ tons}$$

Therefore,
$$x : 17 = 14 : 4$$
$$4x = 238$$
$$\therefore x = 59\tfrac{1}{2}.$$

Hence the cost of 14 tons is $59.50.

2. A man bought a 180-acre farm for $11,000 and sold 30 acres of it at cost. How much did he receive for the part sold?

3. If 55 acres of land yield 1430 bushels of wheat, how many bushels will 132 acres yield at the same rate?

4. If sound travels 825 meters in $2\frac{1}{2}$ seconds, how long will it take the report of a cannon to travel 3.3 Km.?

5. If 24 pounds of maple sugar cost $3.60, how much must be paid for 5 pounds?

6. A motor-boat race of 136.5 miles on the Hudson River was run at the rate of 52.5 miles in 2 hours. How long did it take to complete the course?

7. If it costs 66¢ to have a rug of $8\frac{1}{4}$ square yards cleaned, how much will it cost for a rug of $15\frac{1}{2}$ square yards?

8. A manufacturing plant turns out 7500 concrete building blocks in 6 days. How large an order for concrete blocks can it fill in 20 days?

9. A series of photographs for a moving picture was taken in the New York Subway at the rate of 500 in $33\frac{1}{3}$ seconds. How many photographs were there in the series, if it was completed in 7 minutes?

10. If 10 men can dig a ditch in 12 days, how many days will it take 15 men to dig it?

SOLUTION

Let x represent the number of days it will take 15 men.

Since 15 men can do more work per day than 10 men, they can dig a ditch in less time than 10 men can dig it.

Therefore the ratio x days : 12 days is less than 1.

The ratio, less than 1, between 10 men and 15 men is 10 men : 15 men.

Therefore, $x \text{ days} : 12 \text{ days} = 10 \text{ men} : 15 \text{ men}$

$$x : 12 = 10 : 15$$
$$15\,x = 120$$
$$\therefore x = 8$$

Hence 15 men can dig the ditch in 8 days.

Test. — There are 10×12, or 120, days' work to be done. 15 men can do this work in $(120 \div 15)$ days, or in 8 days.

11. If 16 men can do a piece of work in 15 days, how long will it take 20 men to do it?

12. If 16 men can do a piece of work in 15 days, how many men will be required to do it in 10 days?

13. If a force of men can do a certain piece of work in 20 days by working 8 hours a day, in how many days can they do the work by working 10 hours a day?

14. It is estimated that it will take 24 men 18 days to repair a portion of Second Street. The work must be done in 16 days. How many extra men must be employed?

15. A contractor agrees to do a piece of work in 60 days and employs 40 men for that purpose. Before beginning, however, the time is extended 15 days. How many men may he discharge?

16. If the interest on a sum of money for 6 months is \$33, what will be the interest for 8 months?

17. By counting mile posts, a man found that the train upon which he was riding went 5 miles in 5 minutes 15 seconds. How many miles per hour was the train moving?

18. In a shipment by rail of 450 dozen eggs, 5 eggs out of every 3 dozen were broken. How many dozen eggs were left unbroken?

19. A owned a house assessed at $12,000, on which he paid a tax of $225 one year. The same year B's house, next door, was taxed at $157.50. For how much was B's house assessed?

20. The assessed valuation of the property in a town is $4,650,000, and the tax to be raised is $102,300. If I own property assessed at $23,250, how much tax must I pay?

21. A garrison of 150 soldiers consumed 26 barrels of flour in 9 weeks. At this rate how many days would the same amount of flour last a garrison of 210 soldiers?

22. On a freight train of 18 cars, 4050 barrels of flour are loaded. How many cars must be added so that the train may carry 5175 barrels?

PARTITIVE PROPORTION

507. The process by which a number is separated into parts proportional to two or more given numbers is called **partitive proportion.**

WRITTEN EXERCISES

508. **1.** Separate 225 into parts proportional to 2, 3, and 4.

SOLUTION

Since the parts are proportional to 2, 3, and 4, we may represent them by $2x$, $3x$, and $4x$, respectively.

Since the sum of the parts is the number itself,

$$2x + 3x + 4x = 225$$

$$\therefore x = 25$$

Hence, $2x = 50$, $3x = 75$, and $4x = 100$, the numbers.

Test. $50 + 75 + 100 = 225$; 50, 75, and 100 are proportional to 2, 3, and 4.

2. Separate $24,000 into parts proportional to 3, 4, and 5.

3. Separate $36,000 into parts proportional to 3, 5, and 10.

4. The total receipts of a coal mining company one year were $16,725,000, and the expenses were to the net earnings as 13 is to 2. What were the expenses? the net earnings?

5. The daily ration of a German soldier in the field weighs 1300 grams and consists of bread, meat, rice, salt, and coffee proportional to 6, 3, 1, $\frac{1}{5}$, and $\frac{1}{5}$. Find the weight of each.

6. The freight earnings of two railroads on a train load of grain were $2160. One carried the grain 400 miles, the other 500 miles. Find the earnings apportioned to each road.

7. The annual earnings of a certain railroad company are $78,000,000. Find the amounts received from freight charges, from passenger service, and from other sources (such as mail, express, etc.), if they are proportional to 8, 4, and 1.

8. A quarterly dividend of $6412.50 was divided among the 8 shareholders of a corporation. The holdings of the shareholders were 30, 15, 24, 18, 48, 36, 42, and 72 shares, respectively. What sum did each shareholder receive?

PARTNERSHIP

509. When two or more persons agree to combine their money, goods, labor, or skill, in some business enterprise, and to share the profits and losses of the business in certain proportions, they become **partners**, thus forming a **partnership**.

The partners are collectively called a **firm**, or a **house**.

510. As a rule the legal liability of a partner in a firm is different from that of a stockholder in a company or corporation; for while a stockholder is liable, with few exceptions, for only the par value of his holdings, *a partner is usually liable for the entire indebtedness of the firm.*

511. The investment of a partner is called his **capital.**

The capital may be money or anything that has a money value in the business, as goods, labor, skill, experience, the "good will" of the trade, or some mercantile advantage, etc.

512. *The gains and losses of a firm are shared in proportion to the amount of capital invested by each, and the length of time such capital is invested in the business.*

WRITTEN EXERCISES

513. **1.** A and B engaged in business as partners and gained \$4000. A's capital was \$10,000, and B's was \$6000. Find the profits apportioned to each.

SOLUTION

Entire capital $= \$16000$
A's share of the capital $= \frac{10000}{16000} = \frac{5}{8}$; $\therefore$ A's gain $= \frac{5}{8}$ of $\$4000 = \$2500$
B's share of the capital $= \frac{6000}{16000} = \frac{3}{8}$; $\therefore$ B's gain $= \frac{3}{8}$ of $\$4000 = \$1500$
Test. — A's gain + B's gain $= \$2500 + \$1500 = \$4000$, the entire gain.

2. A and B owned a strawboard factory, A's investment being \$75,000 and B's \$45,000. The net earnings for one year were \$11,200. How much of the earnings did each partner receive?

3. Apportion a loss of \$2400 to the three partners in a business, if their respective investments are \$11,000, \$15,000, and \$6000.

4. A business block worth \$28,000, owned by three men, was insured for $\frac{3}{5}$ of its value. One had \$16,000 invested, one \$7000, and one \$5000. The block was completely destroyed by fire. What was the amount of insurance due each man?

5. As the result of a damage suit, a judgment for \$4600 was obtained against the joint owners of a quarry. The owners' investments were \$10,000, \$3000, \$6500, and \$3500, respectively. How much was each owner obliged to pay?

6. Three men invested $2200, $1800, and $2000, respectively, in a business. After several years, during which the business had grown to $10,500, the first partner withdrew from the firm. How much was his share worth?

The other two bought his part in proportion to their holdings. How much did each pay?

7. Four partners with a capital of $48,000, of which A, B, and C furnished $10,000 each and D the rest, declared a 16 % dividend and used the surplus profits, $2880, to increase their capital. Find each partner's dividend and his increased capital.

8. A and B formed a partnership with a capital of $8000, of which A furnished $5000. After 18 months A withdrew $1000, and at the end of 2 years the partnership was dissolved. If the gain for 2 years was $7440, how much did each partner receive?

SOLUTION

A's capital,	$5000 for 18 mo.	= $ 90000 for 1 mo.
" "	$4000 for 6 mo.	= $ 24000 for 1 mo.
A's total capital		= $114000 for 1 mo.
B's capital,	$3000 for 24 mo.	= $ 72000 for 1 mo.
Total investment for both		= $186000 for 1 mo.

A's gain = $\frac{114000}{186000}$ of $7440 = $4560; B's gain = $\frac{72000}{186000}$ of $7440 = $2880.
Test. $4560 + $2880 = $7440.

9. A and B were partners and divided $3075 in profits. A's investment was $5000 for 1 year; B's was $5000 for 9 months and $6000 for 3 months. How much did each receive?

10. A and B began business with a capital of $10,000, $\frac{3}{5}$ of which A furnished. After 6 months C entered the firm with a capital of $5000. After another 6 months they divided $5500 in profits. How much did each partner receive?

11. A, B, and C were partners for 16 months. A contributed $30,000, B $20,000, and C $40,000, $15,000 of which he withdrew in 12 months. When they dissolved partnership they divided $41,400 in profits. Find the profits allotted to each.

POWERS AND ROOTS

RAISING TO POWERS

514. **1.** $2 \times 2 = 4$ $\quad 2 \times 2 \times 2 = 8 \quad$ $2 \times 2 \times 2 \times 2 = 16$

Or, $\quad 2^2 = 4 \quad$ $2^3 = 8 \quad$ $2^4 = 16$

What is the product when 2 is taken 2 times as a factor? 3 times? 4 times? 5 times? 6 times?

What is the *second power* of 2? the *third* power of 2? the *fourth* power? the *fifth* power? the *sixth* power?

2. What number is the second power of 3? the third power of 3? the fourth power?

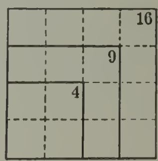
3. Find the second power of 10; the third power.

515. The number of times a number is to be used as a factor may be indicated by using an **exponent** (§ 118).

516. The product arising from using a number a certain number of times as a factor is called a **power** of that number.

4 is the second power of 2, for $4 = 2 \times 2$, or 2^2; 8 is the third power of 2, for $8 = 2 \times 2 \times 2$, or 2^3; 16 is the fourth power of 2; etc.

A number is regarded as the *first* power of itself.



517. If the side of a *square* is 2, the area is the *second* power of 2; if the side is 3, the area is the *second* power of 3; etc.

Therefore the *second* power of a number is called its **square**.

If the edge of a *cube* is 2, the volume is the *third* power of 2; if the edge is 3, the volume is the *third* power of 3; etc.

Therefore the *third* power of a number is called its **cube**.

EXERCISES

518. 1. Find the square of :

1 2 3 4 5 6 7 8 9 10 11 12

2. Find the cube of 1 ; of 2 ; of 3 ; of 4 ; of 5.

3. Find the fourth power of 1 ; of 2 ; of 3.

4. 10^2, 10^3, 10^4, 10^5, are read "ten square," "ten cube," "ten fourth power," and "ten fifth power," respectively. Tell the meaning and value of each indicated power.

5. Multiply $\frac{2}{3}$ by $\frac{2}{3}$, or square $\frac{2}{3}$.

6. Square $\frac{1}{2}$; $\frac{3}{4}$; $\frac{5}{6}$; $\frac{6}{5}$; .3 ; .5 ; .7 ; .12 ; 1.2.

7. Cube $\frac{1}{2}$; $\frac{1}{3}$; $\frac{2}{3}$; $\frac{3}{4}$; .2 ; .3 ; .5; .1 ; .02 ; .05.

519. *The square of a fraction may be obtained by squaring both terms ; the cube, by cubing both terms ; etc.*

WRITTEN EXERCISES

520. Raise each of the following to the power indicated :

1. 20^2	5. 12^3	9. $(\frac{7}{8})^3$	13. 1.5^2
2. 25^2	6. 21^3	10. $(\frac{5}{12})^3$	14. 7.5^2
3. 45^2	7. 15^3	11. $(\frac{15}{28})^2$	15. 1.25^2
4. 52^2	8. 11^4	12. $(\frac{25}{32})^2$	16. $.111^2$

Find the area of a square whose side is :

17. 21 in.	20. 44.7 ft.	23. 0.25 ft.
18. 17 yd.	21. 50.4 ft.	24. 7.56 ft.
19. 32 rd.	22. 0.19 ft.	25. .842 mi.

Find the volume of a cube whose edge is :

26. 11 in.	28. 22 cm.	30. 3.4 ft.
27. 2.5 in.	29. 1.5 m.	31. 12.5 Dm.

32. The number of feet a body will fall, from rest, in any number of seconds is 16.08 times the square of the number of seconds. How far will a body fall in 4 sec. ? in 15 sec. ?

521. **1.** Square 10; 2 tens; 3 tens; 4 tens; 12 tens.

2. Any integer of two or more figures may be regarded as composed of tens and units. Express 35 as tens and units; 45; 92; 106; 125; 432; 2563.

3. You already know the squares of the integers from 1 to 12, inclusive. The square of 13 may be obtained thus:

$$\begin{array}{r}
13 \\
\underline{13} \\
39 \\
\underline{13} \\
169
\end{array}
\qquad\qquad
\begin{array}{l}
\phantom{100+{}}10+3 \\
\phantom{100+{}}\underline{10+3} \\
\phantom{100+{}}30+9 \text{ (Product of } 10+3 \text{ by } 3) \\
\underline{100+\ 30}\ (\text{ “ \quad “ \quad “ \quad by } 10) \\
100+2\,(30)+9 = 10^2 + 2\,(10 \times 3) + 3^2
\end{array}$$

The square of any integer composed of tens and units is equal to the square of the tens, plus twice the product of the tens and the units, plus the square of the units.

EXERCISES

522. **1.** Find the square of the number 14.

SOLUTION

$$14^2 = 10^2 + 2\,(10 \times 4) + 4^2 = 100 + 80 + 16 = 196$$

2. Complete the following table, then commit it to memory:

NUMBER	SQUARE	NUMBER	SQUARE	NUMBER	SQUARE	NUMBER	SQUARE	NUMBER	SQUARE
1	1	6	36	11	121	16	. . .	21	. . .
2	4	7	49	12	144	17	. . .	22	. . .
3	9	8	64	13	169	18	. . .	23	. . .
4	16	9	81	14	196	19	. . .	24	. . .
5	25	10	100	15	. . .	20	. . .	25	. . .

Square the following as in exercise 1, regarding each number as composed of tens and units:

3. 32	**6.** 35	**9.** 65	**12.** 64	**15.** 120
4. 41	**7.** 45	**10.** 75	**13.** 81	**16.** 125
5. 43	**8.** 55	**11.** 85	**14.** 92	**17.** 162

EXTRACTING ROOTS

523. 1. Since $25 = 5 \times 5$, what is one of the two equal factors of 25, or the *square root* of 25?

2. What is the square root of 36? of 49? of 8^2? of 10^2?

3. What is the *cube root* of $2 \times 2 \times 2$, or of 2^3? of 27, or of 3^3? of 64, or of 4^3? of 125, or of 5^3? of 10^3? of 12^3?

524. One of the equal factors of a number is called a **root** of the number.

One of the *two* equal factors of a number is its *second*, or *square*, root; one of the *three* equal factors, the *third*, or *cube*, root; one of the *four* equal factors, the *fourth* root; etc.

525. The **root sign, or radical sign,** is $\sqrt{\ }$. When placed over a number, it indicates that a root is to be taken.

A small figure, written in the opening of the radical sign, as in $\sqrt[2]{25}$, $\sqrt[3]{8}$, $\sqrt[4]{16}$, shows what root is to be taken. It is called the **index** of the root.

The index 2 is usually omitted. Thus, $\sqrt{25}$ means *the square root of* 25.

Roots by Factoring

EXERCISES

526. 1. Find the cube root of 125.

SOLUTION

$$\sqrt[3]{125} = \sqrt[3]{5 \times 5 \times 5} = 5$$

In the same way find the value of each of the following indicated roots:

2. $\sqrt{16}$	6. $\sqrt[3]{8}$	10. $\sqrt{121}$	14. $\sqrt{169}$
3. $\sqrt{36}$	7. $\sqrt[3]{27}$	11. $\sqrt{144}$	15. $\sqrt{289}$
4. $\sqrt{49}$	8. $\sqrt[3]{64}$	12. $\sqrt{225}$	16. $\sqrt{400}$
5. $\sqrt{64}$	9. $\sqrt[4]{16}$	13. $\sqrt{625}$	17. $\sqrt[3]{1000}$

WRITTEN EXERCISES

527. **1.** Find the cube root of 216.

Solution

By factoring, we find that $216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$.
Arranging the factors to give three like groups,

$$216 = (2 \times 3) \times (2 \times 3) \times (2 \times 3)$$
$$\therefore \sqrt[3]{216} = 2 \times 3, \text{ or } 6.$$

Find the roots indicated:

2. $\sqrt{729}$	**6.** $\sqrt{1024}$	**10.** $\sqrt{11664}$	**14.** $\sqrt[5]{243}$
3. $\sqrt{784}$	**7.** $\sqrt{2025}$	**11.** $\sqrt[3]{46656}$	**15.** $\sqrt[3]{343}$
4. $\sqrt[3]{512}$	**8.** $\sqrt[3]{3375}$	**12.** $\sqrt[4]{20736}$	**16.** $\sqrt[7]{128}$
5. $\sqrt[3]{729}$	**9.** $\sqrt[5]{1024}$	**13.** $\sqrt[6]{15625}$	**17.** $\sqrt[4]{625}$

18. The area of a square is 324 sq. in. Find the side.

19. Find the edge of a cube whose volume is 8000 cu. in.

Square Root

528. The square of an integer or a fraction is called a **perfect square.**

529. $\sqrt{1} = 1$ $\quad\sqrt{100} = 10$ $\quad\sqrt{10000} = 100$

$\sqrt{25} = 5$ $\quad\sqrt{3600} = 60$ $\quad\sqrt{160000} = 400$

$\sqrt{81} = 9$ $\quad\sqrt{9801} = 99$ $\quad\sqrt{998001} = 999$

How many figures are there in the square root of any perfect square that is expressed by not more than *two* figures? by *four* figures or by one less than four figures? by *six* figures or by one less than six figures?

The number of figures in the square root of a perfect square is the same as the number of periods of two figures each into which the number can be separated, beginning at units.

The left-hand period may contain only one figure.

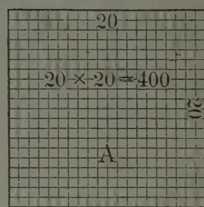
WRITTEN EXERCISES

530. 1. What is the square root of 576, or what is the side of a square whose area is 576 square units?

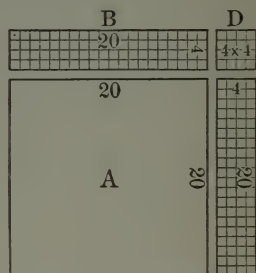
$$\begin{array}{lrl} & 5'76 & \lfloor 20 \\ 20^2 = & \underline{4\ 00} & \underline{\ \ 4} \\ 2 \times 20 = & 40)\overline{1\ 76} & 24 \\ (40+4) \times 4 = & 1\ 76 & \end{array}$$

Since the number of figures in the square root of a number may be determined by separating the number into periods of two figures each, beginning at units, the square root of 576 is seen to be composed of *tens* and *units*.

Since the square of tens is hundreds, 5 hundreds must be the square of at least 2 tens. 2 tens or 20 squared is 400, and 400 subtracted from 576 leaves 176. Therefore the root 20 must be increased by such an amount as will exhaust the remainder.



The square, *A*, already formed from the 576 square units is one whose side is 20 units, but inasmuch as the number of units has not been exhausted, such additions must be made to the square as will exhaust the units and keep the figure a square. The necessary additions are two equal rectangles, *B* and *C*, and a small square, *D*.



Since the square, *D*, is small, the area of the rectangles, *B* and *C*, is nearly 176 square units. This area, divided by the length of the rectangles, will give the width, which is 4 units. The width of the additions is 4 units, and the entire length, including the small square, is 44 units; therefore the area of all the additions is 4 times 44 units, or 176 square units, which is equal to the entire number of square units to be added. Hence the side of the square is 24 units, or the square root of 576 is 24.

2. Extract the square root of 3844.

$$\begin{array}{rrl} & 38'44 & \lfloor 62 \\ & \underline{36} & \\ 2 \times 60 = 120 & 2\ 44 & \\ \underline{2} & & \\ 122 & \underline{2\ 44} & \end{array}$$

Separating the number into periods, it is found that the root is composed of tens and units. Since the largest square in 38 is 6, the tens of the root cannot be greater than 6 tens, or 60. Writing 6 tens in the root, squaring, and subtracting from 3844, there is a remainder of 244.

Since the square of a number composed of tens and units is equal to (*the square of the tens*) + (*twice the product of the tens and the units*) + (*the square of the units*), when the square of the tens has been subtracted, the remainder, 244, is twice the product of the tens and the units, plus the square of the units, or only a little more than twice the product of the tens and the units.

Therefore 244 divided by twice the tens is approximately equal to the units. 2×6 tens, or 120, then, is a *trial*, or *partial*, *divisor*. Dividing 244 by the trial divisor, the units' figure is found to be 2.

Since twice the tens are to be multiplied by the units, and the units also are to be multiplied by the units to obtain the square of the units, to abridge the process the tens and units are first added, forming the *complete divisor* 122, and then multiplied by the units. Thus $(120 + 2)$ multiplied by $2 = 244$.

Therefore the square root of 3844 is 62.

Extract the square root of:

3. 2809	**8.** 5184	**13.** 3721	**18.** 3249
4. 2601	**9.** 1156	**14.** 7056	**19.** 2401
5. 6724	**10.** 6889	**15.** 2116	**20.** 9604
6. 1936	**11.** 1089	**16.** 9025	**21.** 6241
7. 8281	**12.** 8836	**17.** 5776	**22.** 9801

Find the side of a square whose area is:

23. 7225 sq. in.	**26.** 841 sq. ft.	**29.** 7744 sq. m.
24. 1444 sq. in.	**27.** 4225 sq. cm.	**30.** 4761 ares
25. 2209 sq. in.	**28.** 4624 sq. cm.	**31.** 1225 hektares

32. The area of a square field is 12.1 acres. How many rods of fence are required to inclose it?

33. I have a lot 144 feet long and 64 feet wide. What is its area in square feet? Find the side of a square lot that has the same area.

34. An officer has 5625 men and wishes to arrange them in a solid square. How many men shall he place on a side?

WRITTEN EXERCISES

531. 1. Extract the square root of 15625; of 1.5625.

$$\begin{array}{r|l} & 1'56'25 \quad \underline{|125} \\ & 1 \\ 22 & 56 \\ & \underline{44} \\ 245 & 12\,25 \\ & \underline{12\,25} \end{array} \qquad\qquad \begin{array}{r|l} & 1'.56'25 \quad \underline{|1.25} \\ & 1 \\ 22 & 56 \\ & \underline{44} \\ 245 & 12\,25 \\ & \underline{12\,25} \end{array}$$

After two figures of the root in the first process have been obtained, we find that we have subtracted, in all, 10000 + 4400, or 14400, the square of 12 tens, *the part of the root already found*. Therefore, regarding 12 tens as the first part of the root, the units of the root are obtained in the usual way.

When there are decimal figures, as in the second process, they are pointed off into periods of two figures each, *beginning at the decimal point.* The process is then the same as for integers.

Separate the number into periods of two figures each, beginning at units or at the decimal point.

Find the greatest square in the left-hand period, and write its root for the first figure of the required root.

Square this root, subtract the result from the left-hand period, and annex to the remainder the next period for a dividend.

Double the root already found, for a partial divisor, and by it divide the dividend, disregarding the right-hand figure. The quotient, or quotient diminished, will be the second figure of the root.

Annex to the partial divisor for a complete divisor the figure last found, multiply this divisor by the figure of the root last found, subtract the product from the dividend, and to the remainder annex the next period for the next dividend.

Proceed in this manner until all the periods have been used. The result will be the square root sought.

1. When the number is not a perfect square, annex periods of decimal ciphers and continue the process.

2. The square root of a common fraction may be found by extracting the square root of numerator and denominator separately or by reducing it to a decimal and then extracting its root.

Extract the square root of:

2. 15129	**7.** 930.25	**12.** .555025
3. 24336	**8.** 655.36	**13.** .633616
4. 13689	**9.** 282.24	**14.** .994009
5. 30976	**10.** 11.0224	**15.** .00675684
6. 44521	**11.** 14.5924	**16.** .00767376

Extract the square root and express as a common fraction:

17. $\frac{20736}{30625}$ **18.** $\frac{25921}{50625}$ **19.** $32\frac{164}{625}$ **20.** $87560\frac{521}{1024}$

Verify the following:

21. $\sqrt{2} = 1.414^+$ **22.** $\sqrt{3} = 1.732^+$ **23.** $\sqrt{5} = 2.236^+$

Extract the square root, to the nearest thousandth, of:

24. 8	**27.** $\frac{1}{3}$, or $\frac{3}{9}$	**30.** 0.1	**33.** .7854
25. 6	**28.** $\frac{1}{2}$, or $\frac{2}{4}$	**31.** 2.5	**34.** .41265
26. 7	**29.** $\frac{1}{5}$, or .2	**32.** 3.6	**35.** .7400063

36. Find the side of a square whose area is 1 acre.

Solution

Let x = number of feet in one side of the square.
Then, the area = x^2 square feet. But 1 acre = 43,560 square feet.
Therefore, $x^2 = 43,560$
Extracting the square root of both members,
$x = 208.7+$, the number of feet in the side.

Note that a square 209 feet on a side is only a little more than an acre.

Find the side of a square whose area is:

37. 1.5 A. **38.** 12,000 sq. ft. **39.** 375,000 sq. ft.

40. A football field is 110 yd. long and 160 ft. wide. Find the side of a square field having the same area.

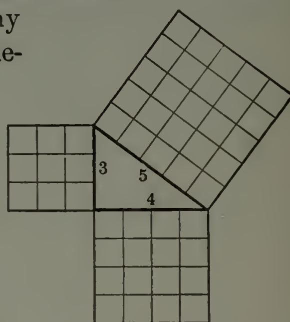
41. Find the dimensions, in rods, of a 20-acre rectangular field whose length is twice its width.

SUGGESTION. — Let x = number of rods in width. Then $2x$ = number of rods in length, and $x(2x)$, or $2x^2$, = number of square rods in area.

42. Find the dimensions, in rods, of a 90-acre rectangular field whose length is 4 times its width.

43. How much more fence is required to inclose the field mentioned in exercise 42 than a square field of equal area?

532. **1.** Since the longest side, or **hypotenuse**, of this **right-angled triangle** is 5 units long, how many square units are there in the square described upon the hypotenuse?



2. Since one of the other two sides, or **legs**, is 3 units long and the other 4 units long, how many square units are there in the square described upon each? in both these squares?

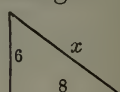
3. How does the number of units in the square of the hypotenuse compare with the number of units in the sum of the squares of the other two sides?

The square of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the other two sides.

WRITTEN EXERCISES

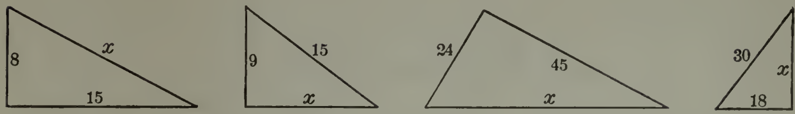
533. **1.** Find the hypotenuse of this right-angled triangle.

SOLUTION. $x^2 = 6^2 + 8^2 = 36 + 64 = 100$
$\therefore x = 10$



2. Draw a right-angled triangle whose legs are 5 in. and 12 in. Compute the hypotenuse. Measure it. Compare results.

3. In the following right-angled triangles find the length of each side marked x:



4. Draw an inch square and a straight line connecting two opposite corners, or a *diagonal*. Compute the length of the diagonal to the nearest .001 of an inch.

5. Draw a rectangle 4 in. by $7\frac{1}{2}$ in. Draw one of its diagonals and describe a square on it. Compute the area of the square.

6. Draw a 5-inch square and one diagonal. How does the area of the square on the diagonal compare with the area of the 5-inch square?

7. Measure the length and width of a rectangular room. Measure a diagonal on the floor as accurately as you can. Then compute the length of the diagonal to the nearest .001 of a foot. Which is the more accurate method of finding the length of the diagonal when the length and width are known exactly?

8. Next measure the height of the ceiling. Compute the distance from the lower corner of the room at one end of the diagonal on the floor to the upper corner at the other end.

9. A 40-foot ladder leans against a wall, with the foot 6 feet from the base of the wall. Draw a sketch and compute the height of the top of the ladder.

10. Two vessels sailed from the same point, one north at the rate of 15 knots an hour, the other east at the rate of 20 knots an hour. How far apart were they after 6 hours?

11. How far apart are the opposite corners of a square farm that contains 360 acres?

MENSURATION

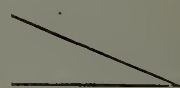
534. The difference in direction of two lines that meet is called an **angle** (§ 241); the lines are called the **sides**, and the point where they meet, the **vertex** of the angle.

535. When a straight line meets another straight line forming two *equal* angles, each angle is called a **right angle**.

Two Right Angles

536. The two lines that form a right angle are said to be **perpendicular** to each other.

537. An angle that is less than a right angle is called an **acute angle**.



Acute Angle

538. An angle that is greater than a right angle but less than two right angles is called an **obtuse angle**.

Obtuse Angle

PLANE FIGURES

539. A surface such that a straight line joining any two points of it lies wholly in the surface is a **plane surface**.

540. A plane surface can be measured in only two directions, and hence has only two *dimensions*, **length** and **breadth**.

541. A portion of a plane surface bounded by four straight lines is called a **quadrilateral**.

You have learned how to measure and find the area (§ 244–§ 246) of squares and rectangles.

542. Lines in the same plane surface that cannot meet, however far they are extended, are called **parallel lines**.

Parallel Lines

543. A quadrilateral whose opposite sides are parallel is called a **parallelogram**.

PARALLELOGRAM

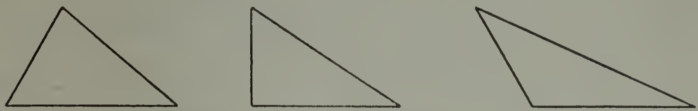
A rectangle is a right-angled parallelogram.

544. A quadrilateral that has only two sides parallel is called a **trapezoid**.

TRAPEZOID

545. A plane figure bounded by three straight lines is called a **triangle**.

A triangle is *acute-angled* if all of its angles are acute; *right-angled* if it has one right angle; *obtuse-angled* if it has one obtuse angle.



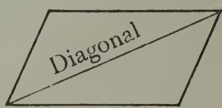
ACUTE-ANGLED TRIANGLE RIGHT-ANGLED TRIANGLE OBTUSE-ANGLED TRIANGLE

546. The side on which a figure is assumed to stand is called its **base**, and the height of the figure measured on a line perpendicular to the base is called its **altitude**.

These figures have different bases but the same altitude:

A parallelogram or a trapezoid has two bases known as the **lower base** and the **upper base**.

547. A straight line joining the vertices of the opposite angles of a quadrilateral is called a **diagonal**.

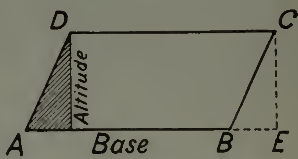


548. Since the length of a rectangle may be considered its base, and the width its altitude (§ 245),

The area of a rectangle is equal to the product of its base and altitude.

For brevity, in this and subsequent similar statements, the *product of lines* means the product of the *numbers* that represent them. Dimensions must be expressed in like units.

549. To find the area of a parallelogram.



1. If the shaded part of the parallelogram $ABCD$ is cut off and placed in the position BCE, what kind of a figure will be obtained?

2. Compare the base of the parallelogram with the base of the rectangle. Compare the altitudes. Compare the areas.

3. Cut several parallelograms out of paper. Proceeding as above, show that each parallelogram has the same area as a rectangle of the same base and altitude.

550. The area of any parallelogram is equal to that of a rectangle having the same base and altitude. Hence,

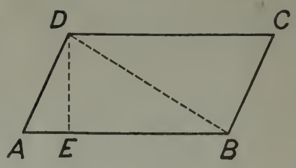
The area of a parallelogram is equal to the product of its base and altitude.

551. Find the area of each of these parallelograms

	Base	Altitude		Base	Altitude
1.	14.5 in.	6 in.	4.	10 Km.	500 m.
2.	320 rd.	4 rd.	5.	$12\frac{3}{4}$ ft.	3 ft. 6 in.
3.	200 m.	9 m.	6.	24 rd.	4 rd. $5\frac{1}{2}$ ft.

7. What is the altitude of a parallelogram whose area is 51 square inches and whose base is $8\frac{1}{2}$ inches long?

552. To find the area of a triangle.



1. Draw a parallelogram $ABCD$; its altitude DE; and one of its diagonals BD. Into how many triangles does BD divide $ABCD$? Cut out the triangles and apply one to the other. What part of $ABCD$ is ABD?

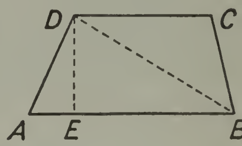
2. Then, since $AB \times DE$ represents the area of the parallelogram, $\frac{1}{2}$ of $AB \times DE$ represents the area of the triangle. Hence,

553. *The area of a triangle is equal to one half the product of its base and altitude.*

554. Find the area of these triangles:

1. Base 24 ft., alt. 12 ft.
2. Base 45 rd., alt. 16 rd.
3. Base 44 m., alt. 26 m.
4. Base 8 Dm., alt. 5.2 m.
5. What is the altitude of a triangle whose area is 84 square feet and whose base is 16 feet?
6. Find the base of a triangle whose area is 147 square inches and whose altitude is 1 ft. 2 in.

555. To find the area of a trapezoid.



1. Draw a trapezoid $ABCD$; its altitude DE; and one of its diagonals DB.
2. What is the base of the triangle ABD? its altitude? its area?
3. If DC, the upper base of the trapezoid, is taken as the base of the triangle BCD, what is its altitude? its area?
4. Then, since $\frac{1}{2}$ $DE \times AB$ represents the area of triangle ABD and $\frac{1}{2}$ $DE \times DC$ the area of BCD, $\frac{1}{2}$ $DE \times (AB + DC)$ represents the area of trapezoid $ABCD$. Hence,

556. *The area of a trapezoid is equal to one half the product of its altitude and the sum of its bases.*

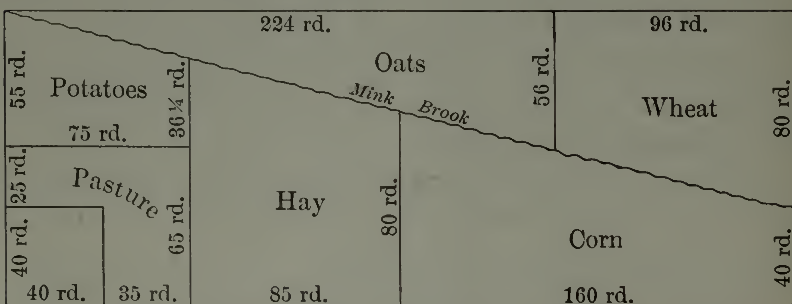
557. **1.** What is the area of a trapezoid whose bases are 7 in. and 9 in., respectively, and whose altitude is $3\frac{1}{2}$ in.?

2. If the sum of the bases of a trapezoid is 32 cm. and its altitude 5 cm., what is its area?

3. A plank 28 ft. long is 1 ft. wide at one end and $1\frac{1}{2}$ ft. wide at the other end. How many square feet are there in one side of the plank? If it is 2 in. thick, how many board feet does it contain?

4. The area of a trapezoid is 36 sq. in. The altitude is 4 in. and the lower base 10 in. How long is the upper base?

One year Mr. Drake's rectangular farm was divided into fields as shown in the diagram:



Find the area, in acres and square rods, of the field devoted to:

5. Corn **7.** Oats **9.** Potatoes

6. Wheat **8.** Hay **10.** Pasture

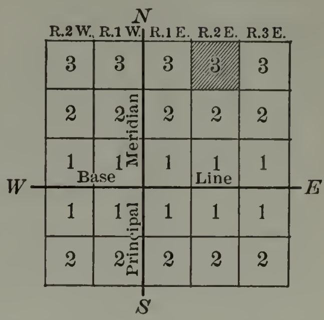
11. How many acres did the field in the lower left-hand corner contain?

12. Find the area of the farm by adding the areas of the different fields. Test your answer by finding the area of the whole farm from its length and width.

13. By applying § 532 find, to the nearest .01 of a rod, the length of Mink Brook that is within Mr. Drake's farm.

558. Government lands are usually surveyed into tracts bounded by lines that run north and south, and east and west.

559. Usually a **principal meridian** (north and south line) and a **base line** (east and west) are first established. Then other lines are run *six miles* apart each way as nearly as possible, forming **townships**.



560. A line of townships extending north and south is called a **range**.

Ranges are designated by their number east or west of the *principal meridian*, and the *townships* in each range by their number north or south of the *base line;* thus, in the first diagram, the township that is shaded is T. 3 N., R. 2 E., which means the third township north of the base line in the second range east of the principal meridian.

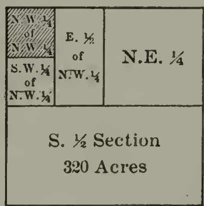
6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

TOWNSHIP

561. A township is divided into **sections**, each being *one mile* square, containing 640 acres, and numbered as in the second diagram.

562. Each section is then subdivided into *half sections*, *quarter sections*, *half-quarter sections*, and *quarter-quarter sections* as shown in the third diagram.

If this section is the shaded section of the second diagram, and that diagram in turn is the shaded township of the first diagram, the shaded part of this section would be designated thus: N.W. $\frac{1}{4}$ of N.W. $\frac{1}{4}$, Sec. 12, T. 3 N., R. 2 E., which means the northwest quarter of the northwest quarter of section 12 of the third township north of the base line in the second range east of the principal meridian.



SECTION

NOTE. — This subject is of most importance in the newer parts of the country, where land is laid out in this way.

563. 1. How many acres of land are there in a township? in a section? in a quarter-quarter section?

Write the description in full and give the number of acres in these tracts of land:

2. N.E. $\frac{1}{4}$, Sec. 14, T. 3 N., R. 5 W.

3. W. $\frac{1}{2}$ of N.W. $\frac{1}{4}$, Sec. 27, T. 5 S., R. 4 E.

4. S.E. $\frac{1}{4}$ of N.E. $\frac{1}{4}$, Sec. 36, T. 2 N., R. 1 W.

5. N.E. $\frac{1}{4}$ of N.W. $\frac{1}{4}$, Sec. 18, T. 4 S., R. 5 E.

6. S. $\frac{1}{2}$, and W. $\frac{1}{2}$ of N.W. $\frac{1}{4}$, Sec. 22, T. 3 S., R. 6 W.

7. Find the value of the tract of land mentioned in exercise 3, at $ 25.50 an acre.

8. How many rods of fence are required to inclose the tract of land mentioned in exercise 4?

9. What is the value of a quarter section and a half-quarter section of land at $18.75 an acre?

Regular Polygons

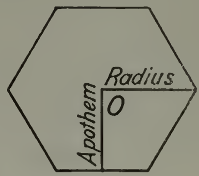
564. Any plane figure bounded by straight lines is a **polygon.**

565. A polygon having all of its sides equal and all of its angles equal is called a **regular polygon.**

EQUILATERAL TRIANGLE SQUARE PENTAGON HEXAGON

566. A regular polygon of *three* sides is called an **equilateral triangle**; of *four* sides, a **square**; of *five* sides, a **pentagon**; of *six* sides, a **hexagon**; of *seven* sides, a **heptagon**; of *eight* sides, an **octagon**; of *nine* sides, a **nonagon**; of *ten* sides, a **decagon**; of *twelve* sides, a **dodecagon.**

567. A straight line drawn from the center of a regular polygon to any vertex is called the **radius** (plural *radii*) of the polygon.



568. A straight line drawn from the center of a regular polygon perpendicular to any side is called the **apothem** of the polygon.

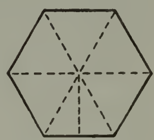
569. The line bounding a polygon, or the sum of its sides, is called its **perimeter**.

570. 1. What is the perimeter of a regular pentagon, each side of which is 5 inches long?

Find the perimeter of each of these regular polygons, having sides as given:

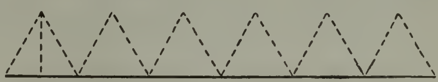
2. Hexagon, 8 in.
3. Heptagon, 4 in.
4. Decagon, $3\frac{1}{2}$ in.
5. Octagon, 2 dm.
6. Nonagon, 3 cm.
7. Dodecagon, $6\frac{1}{2}$ cm.

571. To find the area of a regular polygon.



1. Into how many triangles do the radii of a regular hexagon divide the hexagon?

2. How may the area of each triangle be found? the area of the six triangles, that is, the area of the hexagon?



3. How does the altitude of each triangle compare with the apothem of the polygon? the sum of the bases of the triangles with the perimeter of the polygon?

4. Any regular polygon may be divided in a similar manner into as many triangles as the polygon has sides. Hence,

572. *The area of a regular polygon is equal to one half the product of its perimeter and apothem.*

573. **1.** If the side of a regular hexagon is 6 inches long and the apothem 5.2 inches, what is its area?

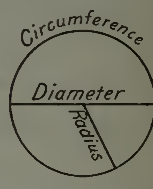
2. The side of a square is 16 inches long. How long is the apothem? Find the area by § 572 and compare with the result found in the usual way.

3. What is the area of a regular octagon whose side is 9 inches long and whose apothem is 10.86 inches?

4. The area of a regular pentagon is 61.8 square inches. Its apothem is 4.12 inches. What is the length of one side?

Circles

574. A plane figure bounded by a curved line every point of which is equally distant from a point within is called a **circle**; the point within is called the **center**; and the bounding line, the **circumference.**



575. A straight line drawn from the center to the circumference of a circle is called a **radius.**

576. A straight line passing through the center of a circle and terminating at both ends in the circumference is called a **diameter.**

A diameter is equal in length to two radii.

577. To find the circumference of a circle.

1. Draw a circle on a piece of stiff cardboard and carefully cut it out.

2. Place one point of its circumference, as D, on the point A of a straight line and roll the cardboard along the line until D again touches the line as at B. Then AB is the length of the circumference.

3. Measure AB and divide its length by the length of the diameter. What is the quotient?

4. Do the same thing with other circles. The result in each case will be nearly $3\frac{1}{7}$.

578. The ratio of the circumference of a circle to its diameter cannot be expressed exactly, but it is proved in geometry to be more nearly 3.1416 than $3\frac{1}{7}$.

The symbol for this ratio is the Greek letter π (pī).

579. *The circumference of a circle is π times the diameter, or 2π times the radius.*

Unless stated otherwise, the approximate value $3\frac{1}{7}$ will be used for π.

580. Find the circumference of a circle whose diameter is:

1. 35 in.	3. 63 yd.	5. 84 m.	7. 126 m.
2. 77 ft.	4. 49 rd.	6. 56 cm.	8. 168 mm.

Find the circumference of a circle whose radius is:

9. 28 ft.	11. 91 rd.	13. 13.3 Hm.	15. 24.5 ft.
10. 42 ft.	12. 11.9 m.	14. 5.88 cm.	16. 36.4 in.

Find the diameter of a circle whose circumference is:

17. 88 yd.	19. 12.1 in.	21. 25.3 m.	23. 990 rd.
18. 55 ft.	20. 3.74 cm.	22. 4.84 Dm.	24. 649 yd.

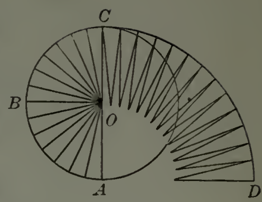
Using $\pi = 3.1416$, find the circumference of a circle whose diameter is:

25. 12 ft.	26. 20 m.	27. $15\frac{1}{2}$ yd.	28. $22\frac{3}{4}$ rd.

29. A wagon wheel is $3\frac{1}{2}$ feet in diameter. How far will the wagon have gone when the wheel has made 100 revolutions?

30. A merry-go-round has two rows of seats. One row is 7 feet from the center and the other $10\frac{1}{2}$ feet from the center. If John sits in an outer seat and Mary in an inner seat, how much farther than Mary does John ride each revolution?

581. To find the area of a circle.



From the accompanying figure, it is evident that a circle may be regarded as composed of a large number of triangles, the sum of whose bases forms the circumference, $ABCD$, of the circle, and whose altitude is the radius, AO, of the circle. Hence,

582. *The area of a circle is equal to one half the product of its circumference and radius.*

583. In § 579 the circumference of a circle is found to be 2π times the radius. Let r denote the radius; then,

$$\text{Circumference} = 2\pi r.$$

Multiplying $2\pi r$ by r and dividing by 2 (§ 582),

$$\text{Area} = \frac{2\pi r \times r}{2} = \pi r^2.$$

Therefore, if the radius of a circle is known its area may be found without finding the circumference.

584. *The area of a circle is equal to the square of the radius multiplied by π, or πr^2.*

535. Find the area of a circle whose radius is:

1. 14 in.	**3.** 2.8 rd.	**5.** 6.3 cm.	**7.** .07 ft.
2. 35 ft.	**4.** 4.2 m.	**6.** .91 dm.	**8.** 10.5 m.

9. If from a hose water can be thrown 42 feet, how many square feet of lawn can be watered from one position?

10. What is the area of a circular flower bed that is 21 feet in diameter?

11. Find the area of a table that is 22 feet in circumference.

12. A cow is tied to a stake with a rope $17\frac{1}{2}$ feet long. Over how many square feet of surface can she graze?

13. Find, to the nearest .01 of a rod, the radius of a circular field containing half an acre.

SOLIDS

586. Anything that can be measured in three directions is said to have three *dimensions*,— *length*, *breadth*, and *thickness*,— and is called a **solid**.

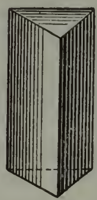
587. The surfaces that bound a solid are called its **faces** and their intersections its **edges**.

588. The number of cubic units that any solid contains is called its **volume**.

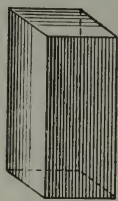
589. What name (§ 247) is given to a solid having six rectangular faces? to a rectangular solid whose faces are equal squares? (§ 248)

590. A solid whose sides are parallelograms and whose two ends are equal polygons, parallel to each other, is called a **prism**.

Prisms are named from the shape of their bases,—*triangular*, *square*, *rectangular*, *pentagonal*, etc.



TRIANGULAR PRISM

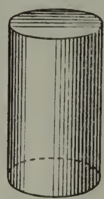


SQUARE PRISM

591. The perpendicular distance between the bases of a prism is called its **altitude**.

592. The parallelograms taken together form the **convex surface** of the prism.

593. A solid bounded by a uniformly curved surface and having for its bases circles that are parallel to each other is called a **circular cylinder**.



CYLINDER

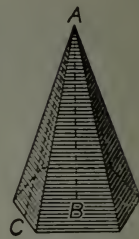
There are other kinds of cylinders, but in this book "cylinder" means "circular cylinder."

A cylinder may be thought of as a prism whose bases are circles and whose convex surface is made up of an infinite number of infinitely narrow parallelograms.

594. A solid whose base is a polygon and whose faces are triangles meeting at a point is called a **pyramid.**

The triangles form the **convex surface** of the pyramid, and the point where they meet is called the **vertex.**

Pyramids, like prisms, are named from their bases, as *triangular*, *square*, *hexagonal*, etc.



PYRAMID

595. The perpendicular distance, as AB, from the vertex to the base of a pyramid is called its **altitude.**

596. The altitude, as AC, of one of the triangles of a pyramid is called the **slant height** of the pyramid.

597. A solid whose base is a circle and whose surface tapers uniformly to a point, called the vertex, is a **circular cone.**

In this book "cone" means "circular cone."

The vertex of a cone is sometimes called its **apex.**

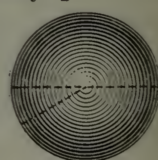
A cone may be thought of as a pyramid whose base is a circle and whose convex surface is made up of an infinite number of infinitely narrow triangles.


CONE

The **altitude** and **slant height** of a cone correspond to the altitude and slant height of a pyramid.

598. A solid bounded by a curved surface every point of which is equally distant from a point within, called the center, is a **sphere.**

599. A straight line passing through the center of a sphere and terminating at both ends in the surface is called its **diameter.**


SPHERE

600. One half the diameter of a sphere, or the distance from the center to the surface, is called its **radius.**

601. A circle of a sphere whose plane passes through the center is called a **great circle** of the sphere.

602. A great circle divides a sphere into two equal parts called **hemispheres**. The circle is the **base** of each hemisphere.

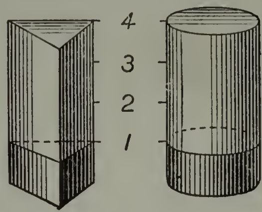
603. The circumference of a great circle of a sphere is called the **circumference** of the sphere.

The circumference of a sphere is the greatest distance around it.

Surfaces of Solids

604. To find the convex surface of a prism or a cylinder.

1. How many square inches are there in the convex surface of a prism or a cylinder 1 inch high, if the perimeter of its base is 6 inches?



2. What is the area of the convex surface, if the altitude of the solid is 2 inches? 3 inches? 4 inches?

3. How, then, may you find the area of the convex surface of a prism or a cylinder?

605. *The convex surface of a prism or a cylinder is equal to the product of its altitude and the perimeter of its base.*

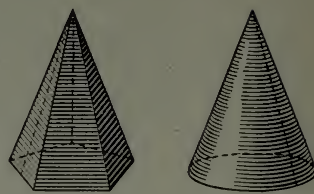
606. 1. What is the convex surface of a cylinder whose diameter is 2 feet and whose height is 5 feet?

2. Find the convex surface of a triangular prism whose base is 2 centimeters on each side and whose altitude is 4 centimeters.

3. What is the convex surface of a square prism whose sides are each $2\frac{1}{2}$ feet and whose altitude is 6 feet?

4. Find the entire surface (convex surface and surface of the two bases) of a cylinder that is 8 feet in height and has a base 3 feet in diameter.

607. To find the convex surface of a pyramid or a cone.



1. You have learned that the convex surface of a pyramid is composed of triangles, and that the convex surface of a cone may be assumed to be made up of an infinite number of triangles. The bases of these triangles form the perimeter of the base of the solid, and their altitude is the slant height of the solid.

2. How, then, may you find the area of the convex surface of a pyramid or a cone?

608. *The convex surface of a pyramid or a cone is equal to one half the product of its slant height and the perimeter of its base.*

609. 1. What is the convex surface of a rectangular pyramid whose base is 6 feet square, and whose slant height is 5 feet?

2. Find the convex surface of a cone having a base 5 centimeters in diameter and a slant height of 3 decimeters.

3. What is the convex surface of a cone whose base is 20 feet in diameter, and whose slant height is 20 feet?

4. At 30 ¢ per square yard, what is the cost of painting a church steeple, the base of which is an octagon 6 feet on each side, and whose slant height is 80 feet?

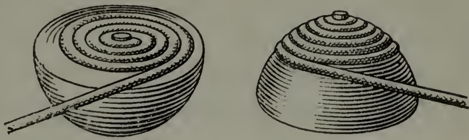
5. How many feet of convex surface are there on a cone, the base diameter of which is 6 feet, and whose slant height is $9\frac{1}{2}$ feet?

6. How many feet of convex surface are there on a pyramid whose base is 10 feet square, and whose slant height is 20 feet?

7. How many feet of convex surface are there on a cone whose base is 8 feet in diameter, and whose slant height is 6 feet?

610. To find the convex surface of a sphere.

1. The length of a waxed cord sufficient to cover the convex surface of a hemisphere, when carefully wound as shown in the picture, is just twice the length of cord required to cover the base of the hemisphere; that is, the area of the convex surface of a hemisphere is twice the area of its base.



2. Then how many times the area of a great circle of a sphere is the convex surface of the *whole* sphere?

3. It may be proved also by geometry that the convex surface of a sphere is equal to 4 great circles.

4. How does the radius of a great circle of a sphere compare with the radius of the sphere? What formula (§ 583) expresses the area of a circle? Then, what formula expresses the area of 4 great circles, or the convex surface of a sphere?

611. *The convex surface of a sphere is equal to* 4 *great circles, or to* $4\pi r^2$.

612. Denote the diameter by d. Then, since $4r^2 = (2r)^2 = d^2$, *the convex surface of a sphere is equal to the square of the diameter times* π, *or* πd^2.

613. 1. What is the convex surface of a baseball 3 inches in diameter?

2. Find the convex surface of a rubber ball having a radius of 12 centimeters.

3. What is the convex surface of a spherical cannon ball 8 inches in diameter?

4. Find the cost, at 12 cents a square foot, of gilding a sphere 28 inches in diameter.

Volume of Solids

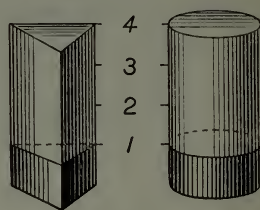
614. 1. What is meant by the volume of a solid?

2. How can you find the volume of a rectangular solid (§ 250) from its three dimensions?

3. Find the volume of a rectangular solid whose base is 5 feet by 4 feet and whose altitude is 5 feet.

615. To find the volume of a prism or a cylinder.

1. If a triangular prism with a base of 9 square inches were 1 inch high, how many cubic inches would it contain? if it were 2 inches high? 3 inches? 4 inches?



2. If a cylinder with a base 9 square inches in area were 1 inch high, how many cubic inches would it contain? if it were 2 inches high? 4 inches high?

3. How, then, can you find the volume of a prism or a cylinder from its altitude and the area of its base?

616. *The volume of a prism or a cylinder is equal to the product of its altitude and the area of its base.*

617. 1. What is the volume of a prism with a base 5 inches square, if its altitude is 3 inches?

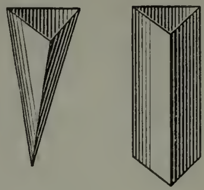
2. Find the volume of a cylinder whose diameter is 8 decimeters and altitude 2 meters.

3. What is the capacity in bushels of a bin 8 feet square and 9 feet high, inside measurements?

4. How many gallons of water will a cylindrical vat hold, if it is 7 feet in diameter and 8 feet high?

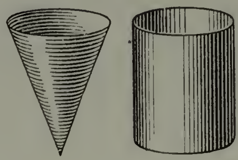
5. Find the value at 63 ¢ a bushel of the wheat that would fill a bin 15 feet square and 12 feet deep.

618. To find the volume of a pyramid or a cone.



1. Take a hollow pyramid and a hollow prism of the same base and altitude. Fill the pyramid with sand and empty it into the prism. How many times must you fill and empty the pyramid of sand to fill the prism?

What is the relation, then, of the volume of the pyramid to that of the prism?



2. Try the same experiment with a cone and a cylinder of the same base and altitude.

What is the relation of the volume of the cone to that of the cylinder?

3. It is proved also in geometry that the volume of a pyramid or a cone is, respectively, one third that of a prism or a cylinder having the same base and altitude.

4. Since the volume of a prism or a cylinder is equal to the product of its altitude and the area of its base, how can you find the volume of a pyramid or a cone?

619. *The volume of a pyramid or a cone is equal to one third the product of its altitude and the area of its base.*

620. 1. What are the solid contents of a cone, the diameter of whose base is 6 feet and whose altitude is 9 feet?

2. Find the solid contents of a pyramid whose base is 10 meters square and whose altitude is 20 meters.

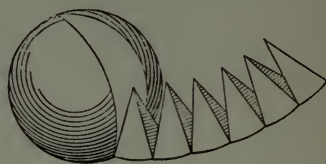
3. If a cubic foot of granite weighs 165 pounds, what is the weight of a granite cone the diameter of whose base is 6 feet and whose altitude is 8 feet?

4. A pile of coal in the shape of a cone is 30 feet high and 132 feet in circumference. Find its volume.

5. What is the weight of a marble pyramid whose base is 4 feet square and whose altitude is 8 feet, if a cubic foot of marble weighs 171 pounds?

6. The pyramid of Cheops has a square base 746 feet on a side and an altitude of 480 feet. Find its volume.

621. To find the volume of a sphere.



1. As indicated in the figure, a sphere may be divided into a great number of figures that are essentially pyramids. The sum of the bases of these pyramids is the convex surface of the sphere, and the altitude of each pyramid is the radius of the sphere.

Then how may the volume of a sphere be found?

2. It is proved also in geometry that the volume of a sphere is equal to $\frac{1}{3}$ the product of its radius and its convex surface.

3. Since (§ 611) $4\pi r^2$ represents the convex surface of a sphere and r its radius, (§ 619) $\frac{1}{3}r \times 4\pi r^2$, or $\frac{4}{3}\pi r^3$, represents the volume of a sphere.

622. *The volume of a sphere is equal to one third the product of its radius and its convex surface, or* $\frac{4}{3}\pi r^3$.

623. 1. Find the volume of a sphere whose radius is $1\frac{1}{2}$ feet.

2. What is the volume of a sphere 21 centimeters in diameter?

3. The circumference of a sphere is 22 inches. Find its volume.

4. A spherical aquarium has a diameter of 14 inches, measured on the inside. How many cubic inches of water will it hold? Find the weight of water that it will hold.

5. Find the weight of a 6-inch cannon ball made of iron that has a specific gravity of 7.21. (Use 1 cu. ft. = $7\frac{1}{2}$ gal.)

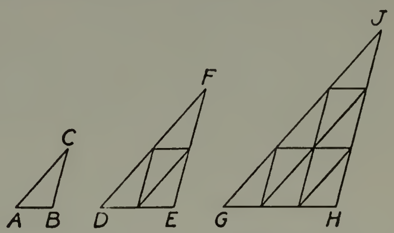
Similar Surfaces

624. Figures that are of exactly the same shape though they differ in size are called **similar figures**.

All circles are similar; also all squares, and all regular polygons having the same number of sides. Two maps of the same country drawn to different scales are similar figures.

In order that polygons may be similar, for every angle of the one there must be a corresponding equal angle of the other and the sides about the equal angles must be proportional.

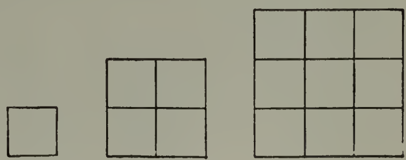
625. 1. Triangles ABC, DEF, and GHJ are similar. How do the sides of ABC compare in length with the corresponding sides of DEF? of GHJ?



How many triangles of the size of ABC are there in DEF? in GHJ?

2. The sides of the first two triangles are in the ratio of 1 to 2, and their areas are in the ratio of 1 to 4 (the squares of 1 and 2). The sides of the first and third triangles are in the ratio of 1 to 3, and their areas in the ratio of 1 to 9.

3. Show in the same way that the sides of these squares are proportional to 1, 2, and 3, and that their areas are proportional to 1, 4, and 9, the squares of the sides.



626. 1. *The corresponding sides or like dimensions of similar plane figures are proportional.*

2. *The areas of similar plane figures are proportional to the squares of their corresponding lines.*

627. **1.** If a rectangle is 4 inches long and 3 inches wide, find the width of a similar rectangle that is 8 inches long.

SUGGESTION. $4 : 8 = 3 : x.$

2. The area of a circle is 5 square inches. Find the area of a circle whose diameter is twice the diameter of the first.

3. The side of a square is 5 inches. Find the side of another square that contains 4 times as much area.

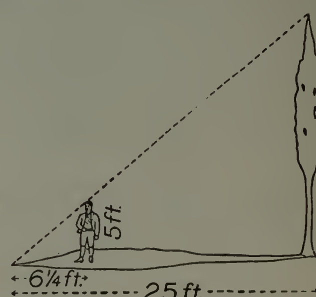
4. The sides of two regular octagons are as 1 to 3. What is the ratio of their areas?

5. A schoolroom has two square blackboards whose sides are 3 feet and 6 feet, respectively. What is the area of the first? Find the area of the second by applying the principle of similar figures.

6. A lady has two circular flower beds, one having a radius of 4 feet and the other a radius of 16 feet. How do they compare in area?

7. The sides of a triangle are 1 centimeter, 2 centimeters, and $2\frac{1}{2}$ centimeters. What are the sides of a similar triangle containing 25 times the area of the first?

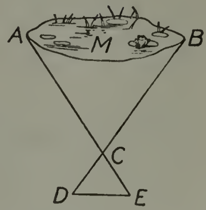
8. If the ratio of two similar triangles is 16, what is the ratio of their bases?



9. By the principle of similar figures, find the height of a poplar tree that casts a 25-foot shadow when a boy 5 feet tall casts a shadow $6\frac{1}{4}$ feet long.

10. When a telephone pole 30 feet high casts a shadow of 60 feet, what is the height of a church steeple that casts a shadow 300 feet long?

11. Suppose that A and B are two points on the opposite sides of the pond M, and we wish to find the distance between them.



Measure the distances AC and BC. Set a stake at E, a short distance from C in line with AC, and set another stake at D in line with BC, making the ratio of CD to CB equal to the ratio of CE to CA. Measure DE.

The triangles DCE and ACB are then similar and $AB : DE = AC : CE$.

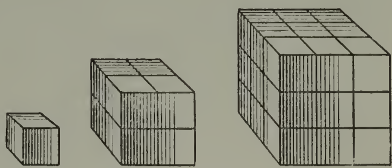
12. I wish to ascertain the distance between A and B on the opposite sides of a lake. From C, I measure the line AC, 2000 feet, and BC, 1500 feet. I set a stake at E in the line AC, 100 feet from C, and another at D in the line BC, 75 feet from C. The distance between D and E is 50 feet. What is the distance between A and B?

Similar Solids

628. Solids that have exactly the same shape though they differ in volume are **similar solids.**

The corresponding edges or other lines of similar solids are proportional.

629. **1.** Draw three cubes as in these figures, having edges proportional to 1, 2, and 3.

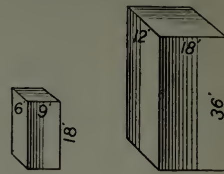


2. How many cubes the size of the first does the second contain? the third?

3. The volumes of cubes with edges proportional to 1, 2, and 3 are proportional to 1, 8, and 27, the cubes of the edges.

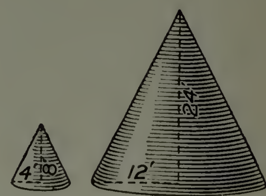
630. *The volumes of similar solids are proportional to the cubes of their corresponding lines.*

631. **1.** In what ratio are the corresponding dimensions of the two similar prisms shown here? Find the volume of the first prism (§ 616). By what number must you multiply this to get the volume of the second prism? What is the volume of the second prism?



Find the volume of the second prism by § 616 and compare results.

2. In what ratio are the corresponding dimensions of these two cones? How can you find the volume of the second cone from that of the first? Find the volume of each cone.



3. If a prism 5 inches high contains 30 cubic inches, how many cubic inches will a similar prism 10 inches high contain?

4. The volume of a sphere is 96 cubic feet. What is the volume of a sphere having a diameter half as long?

5. If the altitude of a cone that weighs 10 pounds is 2 feet, what is the altitude of a similar cone of the same material that weighs 270 pounds?

6. If a bowl 1.2 decimeters in diameter holds a certain quantity of milk, how many times this quantity will a similar bowl 1.8 decimeters in diameter hold?

7. A ball weighs 10 pounds. Find the weight of a ball of the same material, if its diameter is 3 times as great.

8. If a column 2 decimeters in diameter contains 728 cubic decimeters, what will be the volume of a similar column 1 decimeter in diameter?

9. How many more gallons of water can be contained in a tank 21 feet in diameter and 60 feet high than in a similar tank 40 feet high?

GENERAL REVIEW

632. **1.** Mr. Kirk purchased 60 shares of a Philadelphia stock company, engaged in the manufacture of infants' hose, at \$ $6\frac{3}{4}$ above par, brokerage $\frac{1}{8}$ %. How much did the stock cost him?

2. The company was capitalized at \$75,000. What per cent of the stock did Mr. Kirk own?

3. Mr. Kirk inspected the mill in detail. An order was received for 80 doz. pairs of infants' black hose size 4; 132 doz. size $4\frac{1}{2}$; 248 doz. size 5; 280 doz. size $5\frac{1}{2}$; 236 doz. size 6; and 112 doz. size $6\frac{1}{2}$. How many dozen pairs were ordered?

4. He found the weights per dozen pairs to be as follows:

Size	**4**	**$4\frac{1}{2}$**	**5**	**$5\frac{1}{2}$**	**6**	**$6\frac{1}{2}$**
Weight (ounces)	8	9	$10\frac{1}{2}$	$11\frac{3}{4}$	14	$15\frac{5}{8}$

Find the total weight of the yarn used for these stockings, allowing a waste in manufacture of $\frac{1}{2}$ oz. per dozen pairs.

5. The price of the yarn for this order ($832\frac{1}{2}$ lb.) was \$ $1.21\frac{1}{2}$ per pound and for dyeing it $6\frac{1}{2}$ ¢ per pound. If these two items were $62\frac{1}{2}$ % of the total cost of the stockings, what was the cost?

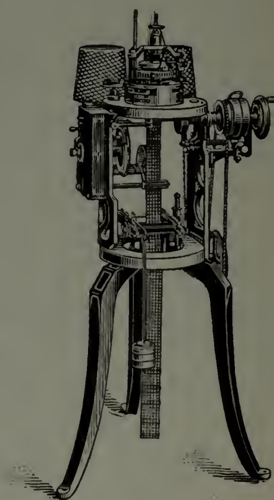
6. The purchaser was given his choice of buying the 1088 dozen pairs at \$1.90 per dozen, or buying size 4 at \$1.60 per dozen, $4\frac{1}{2}$ at \$1.70, 5 at \$1.80, $5\frac{1}{2}$ at \$1.90, 6 at \$2, and $6\frac{1}{2}$ at \$2.10. Which was the better offer and how much?

7. He accepted the second offer and paid \$ 2038 less 5 % discount. How much did the mill gain, the cost being \$1704.96?

8. The hose for this order were the product of $3\frac{2}{5}$ days' work. What was the daily output of the mill?

9. At the rate of 320 dozen per day, how many dozen pairs of hose would be turned out during August, if there are 4 Sundays? how many with 4 Saturday half-holidays?

10. In the knitting room, 48 "ribbers" were used for making the legs. $\frac{1}{4}$ of them could make 810 dozen of sizes 4 or $4\frac{1}{2}$, in a week of 6 days; $\frac{1}{2}$ of them, 1392 dozen of sizes 5 or $5\frac{1}{2}$; and the rest, 612 dozen of sizes 6 or $6\frac{1}{2}$. Find the daily capacity of a single machine on each size.



RIBBER

11. At a winding machine a woman in one day wound enough yarn for 100 dozen pairs of stockings, size 4 (8 oz. plus $\frac{1}{2}$ oz. waste per doz.), and $2\frac{7}{8}$ lb. over. How many pounds did she wind?

12. How much did she receive for winding 56 lb. at $2\frac{1}{2}$¢ per pound?

13. The 45 girls employed to make and knit the feet to the legs, that day footed 330 dozen pairs, earning in all $59.40. How many dozen did each girl foot? at what price? how much did each earn?

14. One of the girls employed to mend stockings in the finishing room averaged 104 dozen per day and received $1\frac{3}{8}$¢ per dozen. Find her wages for a year, deducting 8 days less than 20 % of the year for Sundays and holidays.

15. The mill and stock were insured for $45,000 at $1\frac{2}{5}$ %. What was the premium?

16. The year's sales, in dozens of pairs, were:

Size	4	$4\frac{1}{2}$	5	$5\frac{1}{2}$	6	$6\frac{1}{2}$
White	4256	5248	4912	4032	1312	1024
Black	1664	4288	8896	9744	8120	3696
Colored	1552	2624	3840	2432	2256	912

How many dozen of each size were sold? how many white? how many black? how many colored? Find the total number.

17. Referring to exercise 4, how many pounds of yarn were used for all the white hose, adding $\frac{1}{2}$ oz. per dozen for waste?

18. This yarn, 14,062 lb., was $1\frac{1}{4}\%$ less than the total amount bought for white hose, the shrinkage being due to loss in weight during bleaching. How much of this yarn was purchased?

19. Find the total cost of the 14,240 lb. of yarn for white stockings at an average price of \$1.21 per pound, and 4¢ per pound additional for bleaching.

20. How much yarn was used in the manufacture of the black and the colored hose, allowing as before $\frac{1}{2}$ oz. waste per dozen?

21. With this yarn (37,938 lb.) there was no shrinkage. Find its cost at \$1.21 per pound and $6\frac{1}{2}$¢ per pound for dyeing.

22. What was the cost of materials for all hose sold?

23. The cost of labor in the manufacture of stockings was 35¢ per dozen; the cost of boxes, holding $\frac{1}{2}$ dozen, was $1\frac{1}{2}$¢ each. Find the cost of these items for the 70,808 dozen made during the year; the cost with \$66,170.95 for materials added.

24. These expenses, \$93,077.99, were \$915.21 more than 85% of the entire expenses for the year, including supplies and repairs. What were the company's total expenses?

25. The average price at which these infants' hose were sold during the year was \$1.90 per dozen pairs. If the discount allowed was 5%, find the net receipts from the year's sales.

26. The only other source of income was the sale of waste, amounting to $\frac{1}{2}$ oz. per dozen pairs of hose. How much money was received by selling the waste at 12¢ per pound?

27. The company's expenses being \$108,426.80 and income \$128,073.97, find the gain; the gain per cent, to the nearest tenth.

28. From the net earnings (\$19,647.17) a surplus of \$1647.17 was laid aside for future needs, and the remainder was divided among the stockholders. What was the rate of dividend (capital \$75,000)?

29. Find the amount of dividend on Mr. Kirk's 60 shares.

30. As he had purchased the shares at $6\frac{3}{4}\%$ premium, find, to the nearest tenth, the per cent of profit on his investment.

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31. If a German mason earned 51 marks per week, find the amount he received per day (10 hours); per hour.

32. A British report states that women in a Japanese cotton mill receive 5*d.* per day, men $7\frac{1}{2}d$. How much more does a man earn in 20 days than a woman?

33. One year the average weekly wage of the English laborer was 31 shillings and of the American laborer \$11.679. Which laborer was the better paid, and how much better?

34. The English laborer expended 14 shillings per week for food, the American \$3.893. Which expended the greater amount, and how much greater? What part of his wages did each expend for food?

35. A coal miner's pay in England is 43*d.* per ton of coal raised to the surface, and the average amount raised per year in a certain coal district is 260 tons per man. Find the yearly wage in U. S. money.

36. What is the cost of painting the front, sides, and back of a house 22 feet wide, 32 feet deep, and 24 feet high, one coat, at 30¢ per square yard, no allowance being made for doors and windows?

37. How many pencils 7 inches long can be made from a block of red cedar 7 in. by $10\frac{1}{2}$ in. by $2\frac{1}{4}$ in., if the block is sawed into strips $3\frac{1}{2}$ in. wide and $\frac{3}{16}$ in. thick, each strip making the halves of 6 pencils?

38. The largest paper mill in the world has a roof area of $5\frac{1}{2}$ acres. How many square feet does it cover?

39. The mill uses 225 cords of wood for 200 tons of paper daily. How many cords of wood are used in 310 working days? How much paper is made?

40. A paper machine in this mill turns out a strip of paper 500 feet long and 125 inches wide each minute. Find the area of paper turned out in an hour.

41. The average spruce tree yields enough wood for 500 pounds of paper. The editions of 9 books published recently used 2,000,000 pounds of paper. How many trees were necessary to produce the paper for these editions?

42. In a recent year 1,233,150,000 pounds of paper were used in the United States, 77.6 % of which was used for newspapers, 16.4 % for books and periodicals, and the remainder in job printing. How much was used in each branch?

43. The per capita value of paper used in the United States per year is the greatest in the world, $1.66. When the population was 76,303,387, what was the value of paper used in a year?

44. A large oil company declared a dividend of $43,875,000 on a capital of $97,500,000. What per cent was paid?

45. The National City Bank, N.Y., one year paid a dividend of $3\frac{3}{4}$ % on its capital of $25,000,000. How much was paid?

46. A salesman for a typewriter company received $10 per week salary and 15 % commission on all sales. Find his earnings for one week, if he sold 1 machine for $95 and 3 for $87 apiece.

47. The assessed valuation of property in St. Louis one year was $428,510,340, and the tax rate was $1.47 per $100. Find the amount collected by taxation that year.

48. Mr. Gibson, a real estate agent, leased some property for 8 years at \$1800 per year. His commission was $2\frac{1}{2}$ % the first year and 1 % each succeeding year. Find his total commission.

49. A rice miller sold 560 sacks (100 lb.) of "choice" rice, and received \$220.50 as his commission at $7\frac{1}{2}$ %. For how much did he sell the rice, and at what price per pound?

50. A set of carpenter's tools listed at \$85 is subject to discounts of 60 % and 10 %. Find the net cost of the tools.

51. A grocer paid \$57.60 for 2 cases of cocoa. There were 12 boxes per case, each box holding 6 pounds of cocoa in $\frac{1}{4}$-pound tins. He sold the cocoa at 13¢ per tin. What was his total gain? his per cent of gain? his gain per pound?

52. Using the following ingredients, a baker made 252 lb. of vanilla-cream biscuit, which he sold at 16¢ per pound:

1 bbl. flour @ \$3.85	$3\frac{1}{2}$ gal. milk at 6¢ per qt.
20 lb. butter @ 26¢	2 qt. glycerine @ 75¢
16 lb. lard @ $11\frac{1}{2}$¢	10 oz. soda at 8¢ per lb.
64 lb. powdered sugar @ $7\frac{3}{4}$¢	Ammonia, salt, vanilla, 96¢

2 gal. eggs (10 eggs to a pint) at 24¢ per doz.

Find the cost; amount received; gain; per cent of gain. What part of the total cost was the cost of the eggs?

53. An engine for exhibition at St. Louis reached the grounds on 21 cars, the load of each averaging $34\frac{2}{7}$ tons. Find the entire weight of the engine.

54. A camera has been invented that takes 1100 images of insects in flight every second. How many images can be obtained in 2 min. 31 sec.?

55. A dragon fly's wings beat 28 times per second. The number of beats per second made by a bee's wings is 22 more than 6 times as many as this, and is 8 less than $\frac{3}{5}$ of the number of beats per second made by a house fly's wings. At what rate do a bee's wings beat? a house fly's wings?

56. The projectile from a $9\frac{1}{2}$-inch Krupp gun penetrated 25 cm. of armor plate 4500 m. away. At a distance $33\frac{1}{3}\%$ less, the penetration was 20% deeper. Find this distance, and the depth penetrated.

57. Four cannon on board a Japanese warship fire 2 shots apiece per minute. If used continuously for $\frac{1}{4}$ of an hour, they would discharge shot to the value of $48,000. How much does each shot cost?

58. The charges for wireless telegraphing from the coast of England to passing vessels, by the Marconi system, are $6\frac{1}{2}d.$ per word. Find the cost in United States money of a message of 18 words.

59. The British Admiralty pays £96 10*s.* per year per instrument for a period of 15 years for Marconi wireless telegraph instruments, after which it will own them. Find the cost, in U. S. money, of one instrument.

60. Some of the records of winners in the Olympic games at Athens one year were:

EVENT	NAME AND COUNTRY OF WINNER	RECORD
Throwing the discus	Sheridan, U. S.	41 m. 46 cm.
Throwing the stone	Giorgantas, Greece	19 m. $92\frac{1}{2}$ cm.
Running broad jump	Prinstein, U. S.	7 m. 20 cm.
Single hand lifting	Steinbach, Austria	76.55 Kg.
100-meter dash	Hahn, U. S.	$11\frac{1}{5}$ sec.
1500-meter race	Lightbody, U. S.	4 min. $19\frac{2}{5}$ sec.
400-meter swimming race . . .	Scheff, Austria	6 min. $23\frac{4}{5}$ sec.

Express the first three records in feet and inches to the nearest $\frac{1}{4}$ inch; the fourth, in pounds to the nearest pound.

How much greater is 100 meters than 100 yards?

How many feet less is 1500 meters than 1 mile?

How much greater or less is 400 meters than $\frac{1}{4}$ mile?

61. In Edinburgh 25 million gallons of water per day are supplied to a population of 455,000. Find, to the nearest gallon, the daily supply per person.

62. From the 1200 persons who travel between Berlin and Hamburg every day, 19,200 marks are collected in fares. What is the average fare in our money?

63. German railways issue three classes of mileage books for 1000 Km. each. The first is sold for 60 marks, the second for 40 marks, and the third for 25 marks. What is the price per mile in pfennigs and in cents for each class?

64. The municipal restaurants in Freiburg, Germany, serve breakfast and supper at 5 ¢ each and dinner at $6\frac{1}{2}$ ¢. How much must be paid in German money for 6 days' board?

65. How many of the 10,622 miles of railway in Hungary does the government operate, if it operates 83 %?

66. The copper conductor of the cable between Ireland and Newfoundland weighs $633\frac{3}{4}$ tons, which is 650 pounds per mile. How long is the cable?

67. A cable message was sent at 2:33 P.M. from Washington (75° W.) to London (0°), where it was received $6\frac{3}{4}$ seconds later. At what time was it received?

68. Mexico has one of the tallest steel chimneys in America, 230 feet high. What is its height expressed in meters?

69. The load of a camel is about $4\frac{1}{2}$ metric quintals. Find the cost of transporting 21 camel loads of goods in Turkey from Samsun to Kharput at $4.34 per metric quintal.

70. The propeller shafts of the steamship *Deutschland* are 40 m. in length and 630 mm. in diameter, and terminate in bronze propellers 7 m. in diameter. What are these dimensions in feet and inches?

71. The temperature of the Gulf Stream averages about 28° C. What is its temperature in the Fahrenheit scale?

72. To build the new turbine Cunard steamships, the British Government lent the Cunard Co. £2,600,000 with interest at $2\frac{3}{4}\%$, the term of interest to begin 1 year after completion of the vessels, which occurred 26 months after the loan was made. Find the value of the use of this money at $2\frac{3}{4}\%$ until it began to draw interest.

73. This loan is payable in 20 equal yearly payments. Find the interest for a year on the sum due after the 10th payment.

74. The United States Government pays $4 per statute mile for the transatlantic transportation of mail by American vessels. Over route number 57, from New York to Southampton, the contract price for 47 trips is $690,483.20. What is the distance, to the nearest tenth of a mile?

75. One year the weight of the first-class mail over this route was 350,810 pounds, and of other mail 2,384,406 pounds. When not under contract American vessels receive the postage rate, 5¢ per $\frac{1}{2}$ ounce for first-class mail, and 1¢ per 2 ounces for other mail. How much was saved that year by contracting for the carriage at $4 per statute mile?

76. The first-class mail matter carried to Great Britain one year was 165,148,403 g. This was 29% of all the first-class transatlantic mail. What was the entire amount?

77. A refrigerator car costs about $800. Reckoning interest at 4%, depreciation at 8%, repairs at $3\frac{3}{4}\%$, and general expenses at $4, find the annual cost of operating a car.

78. The monthly rental for the car received by the owners from the railroads is $19. What is the annual income at this rate? What per cent of income is obtained on the cost of the car?

79. The ice for these cars costs $2.50 per ton loaded on the cars. An average price per car on one railroad for icing from New Orleans to Chicago, 923 miles, is $23. Find the number of tons of ice used, and the cost of icing per mile.

80. One firm in the private-car and meat-packing business on a capital of \$110,500,000 cleared \$47,727,412 in one year. What per cent, to the nearest .01%, did the stock pay?

81. The cargo of a sailing vessel carrying 100,000 bushels of wheat worth 72¢ per bushel, from New York to Glasgow, is insured at $1\frac{3}{4}$%, while a steam vessel's cargo of the same value is insured at $\frac{5}{8}$%. What is the difference in the premiums?

82. Gold bullion is insured between London and New York for 25¢ per hundred pounds (avoirdupois). What is the premium on a shipment of \$1,000,000 in gold, worth \$26.87 per avoirdupois ounce?

83. A cargo of grain worth \$80,000 in an American iron-hull steamship sailing between San Francisco and Liverpool is insured at $1\frac{1}{4}$%. What is the premium?

84. The premium on a British vessel carrying an equivalent cargo between these ports is \$200 less. What is the rate for the British steamship?

85. Find the insurance premium on a shipment of coffee consisting of 3750 bags of 60 Kg. each, if the value is 16¢ per kilogram and the insurance $\frac{7}{16}$%.

86. The cost of shipping cotton goods from New York to Shanghai is \$6.87 per 40 cubic feet of space (1 ton). What would be the cost of a shipment occupying 25,000 cubic feet?

87. The charge for piloting a boat from Sandy Hook to New York is \$4.13 per foot of the vessel's draught. What are the charges for piloting the *Deutschland*, which draws 29 feet of water?

88. A wholesale carpet dealer imported 24,000 yards of Brussels carpet, 27 inches wide, paying 83¢ a yard for it. What was the total cost, including duty at 28¢ per square yard and 40% ad valorem?

89. The duty on French plate glass is 30¢ per square foot. How much duty must be paid on a shipment of 9 plates, each $8\frac{1}{2}$ ft. by 11 ft.?

90. One year a large city levied a tax of $38,403,761.18 upon property assessed at $2,016,947,622. What was the tax rate per dollar, to the nearest hundredth of a mill?

91. The assessed valuation was about 40% of the true value. What was the tax of a man owning property worth $250,000?

92. A German department store whose receipts were 450,000 marks was taxed $.8\frac{4}{45}$ mark per hundred. Find the amount of the tax in marks; in United States money.

93. A harbor tax of 2% on the general cargo of a vessel is charged for the use of the harbor at Schiedam, Holland. What was the tax on a vessel carrying a cargo worth $163,180?

94. Elevators in some New York buildings move at the rate of 700 feet per minute. Find the ratio of this speed to that of surface cars that average 10 miles per hour.

95. A double eagle weighs $21\frac{1}{2}$ pennyweights. The ingots from which double eagles are stamped in one mint weigh 72 ounces apiece. What is the value of such an ingot?

96. From one ingot 65 double eagles are stamped, the remaining metal being recast into another ingot. How many ounces from each ingot are recast?

97. The sweepings of the San Francisco mint one year brought $$.0064\frac{1}{4}$ per ounce avoirdupois. The entire receipts from this source were $14,257. How many pounds were sold?

98. The average price of silver in London recently was $26\frac{3}{8}d.$ per ounce (troy). Find the value of 25 pounds of silver in U. S. money.

99. A pile of 9600 new silver dollars weighed $687\frac{1}{2}$ pounds (troy). How many grains did each dollar weigh?

100. The earthquake that destroyed San Francisco in 1906 occurred at 5:13 A.M., the news being received in New York by telegraph at 9:15 A.M. How long was this after the event?

101. The brokerage for buying or selling London exchange is \$5 per £10,000. What is the brokerage on £3500?

102. In London a broker's commission is $\frac{1}{64}$% on continental exchanges. Find in U. S. money a London broker's commission on 48,000 marks.

103. The report of 147 joint stock, fire-insurance companies for a period of five years showed a premium collection of \$196,532,866, 8.61% of which was gain. Find the gain.

104. With U. S. 3s reg. at $102\frac{5}{8}$, what is the annual return on an investment of \$35,962.50, brokerage $\frac{1}{8}$%?

105. How much must I invest in Imperial Japanese $4\frac{1}{2}$s at $91\frac{5}{8}$ to yield an annual income of \$1350, brokerage $\frac{1}{8}$%?

106. One issue of 1000-dollar New York City 3% bonds amounting to \$3,125,000 sold at a premium of \$45,877. What was the selling price per bond?

107. For one issue of \$3,555,000 of New York City bonds $112\frac{1}{7}$ was paid, the highest price paid up to that time for municipal bonds. How much did the entire sale bring?

108. When the shadow of a man 6 feet tall was $4\frac{1}{2}$ feet long, the shadow of a tree beside which he stood was 54 feet long. How tall was the tree?

109. A pencil 4 inches long, held 3 feet from the eye, just covers a tree 45 feet high. How far distant is the tree?

110. A balloon will remain in air as long as its own weight and that of the gas it contains is less than the weight of air displaced. Air weighs 31 grains per 100 cu. in. How many cubic feet of gas are required to support a balloon weighing 1674 lb., if the gas used is $\frac{1}{3}$ as heavy as air?

Measures of Length

12 inches = 1 foot
3 feet = 1 yard
$16\frac{1}{2}$ feet = 1 rod
320 rods = 1 mile (statute)

1 mi. = 1760 yd. = 5280 ft. = 63,360 in.

A **nautical mile** (**knot**) = 6080.27 ft., or approximately 1.15 mi.

A **furlong** = $\frac{1}{8}$ mi.; a **fathom**, used in measuring the depth of water, is 6 ft.; a **hand**, used in measuring the height of horses, is 4 in.

Measures of Surface

144 square inches = 1 square foot
9 square feet = 1 square yard
$30\frac{1}{4}$ square yards = 1 square rod
160 square rods = 1 acre

1 acre = 43,560 sq. ft.

An acre of land in the form of a square is very nearly 209 ft. on a side.

A tract of land 1 mile square is often called a **section**.

100 sq. ft. of roofing, flooring, or slating is called a **square**.

Measures of Volume

1728 cubic inches = 1 cubic foot
27 cubic feet = 1 cubic yard

A pile of wood 8 ft. long, 4 ft. wide, and 4 ft. high, or 128 cu. ft. of wood, is called a **cord**.

Surveyors' Linear Measures

100 links = 1 chain
80 chains = 1 mile

This chain, called **Gunter's chain**, is 4 rd., or 66 ft., long. **1 link** = 7.92 in.

Links are written as hundredths of a chain.

Surveyors' Square Measures

10 square chains = 1 acre
640 acres = 1 square mile

The convenience of this system lies in the easy reduction of square chains to acres, by moving the decimal point one place toward the left.

Liquid Measures

4 gills = 1 pint
2 pints = 1 quart
4 quarts = 1 gallon

1 gal. = 231 cu. in.; 1 cu. ft. = $7\frac{1}{2}$ gal., approximately. A gallon of water weighs about $8\frac{1}{3}$ lb.; a cubic foot of water weighs about $62\frac{1}{2}$ lb., or 1000 oz.

In measuring the capacity of cisterns, etc., $31\frac{1}{2}$ gal. = 1 **barrel**.

Dry Measures

2 pints = 1 quart
8 quarts = 1 peck
4 pecks = 1 bushel

1 bu. = 2150.42 cu. in., or approximately $1\frac{1}{4}$ cu. ft.

Our bushel is the **Winchester bushel**. In form it is a cylinder $18\frac{1}{2}$ in. in diameter and 8 in. deep. This has been displaced in England by the **imperial bushel** of 2218.192 cu. in.

Avoirdupois Weight

16 ounces = 1 pound
100 pounds = 1 hundredweight
2000 pounds = 1 ton
1 **long** or **gross ton** = 2240 pounds
1 av. lb. = 7000 gr.; 1 av. oz. = $437\frac{1}{2}$ gr.

Troy Weight

24 grains = 1 pennyweight
20 pennyweights = 1 ounce
12 ounces = 1 pound
1 troy lb. = 5760 gr. = $\frac{5760}{7000}$ av. lb.
1 troy oz. = 480 gr., or about 1.1 av. oz.

Apothecaries' Weight

This is used to some extent in filling prescriptions. The **grain, ounce,** and **pound** are the same as in troy weight, but the ounce is divided differently.

20 grains (gr.)	= 1 scruple . . .	sc. or ℈
3 scruples	= 1 dram . . .	dr. or ʒ
8 drams	= 1 ounce . . .	oz. or ℥
12 ounces	= 1 pound . . .	lb. or ℔

Apothecaries' Liquid Measures

60 drops (gtt.) or minims (♏)	= 1 fluid dram . . .	*f*ʒ
8 fluid drams	= 1 fluid ounce . . .	*f*℥
16 fluid ounces	= 1 pint	O.
8 pints	= 1 gallon	Cong.

Measures of Time

60 seconds = 1 minute
60 minutes = 1 hour
24 hours = 1 day
7 days = 1 week
365 days = 1 year
366 days = 1 leap year

Thirty days have September,
April, June, and November.
All the rest have thirty-one,
Save February, which alone
Has twenty-eight, and one day more
We add to it one year in four.

10 years = 1 **decade**; 100 years = 1 **century.**

The earth revolves around the sun in 365 days 5 hours 48 minutes 46 seconds. This is the **solar year,** and is nearly $365\frac{1}{4}$ days.

To correct the errors in the calendar, made by disregarding the fraction of a day over 365 days, centennial years divisible by 400 and other years divisible by 4 are lengthened 1 day, Feb. 29. These years are **leap years.**

Measures of Angles and Arcs

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°)
360 degrees = 4 right angles or 1 circumference
90° of angle = 1 **right angle**; 90° of arc = 1 **quadrant.**

Counting Table

2	=	1 pair
20	=	1 score
12	=	1 dozen
12 dozen	=	1 gross
12 gross	=	1 great gross

Stationers' Measures

24 sheets = 1 quire
20 quires = 1 ream

Paper is quite generally sold by the 100, 500, and 1000 sheets; also by the pound.

Metric System

The principal simple units in the metric system are the **meter**, the **gram**, and the **liter**, together with their decimal parts and multiples named by means of the following prefixes:

deci means .1	**deka** means 10
centi means .01	**hekto** means 100
milli means .001	**kilo** means 1000

The unit of area is the **square meter**. In forming its parts and multiples, 100 square units of any order make 1 of the next higher order.

The unit of volume is the **cubic meter**. In forming its parts and multiples, 1000 cubic units of any order make 1 of the next higher order.

1 cu. cm. of water weighs 1 gram; 1 cu. dm., or 1 liter, of water weighs 1 kilogram; 1 cu. m. of water weighs 1000 kilograms, or 1 **metric ton.**

In land measures, a square dekameter is called an **are**; .01 of an are = 1 **centare**; 100 ares = 1 **hektare**, the commonest unit.

In measuring wood, a cubic meter is called a **stere**.

Equivalents

Common to Metric

1 yd.	= .9144 m.
1 mi.	= 1.60935 Km.
1 sq. yd.	= .836 sq. m.
1 A.	= .4047 Ha.
1 cu. yd.	= .765 cu. m.
1 qt. (dry)	= 1.1012 l.
1 qt. (liq.)	= .94636 l.
1 bu.	= .35239 Hl.
1 oz. (troy)	= 31.10348 g.
1 lb. (av.)	= .45359 Kg.
1 T.	= .90718 M. T.

Metric to Common

1 m.	= 39.37 in.
1 Km.	= .62137 mi.
1 sq. m.	= 1.196 sq. yd.
1 Ha.	= 2.471 A.
1 cu. m.	= 1.308 cu. yd.
1 l.	= .908 qt. (dry)
1 l.	= 1.0567 qt. (liq.)
1 Hl.	= 2.8377 bu.
1 Kg.	= 32.1507 oz. (troy)
1 Kg.	= 2.2046 lb. (av.)
1 M. T.	= 1.1023 T.

COMPOUND INTEREST TABLE

Amount of $ 1, at various rates, compound interest, 1 to 20 years

YEARS	1%	1¼%	1½%	2%	2½%	3%
1	1.010000	1.012500	1.015000	1.020000	1.025000	1.030000
2	1.020100	1.025156	1.030225	1.040400	1.050625	1.060900
3	1.030301	1.037971	1.045678	1.061208	1.076891	1.092727
4	1.040604	1.050945	1.061364	1.082432	1.103813	1.125509
5	1.051010	1.064082	1.077284	1.104081	1.131408	1.159274
6	1.061520	1.077383	1.093443	1.126162	1.159693	1.194052
7	1.072135	1.090850	1.109845	1.148686	1.188686	1.229874
8	1.082857	1.104486	1.126493	1.171659	1.218403	1.266770
9	1.093685	1.118292	1.143390	1.195093	1.248863	1.304773
10	1.104622	1.132271	1.160541	1.218994	1.280085	1.343916
11	1.115668	1.146424	1.177949	1.243374	1.312087	1.384234
12	1.126825	1.160755	1.195618	1.268242	1.344889	1.425761
13	1.138093	1.175264	1.213552	1.293607	1.378511	1.468534
14	1.149474	1.189955	1.231756	1.319479	1.412974	1.512590
15	1.160969	1.204829	1.250232	1.345868	1.448298	1.557967
16	1.172579	1.219889	1.268986	1.372786	1.484506	1.604706
17	1.184304	1.235138	1.288020	1.400241	1.521618	1.652848
18	1.196148	1.250477	1.307341	1.428246	1.559659	1.702433
19	1.208109	1.266108	1.326951	1.456811	1.598650	1.753506
20	1.220190	1.281935	1.346855	1.485947	1.638616	1.806111
YEARS	3½%	4%	4½%	5%	6%	7%
1	1.035000	1.040000	1.045000	1.050000	1.060000	1.070000
2	1.071225	1.081600	1.092025	1.102500	1.123600	1.144900
3	1.108718	1.124864	1.141166	1.157625	1.191016	1.225043
4	1.147523	1.169859	1.192519	1.215506	1.262477	1.310796
5	1.187686	1.216653	1.246182	1.276282	1.338226	1.402552
6	1.229255	1.265319	1.302260	1.340096	1.418519	1.500730
7	1.272279	1.315932	1.360862	1.407100	1.503630	1.605782
8	1.316809	1.368569	1.422101	1.477455	1.593848	1.718186
9	1.362897	1.423312	1.486095	1.551328	1.689479	1.838459
10	1.410599	1.480244	1.552969	1.628895	1.790848	1.967151
11	1.459970	1.539454	1.622853	1.710339	1.898299	2.104852
12	1.511069	1.601032	1.695881	1.795856	2.012197	2.252192
13	1.563956	1.665074	1.772196	1.885649	2.132928	2.409845
14	1.618695	1.731676	1.851945	1.979932	2.260904	2.578534
15	1.675349	1.800944	1.935282	2.078928	2.396558	2.759032
16	1.733986	1.872981	2.022370	2.182875	2.540352	2.952164
17	1.794676	1.947901	2.113377	2.292018	2.692773	3.158815
18	1.857489	2.025817	2.208479	2.406619	2.854339	3.379932
19	1.922501	2.106849	2.307860	2.526950	3.025600	3.616528
20	1.989789	2.191123	2.411714	2.653298	3.207136	3.869684

INDEX

ANSWERS

Page 17.—**2.** 254,378. **3.** 2.74993. **4.** 85.33761. **5.** 186,594,945,649.

Page 18.—**6.** $535.45. **7.** $1923.94. **8.** $30,592.94. **9.** $151,189.40. **10.** $2,974,622.52. **11.** 123,492,548. **12.** 969,103.272. **13.** $9932.30. **14.** 2,597,849.

Page 19.—**15.** $4673.50. **16.** $10.25. **17.** $156.75. **18.** 327 vessels. **19.** $3,146,475. **20.** 11,898,000 tons. **21.** 1617 people. **22.** 34,100 lb. **23.** 4618 mi. **24.** 6939 mi.

Page 20.—**25.** $66,654. **26.** 2,186,140 lb. **27.** 905,866 persons. **28.** $3,946,354.63. **29.** $140,907,000.

Page 22.—**2.** 22.4. **3.** 1761. **4.** $65.70. **5.** 21.823. **6.** 4.67. **7.** 516. **8.** $270.67. **9.** 533.29. **10.** .158. **11.** 6676. **12.** $758.55. **13.** .77151. **14.** 4.28. **15.** 3043. **16.** $160.00. **17.** .00727. **18.** 2.6. **19.** 4865. **20.** $299.91. **21.** 2.0566. **22.** 5.59. **23.** 5889. **24.** $183.51. **25.** 82.6.

Page 23.—**27.** 2325. **28.** 3718. **29.** 1001. **30.** $36.35. **31.** $57.20. **32.** 2198. **33.** .1569. **34.** 5.615. **35.** $55.73. **36.** $468.75. **37.** .6049. **38.** 38.22. **39.** .424. **40.** $89.12. **41.** $117.86. **42.** 33,418. **43.** 38,800. **44.** 56,535. **45.** 40,833. **46.** 21,325. **47.** $138.85. **48.** $159.43. **49.** 6.2445. **50.** 69.923. **51.** 3.4614. **52.** 3.3585. **53.** 12.975. **54.** $62,517.12. **55.** $46,226.05.

Page 24.—**64.** 2,197,756 sq. mi. **65.** 239,182. **66.** 13,761 ft. **67.** $3635. **68.** 409,500 lb. **69.** 451,168 mi. **70.** 132,550 boxes. **71.** 10,272 boxes. **72.** 353.8 mi.; Mon., 59.2 mi.; Tues., 61.7 mi.; Wed., 57.8 mi.; Thurs., 93.7 mi.; Fri., 81.4 mi.

Page 25.—**73.** 1670 lb. **74.** 6625 gal. **75.** 117,280 lb. **76.** 202,078. **77.** 3.35 in. **78.** $235.46. **79.** $722. **80.** 369.24 lb.

Page 28.—**1.** Bal. against Geo. Griffin, $1.19. **2.** Bal. in favor of Thomas Hinds & Co., $407.46. **3.** Bal. against Johnson & Mason, $275.90.

Page 31.—**2.** 1840. **3.** 4500. **4.** 26.6. **5.** 66.6. **6.** 336. **7.** 1980. **8.** 4350. **9.** 258. **10.** 520. **11.** 69,000. **12** 429,800. **13.** 22.5. **14.** 24. **15.** 15,420. **16.** 618. **17.** 8,993,600. **18.** 21,800. **19.** 45.

Page 32.—**21.** 31,330. **22.** 62,920. **23.** 45,668. **24.** 47,232. **25.** 276. **26.** 12.4. **27.** 2622. **28.** 760.32. **29.** 472.5. **30.** $1768.80. **31.** $675.00. **32.** $989.33. **33.** $8347.20. **34.** $22,495.40. **35.** $11,340.00. **36.** $23,752.08. **37.** $12,613.32. **38.** $34,018.92. **42.** 298,284. **43.** 233,530. **44.** 7401.24. **45.** 17,228.31. **46.** 6227.97. **47.** 41.85. **48.** 2210.03. **49.** 370,901.22. **50.** 41,861.82. **51.** 47,821.86. **52.** 620.62. **53.** 106,550. **54.** 1,372,800. **55.** 351,960. **56.** 36,124.8. **57.** 67,830. **58.** 347.01. **59.** $77,130. **60.** $64,740. **61.** $284,640. **62.** $962,600. **63.** $147,420. **64.** 211.38. **65.** 33.44.

Page 33. — **66.** 4.776 in. **67.** $32.94. **68.** 148 lb. **69.** $432,000. **70.** $1394. **71.** 13,580 lb. **72.** 2,623,261.5 lb. **73.** $.58. **74.** 25,200 eggs. **75.** $28.80. **76.** $161. **77.** $27.84; 3480 qt.

Page 35. — **2.** 14.4. **3.** 28.7. **4.** 1.74. **5.** .112. **6.** 4.5. **7.** .402. **8.** 42.7. **9.** 1.584. **10.** .07328. **11.** .29995. **12.** .2731. **13.** 2.23884. **15.** 14.08. **16.** 20.72. **17.** 7.98. **18.** 630. **19.** 58.08. **20.** 22.348. **21.** 14.28. **22.** .3074. **23.** 3.542. **24.** 321. **25.** 11.68. **26.** 59.78. **27.** 3.0024. **28.** 552.6. **29.** 529.6. **30.** 5684.9. **31.** 503.03. **32.** 175.392. **33.** 2514.3. **34.** 271.35. **35.** 25.221. **36.** 2.1862. **37.** 4956.864. **38.** 313.053. **39.** 449.8. **40.** 57.5603. **41.** 3320.694.

Page 36. — **42.** 2025.782. **43.** 3713.497. **44.** 854.1354. **45.** 1362.5976. **46.** 225.4875. **47.** 432.5136. **48.** 335.8184. **49.** 47.3008. **50.** 684.74. **51.** 150.3255. **52.** 366.29104. **53.** 47.37555. **54.** .0369. **55.** .01504. **56.** 96.4464. **57.** 124.32574. **58.** 96,144. **59.** 66.528. **60.** 301.176. **61.** .329765. **62.** 2.97242. **63.** 2,500,000. **64.** 8,133,000. **65.** 271,875. **66.** $6,258,845.376. **67.** $795,850. **68.** $2730. **69.** $2.60.

Page 37. — **70.** 443.52 tons. **71.** 864.864 tons. **72.** 487,872 cu. ft. **73.** 60.00040932 ft.; 60.0122796 ft. **74.** Zinc, .005999544 ft. longer. **75.** .2872404 in. **76.** 1,655,500 bbl. **77.** 24,500 tons. **78.** $6747.

Page 39. — **1.** $4566.20. **2.** $5355. **3.** $4190. **4.** $246.50. **5.** $60.75. **6.** $2607.53. **7.** $168.77.

Page 41. — **2.** 444. **3.** 679. **4.** 4.49. **5.** 1288.4. **6.** 15.875. **7.** $1.38\frac{2}{3}$. **8.** 875. **9.** 202.75. **10.** .0313. **11.** $.0130\frac{5}{6}$. **12.** 16.84. **13.** 5.625. **14.** .53275. **15.** 89.9. **16.** .0418. **17.** $166\frac{4}{6}$. **18.** $11.84\frac{6}{7}$. **19.** 1.3215. **20.** $157.7\frac{2}{3}$. **21.** .0362. **22.** 59.5. **23.** .288. **24.** 3.64. **25.** $.602\frac{1}{12}$.

Page 43. — **2.** 96. **3.** 1.305. **4.** 39.5. **5.** $8.46\frac{3}{9}$. **6.** 23.725. **7.** 8.81. **8.** .5264. **9.** .01181. **10.** .00224. **11.** .03205. **12.** 16.71. **13.** 5.2725. **14.** .52025. **15.** .279. **16.** .52.

Page 44. — **18.** 521, rem. 24. **19.** 843. **20.** 427. **21.** 1111, rem. 206. **22.** 529, rem. 605. **23.** 4393, rem. 6. **24.** 375. **25.** 1990, rem. 1200. **28.** $13.27. **29.** $24.95. **30.** $26.86. **31.** $23.56. **32.** $62.18. **33.** $59.75. **34.** $42.10. **35.** 26.48. **36.** 4.235. **37.** 6.414. **38.** .0582. **39.** 5.37. **40.** .4162. **41.** .0000624.

Page 45. — **42.** 6.3−. **43.** 15.6+. **44.** 2.7+. **45.** 3.9+. **46.** 1.6−. **47.** 35.7+. **48.** 76.0−. **49.** 97.3−. **50.** .28+. **51.** 11.11+. **52.** 8.33+. **53.** 7.27+. **54.** 16.98−. **55.** 13.95+. **56.** 10.44−. **57.** .92+. **58.** 2.364−. **59.** .423+. **60.** .862+. **61.** 1.316−. **62.** 1.818+. **63.** .419+. **64.** .042. **65.** .439−. **66.** 750 men. **67.** $27 per month. **68.** 321 mi. **69.** $19.30−. **70.** $35. **71.** 334.32− tons; 13.93− tons. **72.** $342.72. **73.** $.119.

Page 46. — **74.** 356− hundred lb. **75.** 15,000 bbl. **76.** 1325.5 lb. **77.** $1398.14−; $77.67+.

Page 47. — **4.** 101.5625. **5.** 16. **6.** 1024. **7.** .90625. **8.** 4.375. **9.** 2.25. **10.** 5.76. **11.** 56.8. **12.** 4260. **13.** 1.94. **14.** 2109.375. **15.** 1728. **16.** 715. **17.** .206. **18.** 46,215. **19.** 3.0628. **20.** .4173. **21.** .0706. **22.** .8094. **23.** 5.5707. **24.** .1142. **25.** 3 hr. **26.** 3.6 hr. **27.** 1600 bu. **28.** 38.4. **29.** 3.14 times.

Page 48. — **1.** 43. **2.** 15. **3.** 1.12. **4.** 4. **5.** .6. **6.** 20. **7.** 62. **8.** 9. **9.** 10. **10.** 18. **11.** 6. **12.** 10.

Page 49. — **1.** $17.60. **2.** 8 hr. 15 min.; 9 hr. **3.** $48\frac{8}{9}$ mi. per hr. returning. **4.** 302 passengers. **5.** $2845.76. **6.** $140.90. **7.** $15.40.

Page 50. — **8.** 15 days. **9.** 32,916 mi. **10.** 1125 tons. **11.** $63,375. **12.** $2625; $34,125. **13.** $42,832.50.

Page 51. — **14.** $20,542.50; $1580.19+. **15.** 77.042 mi. **16.** 12.10185+ ft. **17.** $408. **18.** $333. **19.** $694.40. **20.** $487.61. **21.** $227.50; $318.50; $682.50; $127.40; $1365.

Page 52. — **22.** 1,247,128 bales. **23.** 213,161 bales. **24.** 322,086 bales. **25.** 509 bales. **26.** 1st, $8.28; 2d, $9.36; 3d, $5.28; 4th, $6.36; yr., $29.28. **27.** $2 44.

Page 53. — **28.** 41.41 million greater than France; 38.411 million greater than Great Britain; 34.51 million greater than Japan; 21.823 million greater than Germany; 60.628 million less than Russia. **29.** 14.215 million. **30.** $10,786,500. **31.** Cost in Paris: butter, $2.50; milk, $.60; beef, $3.20; sugar, $1.20; tea, $2.45. **32.** 83,200. **33.** 4375 sacks; 1,225,000 lb.; 6250 bbl. **34.** $4,706,100. **35.** 26,040.

Page 54. — **36.** $40.45. **37.** $15,300,000. **38.** $2.10. **39.** $389,102,803; to the U.S. **40.** 177,085 visitors. **41.** 6,250,000 lb. to Great Britain; 1,700,000 lb. to U.S. **42.** $248,640,000. **43.** 423.6 lb. **44.** $6900. **45.** 537,105 mi.

Page 58. — **4.** $2^4, 3^2$. **5.** $2^2, 5, 13$. **6.** $3^2, 5, 7$. **7.** $2^6, 3^2$. **8.** $3^4, 11$. **9.** $2^5, 3, 7$. **10.** $2, 3, 5^2, 7$. **11.** $2, 3^2, 7^2, 11$. **12.** $2^2, 3, 5, 7, 11$. **13.** $2^7, 5, 101$. **14.** $2^3, 3^3, 11, 13$. **15.** $2^5, 5^3, 11$. **17.** 24. **18.** 24. **19.** 70. **20.** 32. **21.** 8. **22.** 12.

Page 59. — **2.** 8. **3.** $4\frac{1}{2}$. **4.** 2. **5.** 2. **6.** 168.

Page 60. — **8.** 1.25. **9.** .05. **10.** 27. 5. **11.** 4. **12.** 140. **13.** 1.8. **14.** 5.44. **15.** 42. **16.** 19.8. **17.** 21. **18.** 26.4. **19.** 5.25. **20.** $63. **21.** $1.12. **22.** 20 doz. **23.** 48 bbl. **24.** $.60.

Page 63. — **2.** $\frac{42}{96}$. **3.** $\frac{75}{90}$. **4.** $\frac{42}{108}$. **5.** $\frac{108}{144}$. **6.** $\frac{119}{136}$. **7.** $\frac{125}{150}$. **8.** $\frac{143}{156}$. **9.** $\frac{195}{208}$. **10.** $\frac{285}{300}$. **11.** $\frac{32}{48}$ and $\frac{20}{48}$; $\frac{64}{96}$ and $\frac{40}{96}$; $\frac{96}{144}$ and $\frac{60}{144}$; $\frac{160}{240}$ and $\frac{100}{240}$; $\frac{256}{384}$ and $\frac{160}{384}$; $\frac{352}{528}$ and $\frac{220}{528}$.

Page 64. — **13.** $\frac{7}{12}$. **14.** $\frac{4}{5}$. **15.** $\frac{3}{5}$. **16.** $\frac{1}{3}$. **17.** $\frac{3}{4}$. **18.** $\frac{5}{6}$. **19.** $\frac{2}{5}$. **20.** $\frac{3}{8}$. **21.** $\frac{1}{3}$. **22.** $\frac{7}{8}$. **23.** $\frac{5}{8}$. **24.** $\frac{5}{6}$. **25.** $\frac{2}{3}$. **26.** $\frac{5}{12}$. **27.** $\frac{7}{12}$. **28.** $\frac{1}{3}$. **29.** $\frac{3}{8}$. **30.** $\frac{1}{3}$. **31.** $\frac{1}{4}$. **32.** $\frac{1}{3}$.

Page 65. — **2.** $\frac{113}{4}$. **3.** $\frac{261}{8}$. **4.** $\frac{253}{3}$. **5.** $\frac{484}{5}$. **6.** $\frac{650}{3}$. **7.** $\frac{1299}{4}$. **8.** $\frac{1637}{6}$. **9.** $\frac{3431}{8}$. **10.** $\frac{427}{10}$. **11.** $\frac{821}{12}$. **12.** $\frac{554}{15}$. **13.** $\frac{1435}{16}$. **14.** $\frac{539}{18}$. **15.** $\frac{1363}{24}$. **16.** $\frac{2723}{36}$. **17.** $\frac{4537}{48}$.

Page 66. — **2.** 29. **3.** 23. **4.** $19\frac{4}{5}$. **5.** 49. **6.** $34\frac{5}{8}$. **7.** $216\frac{2}{3}$. **8.** $29\frac{1}{2}$. **9.** $12\frac{1}{3}$. **10.** 24. **11.** $20\frac{3}{5}$. **12.** $68\frac{5}{12}$. **13.** $59\frac{3}{4}$. **14.** 76. **15.** $56\frac{2}{3}$. **16.** $96\frac{1}{2}$.

Page 67. — **2.** $\frac{11}{25}$. **3.** $\frac{17}{20}$. **4.** $\frac{1}{40}$. **5.** $4\frac{1}{4}$. **6.** $8\frac{3}{4}$. **7.** $\frac{1}{125}$. **8.** $\frac{1}{16}$. **9.** $\frac{5}{16}$. **10.** $20\frac{3}{4}$. **11.** $\frac{11}{16}$. **12.** $\frac{9}{16}$. **13.** $9\frac{13}{40}$. **14.** $\frac{5}{32}$. **15.** $\frac{7}{32}$. **16.** $\frac{9}{32}$. **17.** $\frac{11}{32}$. **18.** $\frac{13}{32}$. **19.** $\frac{17}{32}$. **20.** $53\frac{1}{8}$. **21.** $65\frac{5}{8}$. **22.** $7\frac{13}{16}$. **23.** $96\frac{7}{8}$. **24.** $\frac{3}{32}$. **25.** $\frac{1}{160}$.

Page 68. — **4.** .125. **5.** .375. **6.** $.16\frac{2}{3}$. **7.** .625. **8.** $.66\frac{2}{3}$. **9.** $1.33\frac{1}{3}$. **10.** $.11\frac{1}{9}$. **11.** $.57\frac{1}{7}$. **12.** $.55\frac{5}{9}$. **13.** $.41\frac{2}{3}$. **14.** $.53\frac{1}{3}$. **15.** .5625. **16.** .727273 −. **17.** $.21\frac{3}{7}$. **18.** .175. **19.** 1.75. **20.** .9375. **21.** .84. **22.** $.458\frac{1}{3}$. **23.** $1.56\frac{2}{3}$. **24.** .96. **25.** .09375. **26.** .078125. **27.** .425. **28.** $.69\frac{4}{9}$. **29.** .2625. **30.** $1.21\frac{3}{7}$. **31.** $2.08\frac{1}{3}$. **32.** 62.25. **33.** 3.45. **34.** .856364−. **35.** .003125. **36.** .084125. **37.** .04625. **38.** .17675.

Page 70. — **2.** $\frac{12}{16}, \frac{14}{16}, \frac{15}{16}$. **3.** $\frac{21}{24}, \frac{14}{24}, \frac{7}{24}$. **4.** $\frac{33}{36}, \frac{27}{36}, \frac{5}{36}$. **5.** $\frac{10}{32}, \frac{20}{32}, \frac{5}{32}$. **6.** $\frac{20}{24}, \frac{15}{24}, \frac{10}{24}$. **7.** $\frac{4}{24}, \frac{22}{24}, \frac{17}{24}$. **8.** $\frac{48}{60}, \frac{35}{60}, \frac{57}{60}$. **9.** $\frac{42}{60}, \frac{33}{60}, \frac{34}{60}$. **10.** $\frac{8}{16}, \frac{12}{16}, \frac{10}{16}, \frac{7}{16}$. **11.** $\frac{5}{20}, \frac{4}{20}, \frac{2}{20}, \frac{1}{20}$. **12.** $\frac{27}{36}, \frac{18}{36}, \frac{22}{36}, \frac{17}{36}$. **13.** $\frac{36}{60}, \frac{16}{60}, \frac{14}{60}, \frac{57}{60}$.

Page 71. — **2.** $\frac{25}{60}, \frac{28}{60}$. **3.** $\frac{72}{80}, \frac{45}{80}$. **4.** $\frac{52}{72}, \frac{51}{72}$. **5.** $\frac{55}{60}, \frac{46}{60}$. **6.** $\frac{81}{120}, \frac{112}{120}$. **7.** $\frac{10}{48}, \frac{15}{48}$. **8.** $\frac{21}{96}, \frac{56}{96}$. **9.** $\frac{32}{72}, \frac{39}{72}, \frac{46}{72}$. **10.** $\frac{195}{225}, \frac{162}{225}, \frac{155}{225}$. **11.** $\frac{135}{240}, \frac{128}{240}, \frac{408}{240}$. **12.** $\frac{342}{360}, \frac{100}{360}, \frac{165}{360}$. **13.** $\frac{12}{96}, \frac{36}{96}, \frac{17}{96}$. **14.** $\frac{165}{180}, \frac{84}{180}, \frac{50}{180}$. **15.** $\frac{16}{84}, \frac{77}{84}, \frac{78}{84}$. **16.** $\frac{50}{60}, \frac{56}{60}, \frac{27}{60}, \frac{58}{60}$. **17.** $\frac{27}{72}, \frac{42}{72}, \frac{22}{72}, \frac{69}{72}$. **18.** $\frac{16}{42}, \frac{35}{42}, \frac{6}{42}, \frac{13}{42}$. **19.** $\frac{20}{48}, \frac{21}{48}, \frac{38}{48}, \frac{120}{48}$. **20.** $\frac{44}{120}, \frac{64}{120}, \frac{98}{120}, \frac{23}{120}$. **21.** $\frac{45}{300}, \frac{55}{300}, \frac{310}{300}, \frac{38}{300}$. **22.** $\frac{408}{240}, \frac{54}{240}, \frac{110}{240}, \frac{13}{240}$.

Page 73. — **2.** $2\frac{1}{6}$. **3.** $3\frac{5}{12}$. **4.** $3\frac{2}{15}$. **5.** $2\frac{7}{12}$. **6.** $6\frac{1}{5}$. **7.** $9\frac{1}{6}$. **8.** $22\frac{3}{5}$. **9.** $23\frac{7}{15}$. **10.** 13. **11.** 103. **12.** 103. **13.** $85\frac{1}{24}$. **14.** $97\frac{1}{16}$. **15.** $193\frac{5}{12}$.

Page 74. — **17.** $2\frac{5}{8}$. **18.** $2\frac{7}{10}$. **19.** $2\frac{1}{2}$. **20.** $\frac{3}{16}$. **21.** $\frac{3}{16}$. **22.** $\frac{14}{25}$. **23.** $17\frac{3}{4}$. **24.** $2\frac{11}{24}$. **25.** $3\frac{2}{15}$. **26.** $\frac{7}{20}$. **27.** $\frac{19}{32}$. **28.** $\frac{3}{25}$. **29.** $65\frac{1}{4}$; $20\frac{1}{4}$. **30.** $44\frac{1}{24}$; $12\frac{17}{24}$. **31.** $11.0\frac{11}{15}$; $1.7\frac{14}{15}$. **32.** $18.6\frac{3}{8}$; $11.3\frac{3}{8}$. **33.** $38\frac{1}{16}$; $15\frac{3}{16}$. **34.** $167\frac{1}{20}$; $16\frac{11}{20}$. **35.** $116\frac{11}{24}$; $62\frac{19}{24}$. **36.** $6.0\frac{1}{24}$; $.9\frac{7}{24}$. **37.** $7.16\frac{7}{12}$; $2.17\frac{11}{12}$. **38.** $59\frac{19}{24}$; $33\frac{3}{8}$. **39.** $70\frac{2}{3}$; 18. **40.** $159\frac{5}{12}$; $26\frac{1}{12}$. **41.** $11.2\frac{1}{12}$; $5.6\frac{5}{12}$. **42.** $12.80\frac{19}{30}$; $5.00\frac{29}{30}$. **43.** $106\frac{37}{50}$; $48\frac{3}{10}$. **44.** $167\frac{1}{4}$. **45.** $221\frac{7}{8}$. **46.** $20.8\frac{3}{5}$. **47.** $26.71\frac{1}{3}$. **48.** $201\frac{9}{10}$. **49.** $23.66\frac{1}{4}$. **50.** $7.171\frac{2}{3}$. **51.** 91.304.

Page 75. — **1.** $146\frac{3}{5}$ tons. **2.** $3\frac{5}{12}$ ft. **3.** $1\frac{11}{40}$ min. **4.** $12\frac{7}{8}$¢. **5.** $\$21\frac{3}{4}$. **6.** $4\frac{7}{8}$ tons. **7.** British, $52\frac{1}{8}$ lb.; German, $58\frac{3}{4}$ lb. **8.** $265\frac{1}{2}$ lb.

Page 77. — **2.** 58. **3.** 52. **4.** 57. **5.** 72. **6.** $83\frac{1}{3}$. **7.** 81. **8.** $30\frac{5}{8}$. **9.** $65\frac{5}{8}$. **10.** 52. **11.** 600. **12.** 1200. **13.** $777\frac{7}{9}$.

Page 78. — **14.** 45. **15.** 54. **16.** $53\frac{1}{3}$. **17.** $62\frac{1}{2}$. **18.** $49\frac{1}{2}$. **19.** $195\frac{1}{5}$. **20.** $62\frac{1}{2}$. **21.** 74. **22.** 15.4. **23.** 3.72. **24.** .375. **25.** 7.455. **27.** $1023\frac{3}{4}$. **28.** $1490\frac{2}{5}$. **29.** $1157\frac{1}{3}$. **30.** $953\frac{1}{8}$. **31.** $3091\frac{7}{8}$. **32.** 418.5. **33.** 704.7. **34.** 179.58. **35.** 333.25. **36.** 3772.35. **37.** 127.4. **38.** 308. **39.** 28.08. **40.** \$8920.80. **41.** \$7943.75. **42.** 117 pages. **43.** 2,685,000,000 letters. **44.** 150,000 locomotives; 3,000,000 freight cars. **45.** 625,000 tons.

Page 80. — **2.** $6\frac{3}{4}$. **3.** $2\frac{1}{4}$. **4.** $4\frac{4}{5}$. **5.** $9\frac{1}{3}$. **6.** $2\frac{1}{4}$. **7.** $8\frac{1}{4}$. **8.** $28\frac{4}{5}$. **9.** $1\frac{7}{8}$. **10.** $16\frac{1}{5}$. **11.** $5\frac{1}{5}$. **12.** $15\frac{3}{4}$. **13.** $20\frac{5}{12}$. **15.** 3759. **16.** $5740\frac{4}{5}$. **17.** $3037\frac{1}{2}$. **18.** 4375. **19.** \$9.75. **20.** $\$13.12\frac{1}{2}$. **21.** $\$11.60\frac{1}{2}$. **22.** \$18. **23.** $76.03\frac{1}{2}$. **24.** 1110.4. **25.** 285.6. **26.** .8.

Page 81. — **27.** 150 in. **28.** \$13. **29.** 50 gal. **30.** \$4160. **31.** \$881.25. **32.** 69 weeks. **33.** $1177\frac{1}{2}$ lb. **34.** 99 tons. **35.** 114 lb. **36.** \$2862. **37.** \$234. **38.** 12,528,000 gal.

Page 83. — **2.** $\frac{3}{5}$. **3.** $\frac{13}{25}$. **4.** $\frac{5}{12}$. **5.** $\frac{2}{9}$. **6.** $\frac{1}{12}$. **7.** $\frac{3}{20}$. **8.** $\frac{9}{40}$. **9.** $\frac{1}{16}$. **10.** $\frac{5}{12}$. **11.** $\frac{4}{9}$. **12.** $\frac{9}{20}$. **13.** $\frac{5}{16}$.

Page 84. — **14.** $6\frac{3}{4}$. **15.** 5. **16.** $5\frac{3}{5}$. **17.** $8\frac{2}{3}$. **18.** $8\frac{1}{2}$. **19.** 26. **20.** $36\frac{1}{6}$. **22.** 396. **23.** $222\frac{3}{16}$. **24.** $122\frac{1}{2}$. **25.** $87\frac{3}{4}$. **26.** 32. **27.** $447\frac{6}{7}$. **28.** 54 tons. **29.** $1191\frac{2}{3}$ mi. **30.** $4415\frac{5}{8}$ ft. **31.** 7 mi. in Italy ; $5\frac{1}{4}$ mi. in Switzerland. **32.** $102.

Page 86 — **2.** $\frac{3}{5}$. **3.** $\frac{6}{11}$. **4.** $\frac{21}{50}$. **5.** $\frac{3}{40}$. **6.** $\frac{7}{32}$. **7.** $\frac{5}{32}$. **8.** $\frac{3}{40}$. **9.** $\frac{4}{9}$. **10.** $\frac{1}{33}$. **11.** $\frac{3}{10}$. **12.** $\frac{2}{11}$. **13.** $\frac{3}{8}$. **14.** $\frac{5}{12}$. **15.** $\frac{9}{10}$. **16.** $\frac{2}{9}$. **17.** $\frac{5}{28}$.

Page 87. — **19.** $3\frac{2}{3}$. **20.** $2\frac{4}{5}$. **21.** $5\frac{4}{5}$. **22.** $84\frac{5}{9}$. **23.** $66\frac{1}{5}$. **24.** $53\frac{5}{16}$. **25.** $160\frac{2}{3}$. **26.** $40\frac{1}{2}$. **27.** $29\frac{13}{40}$. **29.** $42\frac{1}{4}$. **30.** $31\frac{5}{16}$. **31.** $24\frac{1}{6}$. **32.** $35\frac{3}{8}$. **33.** $42\frac{5}{6}$. **34.** $17\frac{2}{15}$. **35.** $26\frac{3}{10}$. **36.** $18\frac{4}{25}$. **37.** $124\frac{3}{16}$. **38.** $162\frac{7}{10}$. **39.** $85\frac{5}{12}$. **40.** $92\frac{1}{12}$. **41.** $61\frac{1}{2}$ mi. **42.** $\frac{9}{16}$ oz. **43.** $1\frac{3}{8}$ gal. **44.** $\$\frac{1}{4}$. **45.** $\$\frac{5}{8}$. **46.** $\$\frac{7}{40}$.

Page 89. — **2.** 15. **3.** $12\frac{1}{2}$. **4.** 24. **5.** $31\frac{1}{2}$. **6.** 16. **7.** $17\frac{1}{2}$. **8.** 12. **9.** 6. **10.** 6. **11.** $13\frac{3}{4}$. **12.** $6\frac{1}{4}$. **13.** $9\frac{4}{5}$. **14.** $29\frac{1}{3}$. **15.** $12\frac{6}{7}$. **16.** $36\frac{2}{3}$. **17.** 15. **18.** $33\frac{1}{3}$. **19.** $65\frac{5}{11}$. **20.** 48. **21.** 16. **22.** $24\frac{9}{11}$. **23.** $9\frac{3}{8}$. **24.** 112. **25.** 32. **26.** 96. **27.** 84. **28.** 192. **29.** 75. **30.** 108. **31.** 220. **32.** $483\frac{5}{9}$. **33.** 306. **34.** 60. **35.** 136.

Page 90. — **36.** $1880. **37.** $500. **38.** $17,165\frac{5}{28}$ lb. **39.** 6500 gal. **40.** 20 hr. **41.** 18,000,000 matches. **42.** 14,688 acres. **43.** 12,600 lb. **44.** 2400 pictures. **45.** $10\frac{2}{3}$ hr. **46.** 94 hr.

Page 92. — **2.** 2. **3.** $1\frac{1}{5}$. **4.** $\frac{15}{16}$. **5.** 2. **6.** $\frac{15}{16}$. **7.** $2\frac{1}{2}$. **8.** $1\frac{2}{3}$. **9.** $\frac{7}{12}$. **10.** $11\frac{1}{5}$. **11.** $4\frac{7}{8}$. **12.** 15. **13.** $\frac{4}{15}$. **14.** $11\frac{1}{3}$. **15.** $16\frac{7}{20}$. **16.** $36\frac{32}{35}$. **17.** $3\frac{1}{6}$. **18.** $7\frac{1}{18}$. **19.** $4\frac{27}{70}$. **21.** $1\frac{11}{16}$. **22.** 1. **23.** $7\frac{7}{8}$. **24.** $5\frac{5}{6}$. **25.** $\frac{8}{9}$. **26.** $32\frac{2}{5}$. **27.** $26\frac{22}{25}$. **28.** $16\frac{1}{2}$. **29.** $6\frac{2}{3}$. **30.** 18.

Page 93. — **31.** $\frac{2}{3}$; $42\frac{2}{3}$. **32.** $22\frac{2}{5}$. **33.** 18. **34.** $\frac{5}{18}$. **35.** $\frac{7}{36}$. **36.** $\frac{23}{24}$. **37.** $5\frac{2}{5}$. **38.** $5\frac{5}{9}$. **39.** $9\frac{1}{3}$. **40.** $1\frac{1}{2}$. **41.** $10\frac{1}{2}$ cu. yd. **42.** $17\frac{1}{2}$ tons. **43.** 50 gal. **44.** $\$11\frac{3}{5}$. **45.** $\$1\frac{3}{10}$. **46.** 144 days.

Page 94. — **2.** $\frac{5}{12}$. **3.** $1\frac{1}{2}$. **4.** $\frac{8}{9}$. **5.** 3. **6.** $1\frac{1}{9}$. **7.** $14\frac{2}{5}$. **8.** $\frac{3}{25}$. **9.** $5\frac{17}{20}$. **10.** $8\frac{4}{5}$. **11.** $\frac{25}{28}$. **12.** $\frac{1}{3}$. **13.** $\frac{7}{8}$. **14.** $\frac{1}{6}$. **15.** $\frac{15}{16}$. **16.** $\frac{1}{6}$. **17.** $\frac{3}{10}$. **18.** $\frac{5}{9}$. **19.** $\frac{25}{54}$. **20.** $10\frac{2}{9}$. **21.** $13\frac{1}{5}$.

Page 96. — **2.** $\frac{2}{3}$. **3.** $\frac{2}{5}$. **4.** $\frac{1}{2}$. **5.** $\frac{15}{16}$. **6.** $\frac{12}{5}$. **7.** $\frac{5}{12}$. **8.** $\frac{3}{8}$. **9.** $\frac{15}{16}$. **10.** $\frac{4}{5}$. **11.** $\frac{5}{6}$. **12.** $\frac{3}{4}$. **13.** $\frac{16}{25}$. **14.** $\frac{1}{3}$. **15.** $\frac{13}{24}$. **16.** $\frac{1}{3}$. **17.** $\frac{1}{10}$. **18.** $\frac{1}{20}$. **19.** $\frac{4}{3}$. **20.** $\frac{4}{5}$. **21.** $\frac{3}{4}$. **22.** $\frac{1}{2}$. **23.** 66 cars. **24.** $2\frac{1}{2}$ mi. **25.** 7.553 lb. **26.** 1785 baskets. **27.** 330 trees. **28.** $\frac{3}{50}$.

Page 98. — **1.** 126. **2.** $121\frac{1}{3}$. **3.** 280. **4.** 450. **5.** 840. **6.** 1008. **7.** $3578\frac{2}{3}$. **8.** 5400. **10.** $\frac{5}{6}$. **11.** $1\frac{2}{3}$. **12.** $\frac{7}{10}$. **13.** $2\frac{2}{5}$. **14.** 12. **15.** $16\frac{2}{5}$. **16.** 2. **17.** $\frac{5}{8}$. **18.** 2250 mi. **19.** $40,000,000. **20.** 900 acres. **21.** $837,500. **22.** 990 ft. **23.** 1500 tons. **24.** Elephant, 8750 lb. ; camel, 1375 lb.

Page 100. — **2.** $17. **3.** $76.50. **4.** $21.25. **5.** $42.50. **6.** $63.75. **7.** $14.17. **8.** $28.34. **9.** $56.67. **10.** $70.84. **11.** $31.88. **12.** $106.25. **13.** $127.50. **14.** $148.75. **15.** $212.50. **16.** $191.25. **17.** $850. **18.** $2550. **19.** $1530. **20.** $8500. **21.** $7650. **22.** $17,000. **23.** $425. **24.** $1275. **25.** $2125.

Page 101. — **26.** 3.375. **27.** 600. **28.** 53,250. **29.** 2325. **30.** 58.75. **31.** 16,750. **32.** 49,500. **33.** 42,000. **34.** 729. **35.** 487.5 **36.** 20,900. **37.** 92,400. **38.** 52,800. **39.** 383,400. **40.** 1320. **41.** 2; 10; 150; 500; 210; 450. **42.** 4; 20; 300; 1000; 420; 900. 8; 40; 600; 2000; 840; 1800. 5; 25; 375; 1250; 525; 1125. **44.** 180 yd. **45.** 160 yd. **46.** 144 yd. **47.** 96 yd. **48.** 80 yd. **49.** 48 yd. **50.** 188. **51.** 25.6. **52.** 6.4. **53.** 2.96. **54.** .392. **55.** .632. **56.** 1.712. **57.** 1.144. **58.** 19.2. **59.** 3.4. **60.** $144; $146.88; $141.12. **61.** $144; $142.56; $144.72. **62.** $138.24.

Page 102. — **1.** 42,900 trees. **2.** 68,640 lb. **3.** $6\frac{7}{8}$¢. **4.** $1767.48. **5.** $4\frac{3}{10}$¢; $2\frac{23}{40}$¢. **6.** $20.60. **7.** $154.44. **8.** 520 bags. **9.** $193.44. **10.** 58,344 lb.

Page 103. — **11.** 831,000 lb. **12.** 2136 plants. **13.** $1\frac{2}{3}$ ft. **14.** 70¢. **15.** 2403 lb. **16.** $\frac{1}{4}$ lb. **17.** 534 lb.; 128,160 lb.

Page 104. — **18.** 3150 trays. **19.** 2925 lb. **20.** 3204 chests. **21.** 24 chests. **22.** 14¢. **23.** 112,906,000 lb. **24.** 147,517,000 lb. more from India; 42,109,000 lb. less from China. **25.** Great Britain, 6.03 lb.; Russia, .95 lb.; Germany, .12 lb.; France, .06 lb.; United States, 1.3 lb.

Page 105. — **26.** 54 gal. **27.** $121\frac{1}{2}$ lb. **28.** 1944 lb. **29.** $13\frac{1}{2}$ lb. **30.** $4\frac{1}{2}$ lb. **31.** 162 loaves. **32.** 6 chests and 12 loaves. **33.** $1.24. **34.** $16.74.

Page 106. — **35.** 60,000 tons. **36.** 8883 tons. **37.** 49,979,739 pairs. **38.** $22.75. **39.** 450,000. **40.** 1st da., 22,500; 2d da., 168,750; 3d da., 180,000; 4th da., 56,250; 5th da., 22,500. **41.** 180 trees. **42.** 337,500.

Page 107. — **43.** 1350 lb. **44.** 432 lb. **45.** $378. **46.** 108 lb.; 34.56 lb.; $30.24. **47.** 118.8 lb. **48.** $\frac{3}{4}$ lb. **49.** $493.02. **50.** 150 oz. **51.** $337.50. **52.** 42,367,115 lb. **53.** In order: $1056; $948; $1128; $780.

Page 114. — **2.** $405,312.50. **3.** $720. **4.** $21.60. **5.** $61.75. **6.** $550.20. **7.** $397.70. **8.** $503.70. **9.** $1158.30. **10.** $16,983. **11.** $378; $\frac{3}{4}$¢.

Page 115. — **12.** $160.88. **13.** $3703.22. **14.** $1095. **15.** $2531.25. **16.** $291. **17.** $831.79. **18.** $7020. **19.** $410; $1620. **20.** $\frac{3}{10}$. **21.** $56,700. **22.** $21.83. **23.** Neither gained nor lost. **24.** $28.67.

Page 118. — **2.** 36. **3.** 92. **4.** 26. **5.** 496. **6.** 12.5. **7.** $1\frac{1}{4}$. **8.** 108 books. **9.** 13. **10.** 26; 52.

Page 119. — **2.** $x=6$. **3.** $x=6$. **4.** $x=4$. **5.** $x=21$. **6.** $x=35$. **7.** $x=45$. **8.** $x=35$. **9.** $x=24$. **10.** $x=24$. **11.** $x=9$. **12.** $x=72$. **13.** $x=56$. **14.** 24. **15.** 400 pupils. **16.** 24 games. **17.** 640 persons. **18.** 80 ft. **19.** 36 min.

Page 122. — **3.** $x = 2$. **4.** $x = 11$. **5.** $x = 5$. **6.** $x = 9$. **7.** $x = 7$. **8.** $x = 2$. **9.** $x = 6$. **10.** $x = 15$. **11.** $x = 2$. **12.** $x = 7$. **13.** $x = 1\frac{1}{2}$. **14.** $x = 8$. **15.** $x = 5$. **16.** $x = 1$. **17.** $x = 16$. **18.** $x = 4$. **19.** $x = 2$. **20.** $x = 8$. **21.** $x = 4\frac{4}{5}$. **22.** $x = 9$. **23.** $x = 3$. **24.** $x = 11$. **25.** $x = 12$. **26.** $x = 13$. **27.** $x = 4$.

Page 123. — **29.** $x = 24$. **30.** $x = 36$. **31.** $x = 20$. **32.** $x = 320$. **33.** $x = 18$. **34.** $x = 40$. **35.** $x = 28$. **36.** $x = 60$.
2. 29, 87.

Page 124. — **3.** 40 clams, 120 clams. **4.** Steam, 40 cu. ft.; water, 80 cu. ft. **5.** 2. **6.** \$9, \$36. **7.** \$10.50, \$42. **8.** 18, 54. **9.** 13, 65. **10.** 45. **11.** 80. **12.** Jones, \$25,000; Hollis, \$75,000; Frye, \$125,000. **13.** Jones, \$260; Hollis, \$780; Frye, \$1300. **14.** \$.90 per ton. **15.** 1200 lb.; 3000 lb.

Page 125. — **16.** 162 ft. **17.** 400 gal.; 1600 gal. **18.** 250 violins; 500 violins. **19.** 4 16-candle power; 8 20-candle power. **21.** 20 words. **22.** 17 words. **23.** 28 words. **24.** 28 words. **25.** 37 words. **26.** 15 words. **27.** 21 words. **28.** 19 words. **29.** 14 words.

Page 126. — **30.** 8 min. **31.** 140 factories. **32.** 12,000 sq. yd., inside; 52,000 sq. yd., outside. **33.** 36. **34.** 128. **35.** 105. **36.** 133,000 miners. **37.** 320 lb. **38.** \$2.80. **39.** 3300 ft. **40.** 42 mi. **41.** 78 vessels. **42.** 165 ft.

Page 127. — **44.** \$16. **45.** \$7250. **46.** \$150. **47.** 42. **48.** 1725 passengers. **49.** 800 steamers; 225 sailing vessels. **50.** 240 boys.

Page 128. — **52.** $52\frac{1}{2}$, $31\frac{1}{2}$. **53.** 60, 40. **54.** 105 days. **55.** 160 tons; 40 tons; 43.2 tons. **56.** Length, 20 ft.; width, 15 ft. **57.** Ore, 200 car loads; coke, 175 car loads; limestone, 75 car loads.

Page 130. — **2.** $x = 4, y = 1$. **3.** $x = 5, y = 2$. **4.** $x = 8, y = 2$. **5.** $x = 10, y = 2$.

Page 131. — **6.** $x = 3, y = 4$. **7.** $x = 4, y = 3$. **8.** $x = 1, y = 1$. **9.** $x = 5, y = 2$. **10.** $x = 4, y = 3$. **11.** $x = 1, y = 2$. **12.** $x = 6, y = 1$. **13.** $x = 4, y = 5$.

Page 132. — **15.** $x = 2, y = 1$. **16.** $x = 3, y = 2$. **17.** $x = 4, y = 3$. **18.** $x = 5, y = 4$. **19.** $x = 3, y = \frac{1}{2}$. **20.** $x = \frac{1}{2}, y = \frac{1}{3}$. **21.** $x = 6, y = 4$. **22.** $x = 9, y = 2$. **23.** $x = 8, y = 5$. **24.** $x = 4, y = 10$. **25.** $x = 7, y = 13$. **26.** $x = 10, y = 4$. **27.** $x = 4, y = 15$.

Page 133. — **28.** $x = 12, y = 8$. **29.** $x = 20, y = 16$. **30.** $x = 12, y = 18$. **31.** $x = 36, y = 24$. **32.** 11, 3. **33.** 5, 8. **34.** 16, 2. **35.** Raspberries, 12 ¢; cherries, 10 ¢. **36.** 60 3-grain capsules; 160 2-grain capsules. **37.** 600 @ 5 ¢; 250 @ 10 ¢. **38.** 20 @ 12 ¢; 16 @ 10 ¢. **39.** 26 1-dollar bills; 12 2-dollar bills.

Page 134. — **40.** Rate, 30 − 2. **41.** Delaware, 75 ¢; Whitestone, 45 ¢. **42.** 840 men; 160 women. **43.** 100 for adults; 200 for children; **44.** 18 in.; 12 in. **45.** 4 mo.; 8 mo. **46.** 84 tons; 10 tons. **47.** 1200 to grounds; 400 to grand stand.

Page 137. — **3.** 7920 in. **4.** 369,500 lb. **5.** 51 pk. **6.** 1440 min. **7.** 69.6 in. **8.** 8658″. **11.** 7 T. 420 lb. **12.** 8 hr. 20 min. **13.** $\frac{3}{16}$; $\frac{6}{25}$; $\frac{1}{50}$. **14.** $\frac{6}{25}$; $\frac{13}{16}$. **15.** 99 oz. **16.** 301 pt. **17.** $7\frac{71}{125}$ T. **18.** 3310 in. **19.** 173 mo. **20.** 33 hr. 20 min. **21.** 52 wk. 1 da. 6 hr. **22.** 11° 15′. **23.** 2 mi. 665 ft.

Page 138. — **24.** 20 times. **25.** 18,000. **26.** 8 min. 24 sec. **27.** \$5.50. **28.** 14 lb. **29.** 24,000 jars. **30.** \$750,000. **31.** 20 bu. **32.** \$912,109.25. **33.** $1026\frac{2}{3}$ ft. **34.** 185 stamps. **35.** 5.7 mi. **36.** $2\frac{1}{8}$ min.

Page 139. — **37.** 90 gal.

Page 140. — **1.** \$2043.93. **2.** \$120.45. **3.** \$273.13. **4.** \$347.40. **5.** \$27.89. **6.** \$202.48. **7.** \$7.00. **8.** \$9.61. **9.** \$386. **10.** \$2433.25; \$193.

Page 141. — **12.** \$39.74. **13.** \$24.94. **14.** \$35.71. **15.** \$60.24. **16.** \$122.70. **17.** \$323.24. **18.** \$488.63. **19.** \$609.98. **20.** \$2145.83. **22.** £205 9*s.* 9*d.* **23.** £657 11*s.* 2*d.* **24.** £1130 3*s.* 6*d.* **25.** £9 18*s.* 4*d.* **26.** £12 17*s.* 11*d.* **27.** £18 10*s.* 3*d.* **28.** £15 17*s.* **29.** £17 6*s.* 10*d.* **30.** £10 5*s.* 9*d.* **31.** £25 3*s.* 2*d.* **32.** £156 5*s.* 5*d.* **33.** £66 19*s.* 2*d.* **34.** \$369,854. **35.** £2 5*s.*

Page 142. — **36.** 7*s.* 11*d.* **37.** £15 12*s.*; $6.33. **38.** £1 2*s.* 1*d.* **39.** £4 17*s.*; $7.87−. **40.** $2.86−. **41.** 8 M. 32 pf. **42.** 33,694 fr. **43.** $19,902,211.72+. **44.** $1190. **45.** $2219.12. **46.** £17,207,148; $83,738,585.74+.

Page 144. — **2.** 112 sq. ft. **3.** 126 sq. ft. **4.** 247 sq. ft. **5.** 1320 sq. ft. **6.** 400 sq. ft. **7.** $1166\frac{2}{3}$ sq. ft. **8.** 84 sq. ft. **9.** 387 sq. ft. **10.** 152 sq. ft.

Page 145. — **2.** 104 cu. ft. **3.** 105 cu. ft. **4.** 168 cu. ft. **5.** 125 cu. ft. **6.** 147 cu. ft. **7.** 1000 cu. ft. **8.** 941.188 cu. ft. **9.** 1350.972 cu. ft. **10.** 1388.528 cu. ft. **11.** 3065.741 cu. ft.

Page 146. — **12.** £189,000. **13.** 432 cu. ft. **14.** 604,800 bricks. **15.** $30,855. **16.** 1320 lb. **17.** $3093\frac{3}{4}$ lb. **18.** 558,000 lb. **19.** 1944 erasers. **20.** $111\frac{5}{8}$ sq. in. **21.** $3408\frac{1}{8}$ cu. in. **22.** 9 days. **23.** 2420 ft.

Page 147. — **5.** 12 T. **6.** 6 T. 3 cwt.; 2 T. 6 cwt.; 24 lb. 5 oz. **7.** 76 ft.; 11 yd. **8.** 21 ft. 5 in.; 25 ft. $11\frac{1}{2}$ in. **9.** 12° 18′. **10.** 25° 31′ 27″.

Page 148. — **11.** 10 lb. 3 oz. 11.15 pwt. **12.** 142° 49′. **13.** £996 13*s.* **14.** £742 18*s.* $11\frac{1}{2}$*d.* **15.** £223 1*s.* 6*d.*; £16 9*s.* 4*d.*; £14 3*s.* $2\frac{1}{2}$*d.* **16.** 4° 8′ $7\frac{1}{2}$″; 12° 24′ $22\frac{1}{2}$″; 78 ft. 4 in. **17.** 58 lb. 2 oz.; 9 lb. 11 oz. **25.** 26 ft. 2 in. **26.** 62 gal. 1 qt. 1 pt. **27.** 8223 lb. 7 oz. **28.** 7 bu. $3\frac{1}{2}$ pk. **29.** 219 gross.

Page 149. — **30.** 3301 hr. 43 min.; 1375 hr. 56.3 min. **31.** Shrinking cloth, 9 hr. 52 min.; cutting cloth, 28 hr. 47.5 min.; sewing seams, 933 hr. 20 min.; cutting buttonholes, 3 hr. 2.5 min.; working buttonholes, 268 hr. 42.2 min.; all others, 682 hr. 2.5 min.; total difference, 1925 hr. 46.7 min. **32.** 17 hr. $44\frac{1}{4}$ min. **33.** 2712 lb. 8 oz. **34.** 14 oz.; $2\frac{1}{3}$ oz. **35.** 67 A. 100 sq. rd. **36.** 1 mi. 246 rd. 11 ft.

Page 152. — **2.** 23 min. 57 sec. after noon. **3.** 12 min. 11 sec. after noon. **4.** 17 min. 45 sec. after 11 A.M. **5.** 1 min. 47 sec. after 6 P.M. **6.** 17 min. 33 sec. after 5 P.M. **7.** 8 min. 12 sec. after 5 P.M. **8.** 48 min. 6 sec. after 5 P.M. **9.** 27 min. 45 sec. after 5 P.M. **10.** 22 min. 7 sec. after 6 P.M.

Page 153. — **12.** 41° 48′ 6″ W. **13.** 8° 3′ 6″ W. **14.** 21° 33′ 6″ W. **15.** 102° 33′ 6″ W. **16.** 108° 33′ 6″ W. **17.** 164° 48′ 6″ W. **18.** 143° 55′ 36″ W. **19.** 60° 36′ 51″ W. **20.** 137° 6′ 51″ W. **21.** 4 hr. 43 min. 4 sec. **22.** Chicago, 6 min. 48 sec. after midnight; Denver, 57 min. 23 sec. after 10 P.M. day before; San. Fran., 47 min. 37 sec. after 9 P.M. day before. **23.** 4 hr. 53 min. 36 sec.; 1 hr. 34 min. 54 sec.

Page 154. — **24.** 16° 15′ W. **25.** Sat., 8° 45′ E.; Tues., 3° 15′ W.; 12° west. **27.** May 2, 59 min. 54 sec. after midnight. **28.** May 2, 3 min. 16 sec. after midnight. **29.** May 2, 29 min. 8 sec. after midnight. **30.** May 2, 35 min. 56 sec. after 2 A.M. **31.** May 1, 47 min. 17 sec. after 9 P.M. **32.** May 1, 9 min. 56 sec. after 6 P.M. **33.** 15 da. 18 hr. 8 min. 6 sec.

Page 156. — **1.** New York, 11:30 A.M.; New Orleans, 10:30 A.M.; San Antonio, 10:30 A.M.; Portland, 8:30 A.M. **2.** Salt Lake City, 11:15 A.M.; Portland, 1:15 P.M.; Manila, 2:15 A.M. (next day); Bombay, 11:15 P.M.; Berlin, 7:15 P.M.; London, 6:15 P.M.; Tokyo, 3:15 A.M. (next day). **3.** 7 A.M. **4.** Between 4:41 P.M. and 6:40 P.M., Apr. 30. **5.** 20 hr. 39 min.

Page 157. — **6.** 6 P.M. May 2. **7.** 5:15 P.M. **8.** 11:30 A.M. **9.** 7 min. 20 sec. **10.** Tokyo, 11 P.M. Jan. 2; St. P., 4 P.M. Jan. 2; London, 2 P.M. Jan. 2; N.Y., 9 A.M. Jan. 2. **11.** 12 da. 4 hr. 3 min. **12.** 8:03 A.M. **13.** 50 min. 39 sec. after 9 A.M. **14.** In order: Dec. 31, '04, 11 P.M.; Dec. 31, '04, 10 P.M.; Dec. 31, '04, 9 P.M.; Jan. 1, '05, 1 P.M.; Jan. 1, '05, 2 P.M.; Jan. 1, '05, 3 P.M.; Jan. 1, '05. 10 A.M.; Jan. 1, '05, 6 A.M.; Jan. 1, '05, 5 A.M.

Page 163. — 1. 551 cu. cm. 2. 96 steres. 3. 1000 cu. cm. 4. 1600 cu. m. 5. $\frac{1}{60}$ mm. 6. \$175. 7. 320,000,000 cu. m. 8. 1,134,905 cu. cm.

Page 164. — 1. 120 l. 2. \$144. 3. \$60. 4. 17 Hl.

Page 165. — 5. 225 Hl. 6. 5040 cu. m.

Page 167. — 1. 465.878 ft. 2. 457.676 ft. 3. 178.805 ft. 4. 666.009 ft. 5. 9.258 mi. 6. 48.467 mi. 7. 3305.005 mi. 8. 9.055 in. 9. .276 in. 10. 59.055 ft.; 65.617 ft. 11. 2.794 m. 12. Portugal, 6000 sq. Km.; Spain, 3000 sq. Km.; Italy, 800 sq. Km. 13. 12.5 M. T. 14. 13,994.1 cu. m.

Page 168. — 15. 1620 marks. 16. \$1526.25. 17. 45 ¢. 18. 640 g. 19. 81,706.181 oz. 21. .915. 22. 13.598. 23. .24. 24. 21.2 Kg. 25. 434 g. 26. 68,342,600 lb. fresh codfish; 37,478,200 lb. dried codfish.

Page 169. — 27. 336,000 l. 28. 22.4 boxes. 29. 28.65 + cu. cm. 30. 255 francs. 31. 11 ft. 10 in. 32. 1000 francs. 33. 156,600 M. T. 34. Gt. Br., 70,547.2 T.; H., 35,273.6 T.; U.S., 22,046 T. 35. \$20.63. 36. \$1094.80.

Page 171. — 1. 24 lb. 2. American, 451.15 gr.; German, 430.24 gr. 3. 835.576 gr. 4. 40 silver dollars. 5. 100,800 gal.; 13,440 cu. ft. 6. 20,250 gal. 7. 6675 gal. 8. 60,000 bu.

Page 172. — 9. 1,800,000 bu. 10. 5.99− lb. 11. .9. 12. 16 ft. 13. 67.2 cu. ft. 14. 1,280,000 cu. ft.; 1,242,718.447− cu. ft. 15. \$1.03. 16. $64\frac{1}{2}$ gr. 17. 4479 lb. 2 oz. 19. 21,401 lb. 9 oz. 16 pwt. 6 gr. 20. 430 lb. 21. 400,421 eagles and 6.825 pwt. over.

Page 174. — 3. $36\frac{2}{3}°$ C. 4. Olive oil, $-1\frac{1}{9}°$ C.; mercury, $-38\frac{5}{6}°$ C. 5. 25° F.; $13\frac{8}{9}°$ C. 6. $39\frac{1}{5}°$ F. 7. 155° F. 8. 36° F. 9. Highest, 83.66° F.; lowest, 77° F.; difference, 6.66° F.

Page 175. — 1. 1680 ft. 2. 1300 ft. 3. 1000 ft. 4. $3845\frac{1}{3}$ ft. 5. 8640 ft. 6. 2560 ft. 7. 5000 ft. 8. 5760 ft. 9. 1120 ft. 10. 11,520 ft. 11. 1980 ft. 12. 44,100 ft. 13. 12,900 ft. 14. $23,466\frac{2}{3}$ ft.

Page 176. — 15. 20 ft. 16. \$15.29. 17. $1149\frac{1}{2}$ ft. 18. \$521.80. 19. \$125.44. 20. \$76.80. 21. \$113.02. 22. 900 ft.

Page 177. — 1. 6120 pieces. 2. 10,260 lb. 3. \$39.90. 4. \$131.10. 5. \$173.75. 6. 103 bunches. 7. \$153.60. 8. 847 shingles; 47 bunches.

Page 178. — 9. 58 pieces. 10. \$7.11. 11. $43\frac{1}{5}$ days. 12. \$2678.40.
1. 4620 laths. 2. \$18.48. 3. $9\frac{5}{8}$ days. 4. \$101.64.

Page 179. — 5. \$55.42+. 6. $\frac{3}{4}$ gal. 7. \$51.16.

Page 180. — 1. 42 strips. 2. \$7. 3. \$5.46. 4. \$4.20. 5. \$49. 6. \$.50. 7. Short way; \$.90. 8. \$50.96. 9. \$1175. 10. $33\frac{1}{4}$ yd.; \$58.19.

Page 181. — 1. 20,000 sq. yd. 2. \$1160. 3. \$126. 4. 5′ 10″. 5. \$161.12; \$.38. 6. \$4860. 7. \$10,401.60. 8. \$24,921.60. 9. \$21,410.12.

Page 182. — 10. 20,250 bricks. 11. \$151.88. 12. \$243. 13. 14.175 cu. yd. 14. $56\frac{1}{4}$ T. 15. \$120. 16. \$1848. 17. \$10.71. 18. 20 cu. yd. 19. 36 bbl. cement; 10 cu. yd. sand; 20 cu. yd. stone. 20. \$116. 21. 2025 lb. 22. \$31.92+.

Page 183. — 23. 156.25 lb; 160.94− lb. 24. 625 lb. 25. Weight, 937.5 lb.; sp. gr., .75. 26. 25.3 mi.; $37\frac{8}{75}$ ft. 27. $28\frac{7}{40}$ mi.; 676.2 mi. 28. 25.61 mi. 29. 16 knots.

Page 184. — 1. 390 acres. 2. 10 da. 4 hr. 3. \$321.75. 4. 24 bu.; \$162. 5. 9360 bu. 6. $10\frac{2}{3}$ days.

Page 185. — **7.** \$140.40. **8.** 31,200 lb. **9.** 18¢. **10.** \$175.50. **11.** \$3198.25. **12.** \$453.96. **13.** .72. **14.** 4.54 bu. **15.** 1323 loaves. **16.** Gt. Br., 795 bu.; France, 650 bu.; Germany, 485 bu.; Austria, 410 bu.; U. S., 335 bu.; Russia, 225 bu. **17.** 550,000,000 bu.

Page 186. — **18.** \$288.60. **19.** \$11,934. **20.** \$1.90. **21.** \$720. **22.** \$2484. **23.** \$5.25. **24.** \$22,403.55. **25.** \$1849.20.

Page 187. — **26.** Sawyer, \$690; assistant, \$368. **27.** \$1242. **28.** \$1416.80. **29.** Man, \$2.25; boy, \$1.50. **30.** \$1058. **31.** \$6752.80. **32.** \$14,066.80. **33.** 17,480,000 ft. **34.** \$166.25; \$30,590. **35.** \$2104.25. **36.** \$387.182.

Page 188. — **37.** 5760 cu. yd. **38.** \$21,600. **39.** 900 days. **40.** \$850.50. **41.** 24,720 lb. **42.** \$74.25. **43.** 30 sec. **44.** Miner, \$2.53; laborer, \$1.26½. **45.** \$60.72. **46.** \$5.28.

Page 189. — **47.** \$7.23. **48.** \$53.49. **49.** \$128.40. **50.** \$390. **51.** \$261.60. **52.** \$.85. **53.** 315 L. T. **54.** \$306. **55.** \$1700. **56.** 60,242,460 T.

Page 190. — **57.** 3060 tons. **58.** \$357. **59.** \$130.305. **60.** \$49,087.50. **61.** \$1708.50. **62.** 1304 ft. **63.** \$1693.60. **64.** \$78.30.

Page 191. — **65.** 7½¢. **66.** \$881. **67.** \$14,187.52. **68.** \$10.88. **69.** \$3744. **70.** \$4253.40. **71.** \$4.17. **72.** \$2529.60. **73.** \$6.65; \$6783. **74.** 56⅔ lb. (troy). **75.** \$14,055.60. **76.** \$7272.60. **77.** \$19,180.

Page 195. — **2.** \$663.60. **3.** \$6750. **4.** 340 T. **5.** 2455.2 ft. **6.** 6780 gal **7.** .4 cu. in. **8.** 2341. **9.** 13,993. **10.** 106,799. **11.** 95,004. **12.** 1848 **13.** \$90,000. **14.** 1568 lb. **15.** 8052 ft. red fir; 2196 ft. pine; 610 ft. spruce; 610 ft. hemlock; 244 ft. cedar; 488 ft. other kinds. **16.** In order: \$4,051,500,000; \$2,518,500,000; \$2,409.000,000; \$1,642,500,000; \$1.533,000.000; \$876,000,000; \$766.500,000. **17.** 42½%; \$1,023,825,000; \$1,385,175,000.

Page 196. — **2.** 23⅓%. **3.** 80%. **4.** 114²⁄₇%. **5.** 87½%. **6.** 2%. **7.** 600%. **8.** 16⅔%. **9.** 33⅓%. **10.** ⅔%. **11.** 50%. **12.** 62½%. **13.** 16⅔%.

Page 197. — **14.** 41⅔% in N. Y.; 15% in Ill. **15.** 52%. **16.** 45% **17.** Graphite, 59%; clay, 41%. **18.** 62%. **19.** 42.8%. **20.** Population in order: 6.6%; 2.4%; 24.9%; 56.9%; 8.8%; .4%. Area in order: 18.0%; 13.1%; 7.3%; 32.5%; 22.0%; 6.6%; .5%. **21.** Each, of Gt. Brit. and Ire. in order: 62.2%; 59.5%; 40.5%; 37.8%; 21.6%; 18.9%. Each, of U. S. in order: 168.2%; 104.5%; 68.2%; 63.6%; 36.4%; 31.8%.

Page 199. — **3.** \$2400. **4.** \$38,400. **5.** 8800. **6.** \$426. **7.** \$45 **8.** \$17.19. **9.** 1560. **10.** 83,200. **11.** 7240. **12.** 550 acres. **13.** 25,000.000 bbl. **14.** 45,000 T. **15.** 28.000 coaches. **16.** 815,000 immigrants. **17.** 31,200 sq. mi. **18.** \$3,125,000. **19.** 1,100,000,000 T.; 330,000,000 T.; 220,000,000 T.

Page 200. — **1.** 5600. **2.** 1125. **3.** \$1.50. **4.** 25,500 farms. **5.** 26%. **6.** 50%. **7.** 18,291 T. **8.** 24 lb.

Page 201. — **9.** 58½%. **10.** Ill., \$1.88; N. Y., \$8.80.

Page 202. — **3.** 120. **4.** 292. **5.** 2680. **6.** 2480. **7.** 280 oranges. **8.** \$6400. **9.** 220,000,000 bu. **10.** 4,500,000 T. **11.** 55,000 cu. yd.

Page 203. — **1.** \$60. **2.** \$18. **3.** 16⅔%. **4.** $\frac{3}{10}$. **5.** 5.91. **6.** 20%. **7.** 50. **8.** 16⅔%. **9.** 8938.3. **10.** \$5000. **11.** 1560 mi. **12.** 1229.76 lb. **13.** \$1642. **14.** 1006. **15.** \$14,401.20. **16.** 11,968 lb. **17.** 4,900,000 mi. **18.** 69%; 22%. **19.** 25,000,000 lb. from U. S.; 210,000,000 lb. from Canada. **20.** U. S., 2,875,000 T.; Eng., 900,000 T.

Page 205. — **1.** 20% gain. **2.** $12\frac{1}{2}$% gain. **3.** $16\frac{2}{3}$% gain. **4.** 20% loss. **5.** 10% loss. **6.** $8\frac{1}{3}$% loss. **7.** 20% gain. **8.** $12\frac{1}{2}$% loss. **9.** 20% gain. **10.** 25% gain. **11.** $\frac{\text{⊐△V}}{\text{□△V}}$ **12.** $\frac{\text{⊏⊐⊓}}{\text{⊓┘△}}$ **13.** $\frac{\text{⊔⊐□⊏}}{\text{└△┐△}}$ **14.** $\frac{\text{┘□⊔△}}{\text{┘┌△V}}$ **15.** $\frac{\text{┌⊏}}{\text{┘⊔△}}$ **16.** $\frac{\text{┘⊔△}}{\text{┘△□}}$ **17.** $\frac{\text{└⊏△}}{\text{└┘□}}$ **18.** $\frac{\text{⊐⊓┐⊔}}{\text{⊐⊔⊏└}}$ **19.** $\frac{\text{┘△VV}}{\text{⊓┐□}}$ **20.** $\frac{\text{⊓V}}{\text{┐V}}$ **21.** $\frac{\text{△□}}{\text{△⊏}}$ **22.** $\frac{\text{┘⊔}}{\text{┘⊐}}$ **23.** $\frac{\text{△⊐}}{\text{△□}}$ **24.** $\frac{\text{┘⊔⊓}}{\text{┘⊐V}}$ **25.** $\frac{\text{⊔□△}}{\text{└┐□}}$ **26.** $\frac{\text{⊔┐□}}{\text{└⊓□}}$ **27.** $37\frac{1}{2}$%. **28.** 100%. **29.** $525.10. **30.** $78. **31.** 20¢. **32.** $33\frac{1}{3}$%.

Page 206. — **33.** 50%. **34.** 25%. **35.** $2.40. **36.** $5.25. **37.** Lost; $100.

Page 207. — **2.** $60; $1049.50. **3.** $1.25; $708.75. **4.** $55.50; $1738.50. **5.** $15; $280. **6.** $157.50; $225.75. **7.** $12.77; $452.11. **8.** $122.50; $1602.50. **9.** $2550. **10.** $25. **11.** $12.50. **12.** 120 books. **13.** 4%.

Page 208. — **14.** $262.20. **15.** $40.95. **16.** $8106. **17.** 25%. **18.** $7\frac{1}{2}$%. **19.** $2800. **20.** $200; $900. **21.** $281.66. **22.** $2752; 34.4¢.

Page 210. — **2.** Results the same. **3.** $360; $140. **4.** $420; $330. **5.** $574.56; $265.44. **6.** $557.80; $374.20. **7.** 191.09\frac{1}{4}$; 181.40\frac{3}{4}$. **8.** $26.60. **9.** $216. **10.** $4.50. **11.** $13.20. **12.** Direct disc.; $33.375. **13.** 16%. **14.** $66\frac{2}{3}$%. **15.** $23\frac{1}{2}$%.

Page 213. — **2.** 6.72 mills. **3.** $84.84. **4.** $1\frac{3}{4}$ mills. **5.** $8\frac{2}{5}$ mills. **6.** $549.74. **7.** $54,000. **8.** $1,926,960. **9.** In order: $56,995,285.65; $3,699,980.18; $14,202,003.16; $1,975,889.69; $735,375.96.

Page 214. — **10.** $154.80. **11.** $1,191,677.51. **13.** $5.36. **14.** $6.19. **15.** $6.98. **16.** $122.41. **17.** $155.93. **18.** $1119.17. **19.** $906.54. **20.** $1305. **21.** $1115.34.

Page 215. — **1.** $144. **2.** $1650. **3.** $150. **4.** $535.50. **5.** $1286. **6.** $282. **7.** $3750. **8.** $2500.

Page 216. — **9.** $45. **10.** $330. **11.** $354. **12.** $1867.20. **13.** 75¢. **14.** $547.49. **15.** $1793.53; $1956.82.

Page 217. — **16.** $209.33. **17.** $29,127.14. **18.** $137.10. **19.** $1512.21. **20.** $4556.16. **21.** $4.56.

Page 218. — **1.** $24.50. **2.** $35. **3.** $12.50. **4.** $50. **5.** $1600. **6.** $3900. **7.** 40¢; $\frac{2}{5}$%. **8** $500; $4500. **9.** Phœnix, $2000; Firemen's, $1200; Protective, $800. **10.** $42.

Page 219. — **11.** $444.99. **12.** $382.

Page 220. — **1.** $41.40. **2.** $147. **3.** $120.25. **4.** 36 premiums; $638.64.

Page 221. — **5.** $1764. **6.** $1585.02. **7.** $408. **8.** Lost; $1363.33. **9.** $454.44+. **10.** $630. **11.** $660. **12.** $38.25. **13.** $23,000.

Page 222. — **1.** 7200 ft. **2.** 132 rows. **3.** 690 T. **4.** $19\frac{1}{2}$ T. **5.** $1.90. **6.** $6552. **7.** $2106; $17.55. **8.** 1755 T. **9.** 234 T. **10.** $\frac{9}{10}$. **11.** 9%.

Page 223. — **12.** 180 lb.; 3510 lb. **13.** 117 T.; $8190. **14.** 585 T. **15.** 65¢. **16.** 237,500 lb. **17.** $17\frac{1}{2}$¢. **18.** 200 lb. **19.** 25 lb.; 5¢. **20.** $12\frac{1}{2}$¢. **21.** $5937.50. **22.** La., 1.44 T.; Cuba, 4.08 T.; Hawaii, 6.6 T.

Page 224. — **23.** $72. **24.** $17.05; $409.20. **25.** $744; $31. **26.** 120 loads. **27.** 276 T. **28.** $1242. **29.** $498.

Page 225.—**30.** 80%. **31.** $2859.36. **32.** $1.40. **33.** 431.8 T. **34.** 508 T. **35.** $24,150. **36.** $6118. **37.** $472.50. **38.** $1252.50. **39.** $1737. **40.** $6355.55. **41.** 4,050,000 T. **42.** 972,000 T. **43.** 1,750,000 T. **44.** 197,200 T. **45.** 57%.

Page 226.—**46.** $31\frac{1}{2}$ cu. ft. **47.** 10 compressed bales. **48.** 475 lb. **49.** India, 80%; Brazil, 46%; Peru, $36\frac{2}{3}$%; Egypt, 147%.

Page 227.—**50.** 32%; 68%. **51.** 9 bales. **52.** 45%. **53.** 12 days. **54.** $7\frac{1}{2}$ sec. **55.** 1 hr. **56.** 8640 lb. **57.** 45¢. **58.** 192 bu.; 2880 lb. **59.** 6 bales. **60.** $218.04. **61.** $54.51.

Page 228.—**62.** $67.19. **63.** 50¢. **64.** $318.75. **65.** 27,500 lb. **66.** 68,500 lb. **67.** Egypt, 1,200,000 bales; Brazil, 300,000 bales; East Indies, 3,000,000 bales; U. S., 13,500,000 bales. **68.** In order: 20%; $28\frac{1}{3}$%; 24%.

Page 229.—**69.** $1\frac{1}{4}$ T. **70.** 210 days. **71.** $3\frac{1}{2}$¢. **72.** $2.80. **73.** 20¢. **74.** $1. **75.** $17.13. **76.** $3.38. **77.** $21,294. **78.** 100 lb. **79.** 3500 lb.

Page 231.—**2.** $25. **3.** $75. **4.** $57.75. **5.** $450. **6.** $1500. **7.** $13.125. **9.** $225. **10.** $350. **11.** $161.25. **12.** $511.44.

Page 232.—**14.** $377.50; $5377.50. **15.** $379.17; $5379.17. **16.** $284.37½; $2784.37½. **17.** $421.27; $4071.27. **18.** $965; $8465. **19.** $129.25; $1004.50. **20.** $23.43; $193.18. **21.** $112.50; $15,112.50. **22.** $2500; $502,500. **23.** $28.56; $3292.74. **24.** $97.40; $4967.61. **25.** $43.75; $3043.75. **26.** $414.17; $5414.17. **27.** $157.61; $623.91. **28.** $18.98; $193.63. **29.** $618.06; $50,618.06. **30.** $4984.37½; $129,984.37½; **31.** $50,177.78; $370,177.78.

Page 233.—**1.** $2.25. **2.** $11.25. **3.** $5. **4.** $8.66. **5.** $9. **6.** $50. **7.** $750. **8.** $1562.50.

Page 234.—**1.** $50. **2.** $41.25. **3.** $16.50. **4.** $100. **5.** $45. **6.** $111.99. **7.** $88.20. **8.** $32. **9.** $.12. **10.** $.30. **11.** $35. **12.** $133.33. **13.** $1208.33. **14.** $7333.33.

Page 235.—**2.** $18.84. **3.** $12.62. **4.** $164.37. **5.** $195.13. **6.** $260.90. **7.** $269.60. **8.** $94.75. **9.** $424.80. **10.** $1571.20. **11.** $21.95. **12.** $15,700. **13.** $9454.67. **14.** $108.55. **15.** $400.43. **16.** $18,262.50. **17.** $15,000.

Page 236.—**1.** $2.92; $252.92. **2.** $2.08; $502.08. **3.** $8; $408. **4.** $3.33; $503.33. **5.** $.84; $160.84. **6.** $2.84; $777.84. **7.** $2.10; $842.10. **8.** $.84; $450.84. **9.** $.83; $750.83. **10.** $3; $803. **11.** $3.12½; $903.12½. **12.** $7.50; $507.50. **13.** $861.11; $5861.11. **14.** $848: $4848. **15.** $717.08; $5717.08. **16.** $135.10; $1535.10. **17.** $50.83; $5050.83; $59.31; $5059.31. **18.** $67.71; $7567.71; $74.48; $7574.48. **19.** $121.19; $2561.19. **20.** $256.93; $3941.93. **21.** $2.47; $77.97. **22.** $344.71; $1169.96. **23.** $933.33; $25,933.33.

Page 237.—**2.** $6. **3.** $4.44. **4.** $9.37½. **5.** $36.37½. **6.** $8.89. **7.** $72. **8.** $23.94. **9.** $28.12½. **10.** $65.10. **11.** $30. **12.** $42. **13.** $59.80. **14.** $68.85. **15.** $120. **16.** $48.82½. **17.** $1.42. **18.** $.32. **19.** $1.50.

Page 239.—**2.** $5. **3.** $7.47. **4.** $8.25. **5.** $5. **6.** $23.20. **7.** $36.25. **8.** $92.94. **9.** $30.84. **10.** $237.68. **11.** $5.33. **12.** $6.78. **13.** $5.56. **14.** $222.78. **15.** $1080.04. **16.** $751.34. **17.** $1443.04. **18.** $15.19.

Page 240.—**1.** $67.62; $45.08; $50.71½; $33.81; $56.35. **2.** $40; $26.67; $30; $20; $33.33. **3.** $1.67; $1.11; $1.25; $.83; $1.39. **4.** $4.17; $2.78; $3.12½; $2.08; $3.47. **5.** $8.33; $5.56; $6.25; $4.17; $6.94. **6.** $25; $16.67; $18.75; $12.50; $20.83. **7.** $50; $33.33; $37.50; $25; $41.67.

8. $75 ; $50 ; $56.25 ; $37.50 ; $62.50. **9.** $100 ; $66.67 ; $75 ; $50 ; $83.33. **10.** $104.17 ; $69.44 ; $78.12½ ; $52.08 ; $86.81. **11.** $.47 ; $.31 ; $.35 ; $.23 ; $.39. **12.** $3.54 ; $2.36 ; $2.65 ; $1.77 ; $2.95. **13.** $.79 ; $.53 ; $.59 ; $.40 ; $.66. **14.** $6.04 ; $4.03 ; $4.53 ; $3.02 ; $5.03. **15.** $11.66 ; $7.77 ; $8.74 ; $5.83 ; $9.72. **16.** $10.14 ; $6.76 ; $7.60 ; $5.07 ; $8.45. **17.** $7.75 ; $5.17 ; $5.81 ; $3.88 ; $6.46. **18.** $71.50 ; $47.67 ; $53.62½ ; $35.75 ; $59.58. **19.** $186.60 ; $124.40 ; $139.95 ; $93.30 ; $155.50. **20.** $72 ; $48 ; $54 ; $36 ; $60. **21.** $75 ; $50 ; $56.25 ; $37.50 ; $62.50. **22.** $2000 ; $1333.33 ; $1500 ; $1000 ; $1666.67. **23.** $25,000 ; $16,666.67 ; $18,750 ; $12,500 ; $20,833.33. **24.** $66,000 ; $44,000 ; $49,500 ; $33,000 ; $55,000. **25.** $2750. **26.** $2250. **27.** $3.28. **28.** $3200 better off.

Page 241. — **1.** $2.96. **2.** $2.71. **3.** $6.16. **4.** $2.37. **5.** $49.32. **6.** $77.85. **7.** $211.07 **8.** $178.75. **9.** $3.33. **10.** $8.90. **11.** $23.14. **12.** $14.44. **13.** $150.41. **14.** $151.23. **15.** $118.99. **16.** $99.37. **17.** $1.05. **18.** $522.26.

Page 242. — **2.** 5%. **3.** 6%. **4.** 4%. **5.** 4%. **6.** 6%.

Page 243. — **7.** 5%. **8.** 6%. **9.** 4%. **10.** 4½%. **11.** 3½%. **12.** $50 ; 5%. **13.** 5%. **14.** $418.50 ; 7¾%. **15.** 32¾%.

Page 244. — **2.** 8 yr. 4 mo. **3.** 5 yr. **4.** 2 yr. 9 mo. 10 da. **5.** 3 yr. 2 mo. 3 da. **6.** 13 yr. 4 mo. **7.** 4 yr. 1 mo. **8.** 25 yr. ; 20 yr. ; 50 yr. ; 16⅔ yr. ; 22$\frac{2}{9}$ yr. **9.** 40 yr.
2. $2500. **3.** $12,500. **4.** $12,500. **5.** $20,000. **6.** $50,000. **7.** $40,000.

Page 245. — **8.** $1875. **9.** $7500. **10.** $4000. **11.** $5000. **12.** $2400. **13.** $120,000. **14.** $75,000. **15.** $100,000.

Page 246. — **2.** $792.08. **3.** $869.57. **4.** $121.95. **5.** $1600. **6.** $4761.90. **7.** $4761.90.

Page 247. — **8.** Cash ; $12.75. **9.** Mr. White's by $30.19 ; Mr. Brown's by $22.22. **2.** $6910.02. **3.** $28,171.

Page 249. — **2.** $441 ; $41. **3.** $1124.86 ; $124.86. **4.** $1950.53 ; $450.53. **5.** $5627.54 ; $627.54. **6.** $5632.46 ; $632.46. **7.** $8548.78 ; $548.78. **9.** $1628.90. **10.** $9948.94½. **11.** $5511.61. **12.** $1842.57. **13.** $5529.02. **14.** $7074.13. **15.** $10,826.36. **16.** $259.80. **17.** $138.74. **18.** $8023.53. **19.** $7732.19.

Page 256. — **2.** $529.92. **3.** $258.68. **4.** $1246.56. **5.** $3798.19. **6.** $546.86. **7.** $34.44.

Page 257. — **1.** $900.65. **2.** $192.51. **3.** $567.31. **4.** $76.90. **5.** $1274.52 ; $1276.39.

Page 260. — **1.** $406.98. **2.** $365.99.

Page 261. — **3.** $3330.92. **4.** $715.07. **5.** Check for $1757.50. **6.** Total deposit, $1849.13.

Page 263. — **2.** May 3 ; 57 da.; $47.50 ; $4952.50.

Page 264. — **3.** Sept. 26 ; 30 da. ; $25 ; $4975. **4.** May 16 ; 15 da.; $20.83 ; $9979.17. **5.** Feb. 16 ; 89 da. ; $111.25 ; $7388.75. **6.** Aug. 7 ; 10 da. ; $25 ; $17,975. **7.** Jan. 28 ; 120 da. ; $74.67 ; $3125.33. **8.** Dec. 11 ; 39 da. ; $5.51 ; $842.89. **9.** Aug. 2 ; 28 da. ; $14.06 ; $2998.44. **10.** Feb. 11 ; 40 da. ; $335 ; $59,965. **11.** May 2 ; 90 da. ; $145.65 ; $8177.35. **12.** Apr. 1 ; 31 da.; $86.97 ; $25,163.03. **13.** May 1 ; 111 da.; $75.48 ; $4004.52. **14.** June 6 ; 60 da. ; $375 ; $74,625. **15.** Oct. 17 ; 46 da. ; $37.35 ; $7270.65. **16.** 10 da. after date ; 10 da. ; $.97 ; $999.03.

Page 265. — **17.** $42,851.89. **18.** $10,000. **19.** $3000. **20.** $20,000.

Page 267. — **1.** $487.59. **2.** $482.68. **3.** $272.56. **4.** $268.97. **5.** $269.63. **6.** $638.17. **7.** $221.90. **8.** $1663.61.

Page 273. — **1.** $750.75. **2.** $125,256.44. **3.** $3495.62. **4.** $1988. **6.** $1862.70. **7.** Draft for bal., $757.50. **8.** $1024.31.

Page 274. — **9.** $4001.65. **10.** $862.84. **11.** $938.83. **12.** $2571.01. **13** $8680.50.

Page 277. — **1.** $51.27 –. **2.** $59.06 +. **3.** $365.47 +. **4.** $5803.80. **5.** $7384.05. **6.** $465.81. **7.** $1090.61. **8.** $1930.50. **9.** $5941.41. **10.** $235.59. **11.** $1070.83. **12.** $5201.66. **13.** $385.36. **14.** $24.65. **15.** $4020. **16.** £142 10*d.*; 4612.50 fr. (French); 1650 M.; 733.9 crowns; 2562.50 fr. (Belgian); 490.2 florins; 3075 lire; $500.

Page 278. — **17.** $72,975. **18.** $7.50. **19.** $6047.05. **20.** $120,956.25. **21.** $1945.63. **22.** $400. **23.** $700. **24.** Face, 187,290 fr.

Page 283. — **2.** $4368.75. **3.** $25,718.75. **4.** $10,650. **5.** $38,500. **6.** $68,812.50. **7.** $32,000. **8.** $212.50. **9.** $125; $250. **10.** $27,775.

Page 284. — **12.** 50 shares. **13.** 80 shares. **14.** 440 shares. **16.** $531.25. **17** Increased, 4.96\frac{1}{2}$.

Page 285. — **18.** $536.25. **20.** $39,900. **21.** $10,550. **22.** $20,000. **23.** 5%. **24.** 8%. **25.** $80 per share. **26.** $120 per share.

Page 286. — **27.** 37\frac{1}{2}$ per share. **28.** 2%. **29.** $80. **30.** $1000. **31.** $252. **32.** $81.25. **33.** $33,406.25. **34.** 6.42%. **35.** $4\frac{684}{829}$%. **36.** $100.

Page 288. — **1.** 2. **2.** $1\frac{1}{2}$. **3.** $2\frac{1}{2}$. **4.** $\frac{7}{8}$. **5.** $\frac{1}{25}$. **6.** $\frac{5}{8}$. **7.** $2\frac{2}{9}$. **8.** $8\frac{8}{17}$. **9.** $\frac{1}{3}$. **10** 3. **11.** 4. **12.** 16. **13.** 1 : 5; 2 : 5; 2 : 25; 20%; 40%; 8%.

Page 289. — **14.** 1700 : 10,000; 1330 : 10,000; 1070 : 10,000.

Page 290. — **3.** $x = 2$. **4.** $x = 14$. **5.** $x = 30$. **6.** $x = 5$. **7.** $x = 3$. **8.** $x = 8$. **9.** $x = \frac{2}{3}$. **10.** $x = 1\frac{1}{3}$. **11.** $x = \frac{3}{4}$. **12.** $x = 2\frac{11}{12}$. **13.** $x = 2$. **14.** $x = 9$. **15.** 8 tons. **16.** $2.70. **17.** 40 bu. **18.** 15 days. **19.** 20 ft.

Page 291. — **2.** $1833.34. **3.** 3432 bu. **4.** 10 sec. **5.** 75¢. **6.** 5 hr. 12 min. **7.** $1.24. **8.** 25,000 blocks. **9.** 6300 photographs.

Page 292. — **11.** 12 days. **12.** 24 men. **13.** 16 days. **14.** 3 extra men. **15.** 8 men. **16.** $44. **17.** $57\frac{1}{7}$ mi.

Page 293. — **18.** $387\frac{1}{2}$ doz. **19.** $8400. **20.** $511.50. **21.** 45 days. **22.** 5 cars.

Page 294. — **2.** $6000; $8000; $10,000. **3.** $6000; $10,000; $20,000. **4.** $14,495,000; $2,230,000. **5.** Bread, 750 g.; meat, 375 g.; rice, 125 g.; salt, 25 g.; coffee, 25 g. **6.** 1st, $960; 2d, $1200. **7.** In order: $48,000,000; $24,000,000; $6,000,000. **8.** In order: $675; $337.50; $540; $405; $1080; $810; $945; $1620.

Page 295. — **2.** A, $7000; B, $4200. **3.** In order: $825; $1125; $450. **4.** In order: $9600; $4200; $3000. **5.** In order: $2000; $600; $1300; $700.

Page 296. — **6.** $3850; 1st, $1823.68; 2d, $2026.32. **7.** A, $1600; $10,600; B, $1600; $10,600; C, $1600; $10,600; D, $2880; $19,080. **9.** A, $1500; B, $1575. **10.** A, $2640; B, $1760; C, $1100. **11.** A, $14,400; B, $9600; C, $17,400.

Page 298. — **1.** 400. **2.** 625. **3.** 2025. **4.** 2704. **5.** 1728. **6.** 9261. **7.** 3375. **8.** 14,641. **9.** $\frac{343}{512}$. **10.** $\frac{125}{1728}$. **11.** $\frac{225}{784}$. **12.** $\frac{625}{1024}$. **13.** 2.25. **14.** 56.25. **15.** 1.5625. **16.** .012321. **17.** 441 sq. in. **18.** 289 sq. yd. **19.** 1024 sq. rd. **20.** 1998.09 sq. ft. **21.** 2540.16 sq. ft. **22.** .0361 sq. ft. **23.** .0625 sq. ft. **24.** 57.1536 sq. ft. **25.** .708964 sq. mi. **26.** 1331 cu. in. **27.** 15.625 cu. in. **28.** 10,648 cu. cm. **29.** 3.375 cu. m. **30.** 39.304 cu. ft. **31.** 1953.125 cu. Dm. **32.** 257.28 ft.; 3618 ft.

Page 301. — **2.** 27. **3.** 28. **4.** 8. **5.** 9. **6.** 32. **7.** 45. **8.** 15. **9.** 4. **10.** 108. **11.** 36. **12.** 12. **13.** 5. **14.** 3. **15.** 7. **16.** 2. **17.** 5. **18.** 18 in. **19.** 20 in.

Page 303. — **3.** 53. **4.** 51. **5.** 82. **6.** 44. **7.** 91. **8.** 72. **9.** 34. **10.** 83. **11.** 33. **12.** 94. **13.** 61. **14.** 84. **15.** 46. **16.** 95. **17.** 76. **18.** 57. **19.** 49. **20.** 98. **21.** 79. **22.** 99. **23.** 85 in. **24.** 38 in. **25.** 47 in. **26.** 29 ft. **27.** 65 cm. **28.** 68 cm. **29.** 88 m. **30.** 69 Dm. **31.** 35 Hm. **32.** 176 rd. **33.** 9216 sq. ft.; 96 ft. **34.** 75 men.

Page 305. — **2.** 123. **3.** 156. **4.** 117. **5.** 176. **6.** 211. **7.** 30.5. **8.** 25.6. **9.** 16.8. **10.** 3.32. **11.** 3.82. **12.** .745. **13.** .796. **14.** .997. **15.** .0822. **16.** .0876. **17.** $\frac{144}{175}$. **18.** $\frac{161}{225}$. **19.** $5\frac{17}{25}$. **20.** $295\frac{29}{32}$. **24.** 2.828+. **25.** 2.449+. **26.** 2.646−. **27.** .577+. **28.** .707+. **29.** .447+. **30.** .316+. **31.** 1.581+. **32.** 1.897+. **33.** .886+. **34.** .642+. **35.** .860+. **37.** 255.6+ ft. **38.** 109.5+ ft. **39.** 612.4− ft.

Page 306. — **40.** 229.8− ft. **41.** 80 rd. long; 40 rd. wide. **42.** 240 rd. long; 60 rd. wide. **43.** 120 rd. **2.** 13 in.

Page 307. — **3.** In order: $x=17$; $x=12$; $x=51$; $x=24$. **4.** 1.414+ in. **5.** 72.25 sq. in. **6.** As 2 : 1. **9.** 39.55− ft. **10.** 150 knots. **11.** 339.41+ rd.

Page 310. — **1.** 87 sq. in. **2.** 8 A. **3.** 1800 sq. m. **4.** 5 sq. Km. **5.** $44\frac{5}{8}$ sq. ft. **6.** 104 sq. rd. **7.** 6 in.

Page 311. — **1.** 144 sq. ft. **2.** $2\frac{1}{4}$ A. **3.** 572 sq. m. **4.** 208 sq. m. **5.** $10\frac{1}{2}$ ft. **6.** 21 in.

Page 312. — **1.** 28 sq. in. **2.** 80 sq. cm. **3.** 35 sq. ft.; 70 board feet. **4.** 8 in. **5.** 60 A. **6.** 40 A. 128 sq. rd. **7.** 39 A. 32 sq. rd. **8.** 48 A. $23\frac{1}{8}$ sq. rd. **9.** 21 A. $61\frac{7}{8}$ sq. rd. **10.** 20 A. 75 sq. rd. **11.** 10 A. **12.** 240 A. **13.** 329.85 rd.

Page 314. — **1.** 23,040 A.; 640 A.; 40 A. **2.** 160 A. **3.** 80 A. **4.** 40 A. **5.** 40 A. **6.** 400 A. **7.** \$2040. **8.** 320 rd. **9.** \$4500.

Page 316. — **1.** 93.6 sq. in. **2.** 8 in.; 256 sq. in. **3.** 390.96 sq. in. **4.** 6 in.

Page 317. — **1.** 110 in. **2.** 242 ft. **3.** 198 yd. **4.** 154 rd. **5.** 264 m. **6.** 176 cm. **7.** 396 m. **8.** 528 mm. **9.** 176 ft. **10.** 264 ft. **11.** 572 rd. **12.** 74.8 m. **13.** 83.6 Hm. **14.** 36.96 cm. **15.** 154 ft. **16.** 228.8 in. **17.** 28 yd. **18.** $17\frac{1}{2}$ ft. **19.** 3.85 in. **20.** 1.19 cm. **21.** 8.05 m. **22.** 1.54 Dm. **23.** 315 rd. **24.** $206\frac{1}{2}$ yd. **25.** 37.6992 ft. **26.** 62.832 m. **27.** 48.6948 yd. **28.** 71.4714 rd. **29.** 1100 ft. **30.** 22 ft.

Page 318. — **1.** 616 sq. in. **2.** 3850 sq. ft. **3.** 24.64 sq. rd. **4.** 55.44 sq. m. **5.** 124.74 sq. cm. **6.** 2.6026 sq. dm. **7.** .0154 sq. ft. **8.** 346.5 sq. m. **9.** 5544 sq. ft. **10.** 346.5 sq. ft. **11.** 38.5 sq. ft. **12.** $962\frac{1}{2}$ sq. ft. **13.** 5.05 rd.

Page 321. — **1.** $31\frac{3}{7}$ sq. ft. **2.** 24 sq. cm. **3.** 60 sq. ft. **4.** $89\frac{4}{7}$ sq. ft.

Page 322. — **1.** 60 sq. ft. **2.** $235\frac{5}{7}$ sq. cm. **3.** $628\frac{4}{7}$ sq. ft. **4.** \$64. **5.** $89\frac{4}{7}$ sq. ft. **6.** 400 sq. ft. **7.** $75\frac{3}{7}$ sq. ft.

Page 323. — **1.** $28\frac{2}{7}$ sq. in. **2.** $1810\frac{2}{7}$ sq. cm. **3.** $201\frac{1}{7}$ sq. in. **4.** \$2.06.

Page 324. — **1.** 75 cu. in. **2.** $1005\frac{5}{7}$ cu. dm. **3.** 460.8 bu. **4.** 2310 gal. **5.** \$1360.80.

Page 325. — **1.** $84\frac{6}{7}$ cu. ft. **2.** $666\frac{2}{3}$ cu. m. **3.** $12{,}445\frac{5}{7}$ lb. **4.** 13,860 cu. ft.

Page 326. — **5.** 7296 lb. **6.** 89,042,560 cu. ft. **1.** $14\frac{1}{7}$ cu. ft. **2.** 4851 cu. cm. **3.** $179\frac{2}{3}$ cu. in. **4.** $1437\frac{1}{3}$ cu. in.; 51.9 − lb. **5.** 29.505+ lb.

Page 328. — **1.** 6 in. **2.** 20 sq. in. **3.** 10 in. **4.** 1 : 9. **5.** 9 sq. ft.; 36 sq. ft. **6.** As 1 : 16. **7.** 5 cm., 10 cm., $12\frac{1}{2}$ cm. **8.** 4. **9.** 20 ft. **10.** 150 ft.

Page 329. — **12.** 1000 ft.

Page 330. — **1.** 1 : 2; 972 cu. ft.; 8; 7776 cu. ft. **2.** 1 : 3; by multiplying by 27; 1st, $134\frac{2}{21}$ cu. ft.; 2d, $3620\frac{4}{7}$ cu. ft. **3.** 240 cu. in. **4.** 12 cu. ft. **5.** 6 ft. **6.** $3\frac{3}{8}$ times. **7.** 270 lb. **8.** 91 cu. dm. **9.** 109,725 gal.

Page 331. — **1.** \$6412.50. **2.** 8%. **3.** 1088 doz. pairs. **4.** $832\frac{1}{2}$ lb. **5.** \$1704.96. **6.** Second; \$29.20. **7.** \$231.14. **8.** 320 doz. pairs.

Page 332. — **9.** 8640 doz.; 8000 doz. **10.** 4 or $4\frac{1}{2}$, $11\frac{1}{4}$ doz.; 5 or $5\frac{1}{2}$, $9\frac{2}{3}$ doz.; 6 or $6\frac{1}{2}$, $8\frac{1}{2}$ doz. **11.** 56 lb. **12.** \$1.40. **13.** $7\frac{1}{3}$ doz.; 18 ¢ per doz.; \$1.32. **14.** \$429. **15.** \$630. **16.** 4's, 7472 doz.; $4\frac{1}{2}$'s, 12,160 doz.; 5's, 17,648 doz.; $5\frac{1}{2}$'s, 16,208 doz.; 6's, 11,688 doz.; $6\frac{1}{2}$'s, 5632 doz.; white, 20,784 doz.; black, 36,408 doz.; colored, 13,616 doz.; total, 70,808 doz.

Page 333. — **17.** 14,062 lb. **18.** 14,240 lb. **19.** \$17,800. **20.** 37,938 lb. **21.** \$48,370.95. **22.** \$66,170.95. **23.** \$26,907.04; \$93,077.99. **24.** \$108,426.80. **25.** \$127,808.44. **26.** \$265.53. **27.** \$19,647.17; 18.1%.

Page 334. — **28.** 24%. **29.** \$1440. **30.** 22.5%. **31.** $8\frac{1}{2}$ M.; 85 pf. **32.** 4*s.* 2*d.* **33.** Am.; \$4.136−. **34.** Am.; \$.486+; Eng., $\frac{14}{31}$; Am., $\frac{1}{3}$. **35.** \$226.70−. **36.** \$86.40. **37.** 108 pencils.

Page 335. — **38.** 239,580 sq. ft. **39.** 69.750 cords; 62,000 T. **40.** 312,500 sq. ft. **41.** 4000 trees. **42.** In order: 956,924,400 lb.; 202,236,600 lb.; 73,989,000 lb. **43.** \$126,663,622.42. **44.** 45%. **45.** \$937,500. **46** \$63.40. **47.** \$6,299,102.

Page 336. — **48.** \$171. **49.** \$2940; $5\frac{1}{4}$ ¢. **50.** \$30.60. **51.** \$17.28; 30%; 12 ¢. **52.** \$22.40; \$40.32; \$17.92; 80%; $\frac{1}{7}$. **53.** 720 tons. **54.** 166,100 images. **55.** 190 beats per sec.; 330 beats per sec.

Page 337. — **56.** 3000 m.; 30 cm. **57.** \$400. **58.** \$2.37+. **59.** \$7044.26−. **60.** In order: 136 ft. $\frac{1}{4}$ in.; 65 ft. $4\frac{1}{2}$ in.; 23 ft. $7\frac{1}{2}$ in.; 169 lb.; 9 yd. 1 ft. 1 in.; 358.75 ft.; $7\frac{2}{3}$ ft. less.

Page 338. — **61.** 55 gal. **62.** \$3.81−. **63.** 1st class, 10− pf. or 2.3− ¢; 2d class, 6+ pf. or 1.5+ ¢; 3d class, 4+ pf. or 1− ¢. **64.** 4 M. 16− pf. **65.** 8816.26 mi. **66.** 1950 mi. **67.** 33 min. $6\frac{3}{4}$ sec. after 7 P.M. **68.** 70.104 m. **69.** \$410.13. **70.** In order: 131 ft. 2.8 in.; 2 ft. .803+ in.; 22 ft. 11.59 in. **71.** $82\frac{2}{5}$° F.

Page 339. — **72.** £226,416$\frac{2}{3}$. **73.** £35,750. **74.** 3672.8 mi. **75.** \$61,565.28. **76.** 569,477.252− Kg. **77.** \$130. **78.** \$228; $28\frac{1}{2}$%. **79.** 9.2 T.; \$.025.

Page 340. — **80.** 43.19%. **81.** \$810. **82.** \$5.82. **83.** \$1000. **84.** 1%. **85.** \$157.50. **86.** \$4293.75. **87.** \$119.77. **88.** \$32,928.

Page 341. — **89.** \$252.45. **90.** 19.04 mills. **91.** \$1904. **92.** 3640 M.; \$866.32. **93.** \$3263.60. **94.** 35 : 44. **95.** \$1339.535−. **96.** $2\frac{1}{8}$ oz. **97.** 138,686.77+ lb. **98.** \$160.442+. **99.** $412\frac{1}{2}$ gr.

Page 342. — **100.** 1 hr. 2 min. **101.** \$1.75. **102.** \$1.78$\frac{1}{2}$. **103.** \$16,921,479.76+. **104.** \$1050. **105.** \$27,525. **106.** \$1014.68. **107.** \$3,986,678.57$\frac{1}{7}$. **108.** 72 ft. **109.** 405 ft. **110.** $32{,}812\frac{1}{2}$ cu. ft.